A Cognitive Basis for Cardinal Utility

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Abstract

This paper suggests a cognitive basis for the isomorphism of a utility function and a distribution function. With only a minimal set of cognitive tools - ordinal comparison and frequency processing - an object can be evaluated by its relative position in a reference set. This cognitive evaluation, coupled with "more is better" preferences, is consistent with possession of a cardinal utility function, which is determined by one's environment, and thus adapts to changes in the environment. The effects of cognitive imperfections are considered. The model provides a framework which may accommodate a number of developments both in psychology and economics.

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An American male 5 ft. 9 in. in height living in the United States, though tall on an international scale, is not likely to feel tall.

Richard E. Easterlin [1974]

...however high [the house] may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.

Karl Marx [1847]

1 Introduction

How big is an apple? How large is a salary? How intense is an experience? How long is a piece of string? How can we tell?

When it comes down to money, there is plenty of evidence that people compare what they have relatively to what others have (see, for example, Frank [1985a], Clark and Oswald [1996], Solnick and Hemenway (1998), Neumark and Postlewaite [1998]). Nor are moral judgements absolute - as students were found to be appalled by the idea of poisoning a neighbour's dog, yet judged it as a petty crime when poisoning the neighbour herself was on the list (Parducci [1968]). Nor is evaluation of painful experience absolute - it may vary with different sequences of pain (Redelmeier and Kahneman [1996]). And so are the judgements of size, weight, or numerousness (Parducci [1963, 1965]). In other words, nearly everything coming into the human "in-box", is subject to an evaluation against a collection of reference points, a reference set - or at least against specific elements of it. So one may ask - if it is truly the case that all human evaluations are relative, can economists even dream about a cardinal measure?

The answer, surprisingly, is "maybe" - if one considers more carefully the cognitive tools that Nature has endowed humans. As this paper argues, it is possible to construct an evaluation which is entirely built upon *ordinal comparisons* and yet, by means of a *frequency tool*, it can be keyed into a cardinal scale. This is done by calculating how often a given object came out a winner from a pairwise ordinal "tournament" against every other object in the reference set.

In contrast to the previous literature, this paper focuses on the subjective evaluation, or judgement, or *measurement*, of magnitudes, rather than on *preferences* over magnitudes. In other words, instead of looking into whether "more is better", it looks into "how more is more", into subjective measurement of magnitudes.[1] Now suppose that,

[1]The issue of measurement is not new (see, for example, Stevens [1968]). Interestingly, Lazear and Rosen [1981] used the theory of measurement to make a case for the use of rank-order tournaments by arguing that since "a cardinal scale is based on an underlying ordering of objects or on an ordinal

in addition to the above-mentioned cognitive tools used to evaluate magnitude of an object, an individual possesses "more is better" preferences.[2] In this case, the cognitive evaluation coupled with "more is better" preferences is consistent with possession of a cardinal utility function. This cardinal utility function is determined by one's environment, and thus adapts to changes in the environment, so that a given object of a fixed size will be evaluated differently in different contexts.

Following the approach of Robson [2001, 2002], Samuelson and Swinkels [2002], and Samuelson [2004], the present work seeks to identify those "tools" that humans may use to "measure" magnitudes. There are a few experimental studies in support of the use of the two cognitive tools used in the paper. As development psychologists show, human infants seem to able to make ordinal comparisons as early as before reaching their first birthday (Brannon [2002], Feigenson, Carey and Spelke [2002], and Xu [2003] and references in these works). Furthermore, cognitive and evolutionary psychologists has been arguing that humans are endowed with well-working system of dealing with frequency information (Hintzman and Stern [1978], Hasher and Zack [1979, 1984], Jonides and Jones [1992], Gallistel and Gelman [1992]) - an observation which prompted researchers to reconsider human abilities to deal with uncertainty (Gigerenzer [1991, 1998], Hoffrage, Lindsey, Hertwig and Gigerenzer [2000], Cosmides and Tooby [1996]).

The paper strives to further explore the implications of the candidate tools for human evaluation of magnitudes. Were humans able to make perfect ordinal comparisons (that is, able to tell apart any two objects even of slightly different magnitudes), the evaluation function (and, thus, a cardinal utility function representing "more is better" preferences) built upon these two simple cognitive tools would be equivalent to a cumulative density function (CDF) of the reference set. Simply put, if the smallest object in the reference set is assigned a value of a and the largest object is assigned a value of b, the object with a median size would be valued at $(a+b)/2$, the object with the size corresponding to the fourth quintile would be valued at $0.8(a+b)$, and so on. In other words, for the case of perfect comparison tool, the present model provides a cognitive basis for the isomorphism of a utility function and a distribution function, and thus may be thought as an alternative justification to the neo-cardinal models of Van Praag [1968] and Kapteyn's [1985 and references therein].

Importantly, experiments by a psychologist Parducci [1963, 1965] and, more recently, by Morone, Seidl and Levati [2001], showed that subjects' evaluations of magnitudes do vary systematically with the distribution of the magnitudes presented to subjects, and thus provide evidence that the composition of reference set affects subjective evaluations

scale", the ordinal measure is weaker and thus less costly measure. What the present paper points out is that a rank-order tournament, besides providing an ordinal measure for performance, also can serve as a reference-based cardinal measure. The key is *metrics*, and here the cardinal measure is based on *relative frequencies*, rather than on centimeters, kilograms, or dollars.

[2] Obviously, in order to identify whether "more" is better or worse, or what exactly is better, an individual has to have additional cognitive tools. For some possible ways for individuals to acquire preferences, see for example, Samuelson and Swinkels [2002] and Samuelson [2004].

of magnitudes in a systematic way. However, while the experimentally observed evaluation functions have, in general, the same shape as the distribution of magnitudes in the reference set, they are nevertheless quite distinct from the "true" distributions - in particularly, the evaluation functions are flatter, less curved, than the "true" distributions. In other words, these experimental findings only partially support the Leyden approach of Van Praag and Kapteyn, or, indeed, any theory suggesting that evaluation functions are equal to the true distribution functions. Instead, human judgements of magnitudes have been better explained by the range-frequency theory of Parducci [1963, 1965, 1968], who suggested that human evaluations of sizes, weights, abstract numbers, and so on, can be modelled as a weighted sum of the CDF of the underlying distributions and of the CDF of a uniform distribution with the same support.

Yet, this systematic but distinct behavior of the evaluation function may instead arise because people's judgements of the relative magnitude of two objects may not be perfectly accurate. Simply put, most people would notice the difference between having one dollar bill and two dollar bills in the wallet, but only a few would notice the difference between 67 dollar bills and 68 dollar bills in the wallet. These imperfections in human ordinal comparison tool are important because, as this paper shows, if any two distinct objects might be judged to be similar, the resulting evaluation function is only partly based on the true distribution function. While it tends to have the same shape as the true distribution, it is quite distinct, and may even come from a family of distributions different from the true distribution.

Take, for example, an ordinal tool which is subject to the similarity relation proposed by Rubinstein [1988], which implies that two distinct objects might be judged to be similar if the ratio of their magnitudes is neither large enough nor small enough.[3] In this case, a given object is evaluated by a weighted sum of the rank of an object relatively a bit below and of the rank of an object relatively far above. For some parameters of the model, an object on a concave portion of the true distribution might be overvalued, while an object on a convex portion might be undervalued, leading to an overall flatter evaluation. Alternatively, an evaluation function may be a member of a different family of distribution when the ordinal tool is noisy - as it is partly determined by the distribution of error as well. For example, when the error enters multiplicatively and is gamma-distributed, a noisy cognitive evaluation of exponentially distributed objects will be described by a function from a family of Pareto distributions.

Does the above cognitive model solve the problem of interpersonal utility comparisons? Unfortunately, not really - as individuals tend to have different reference sets. Yet this approach suggests a condition under which interpersonal utility comparisons

[3]Rubinstein's similarity relationship is, in essence, a form of Weber's law of "just noticeable differences". This empirical law is widely known in psychophysics (see, for example, Ekman [1959] or Laming [1973]) and postulates that the accuracy of discrimination of magnitudes depends on the actual magnitudes; moreover, to be noticed, the difference between the magnitudes of objects must be bigger for bigger magnitudes. See also recent work by Rubinstein [2003] on similarity relationship as an alternative explanation to the phenomena widely known as hyperbolic time discounting.

can be made - namely, when individuals have the same reference set (which could be achieved in a laboratory setting or might be due to the media). Thus, if all individuals have the same reference set which coincides with the true distribution, then individual evaluations can be compared across persons, and one can make evaluations of societal welfare.

Furthermore, if each individual's welfare is entirely determined by the cognitive evaluation of what she has (which can be income, consumption, happiness, etc.) and if the reference set consists of what other people have (as it is common in private ownership economies), an individual's ordinal comparisons are necessarily interpersonal (yet may have nothing to do with concern with social status). As the result of this interdependency of evaluations, when one individual's possessions change, other people's evaluations of their possessions change as well.

Moreover, if each individual in the economy has the same reference set and uses the same ordinal comparison tool subject to a zero-sum condition (conditional on suitably chosen parameters, most of the ordinal comparison tools considered in the paper do satisfy this condition), a utilitarian measure of welfare may be identical for all distributions of possessions, so that the distributions are welfare-neutral. Thus, the famous paradox of happiness first observed by Easterlin [1974] - that is, despite self-reported happiness is increasing in one's own income, average happiness is almost identical across time and nations - may be a consequence of human cognitive evaluation.

The paper is organized as follows. Section 2 presents the cognitive evaluation procedure and the comparative statics predictions. Section 3 surveys the related literature and provides experimental motivation for the subsequent section. Section 4 demonstrates how imperfect ordinal comparisons may affect the cognitive evaluation and how these imperfections may help to interpret experimental data. Section 5 looks into the interdependent cognitive evaluations and presents the welfare-neutrality of distributions. Section 6 is devoted to the relationship between cognitive evaluation and concern with status. Last section concludes.

2 The Basic Model of Cognitive Evaluation

Neoclassical theory assumes that a utility function represents preferences that are exogenous, fixed, and independent across individuals. This is an abstraction from the real world where, at least in part, people's preferences and decisions change as their situation, experiences and physical and social environments change. This apparent inconsistency between theory and the real world has lead researchers to look into possible alternatives, often inspired by insights from neighbouring sciences. Since the seminal work by Kahneman and Tversky [1979], psychologists have influenced economists' thinking. More recently, a number of researchers have turned to evolutionary explanations of the structure of human preferences (see, for example, Robson [2001, 2002], Samuelson and Swinkels [2002], Samuelson [2004]), while others searched for insights from neurobiological sciences (see, for example, Zizzo [2002a, b]).

Let us take the following approach to human evaluations of objects. Rather than endowing humans with necessary information, Nature gives humans "tools" that allow to extract information from one's environment or/and experience (Samuelson and Swinkels [2002], Samuelson [2004]). While individuals may have hard-wired "tools" to evaluate quantities of food, drink and shelter in order to survive, it is questionable that individuals are born with abilities to evaluate evolutionary novel goods - such as exotic fruits, rare wines, designer clothes, the newest technological gadgets, or money (Robson [2001, 2002]). Thus, the ancient tools that our hunter-gatherer ancestors had long ago must have been adapted to deal with more recent tasks (Cosmides and Tooby [1994]). As we will see later, all few as only two primitive tools are sufficient for constructing an evaluation of a magnitude of an object against a reference set, and, when the ordinal comparison tool is perfectly accurate, this evaluation is equivalent to the cumulative density function of the distribution of magnitudes of reference elements.

One such tool - ordinal comparisons - has been long known to economists. The ordinal comparison tool is simply an ability to tell whether one object is larger, smaller, or equal to the other object. As recent research by cognitive psychologists suggest, human infants seem to be able to make ordinal discriminations as early as before reaching their first birthday. While Brannon [2002] suggests that human infants seem to be able to discriminate between small arrays of objects even at the age of 9-11 months, Feigenson, Carey and Spelke [2002] suggest that human infants develop abilities to discriminate continuous variables (such as areas, sizes, densities) earlier than ability to discriminate small numerosities, or numbers. The precise mechanism of ordinal discrimination is still under close scrutiny by cognitive psychologists as Xu [2003] reports that small and large numerosities are discriminated differently, with large numerosities being subject to the Weber's law of just noticeable differences.

The other tool, which is used here as a primitive cardinal measure, is frequency-of-occurrence. It has long been known by cognitive psychologists that human mind is capable of easy and constant updating of frequencies (Hintzman and Stern [1978]) and furthermore, people seem to encode frequency information automatically and very accurately (Hasher and Zacks [1979, 1984]). Gigerenzer [1998] and Hoffrage, Lindsey, Hertwig and Gigerenzer [2000] advocate that humans are particularly well equipped for dealing with natural frequencies (i.e. the ratios of the number of observations satisfying a particular criterion over the total number of all observations) given that frequency information seems to be stored in a numerical format (Jonides and Jones [1992]), and that human children and even animals seem to be equipped with a mental system for counting (Gallistel and Gelman [1992]). This second tool, perhaps, escaped economists' attention as it seems to require a significant level of sophistication. Yet, as Gigerenzer [1991, 1998], Cosmides and Tooby [1996] and Hoffrage, Lindsey, Hertwig and Gigerenzer [2000] pointed out, humans are reasonably "good statisticians" when faced with frequency-of-occurrence problems rather than, as it is tradition in economics, with probability-of-a-single-event problems.

To see how these cognitive tools are used to build up a cardinal evaluation, take Robinson Crusoe on a desert island.[4] Suppose that there is a coconut tree on his island. The tree bears coconuts in a variety of sizes, and Crusoe can see all coconuts at once. One day, a coconut falls off the tree. How would Crusoe evaluate this coconut? Let us construct Crusoe's evaluation in the following way. If the coconut is the smallest one, it is the least valuable. If instead it is the largest one, it is the most valuable. If it is neither smallest nor largest, its evaluation is in-between.

More formally, let us construct a simple procedure whereby an individual can use ordinal reasoning to build up a cardinal measure. Suppose that there is a set X of the magnitudes of observable objects of the same type (which can be sizes of either a physical good (coconuts, houses, cars, incomes, etc.) or magnitudes of a characteristic such as height, beauty, intelligence, etc.). An individual is endowed with two simple cognitive tools on the set X. First, she is able to make ordinal comparisons of magnitudes of any two items in the set X, i.e. for any two items in the set X with magnitudes x and x', she is able to tell that either $x > x'$, or $x < x'$, or $x = x'$. The other cognitive tool involves an ability to process natural frequencies of occurrence. Natural frequencies here are taken to be in a sense of Gigerenzer [1998], i.e. let us define N_{yes} as the number of occurrences of items with the a specific characteristics present and let N_{no} is the number of occurrences of items which do not have this specific characteristics. Then the *natural frequency* of an item with a specific characteristics is simply $N_{yes}/(N_{yes} + N_{no})$. Here, the specific characteristics is the ordinal relation of a fixed item to some other item in the set. Thus the (natural) frequency tool is used to aggregate the results of pairwise ordinal comparisons, i.e. to tell the frequencies of encountering of a larger, an equal, or a smaller object.

Evaluation Algorithm *Suppose an individual is endowed with ordinal comparison and frequency processing tools only. Than the cognitive evaluation of an object of a given magnitude x may be constructed as follows:*

Step 1 *Using the ordinal comparison tool, compare the magnitude of a given object x to magnitudes of every other object x' in the reference set X - that is, observe whether $x > x'$, or $x < x'$, or $x = x'$.*

Step 2 *Using the frequency processing tool, estimate how often $x > x'$, $x < x'$, and $x = x'$.*

Step 3 *Evaluate the magnitude of a given object x as follows.*

$$I(x) = 1 \cdot [\textit{Frequency of } x > x'] + C \cdot [\textit{Frequency of } x = x'] + 0 \cdot [\textit{Frequency of } x < x'] \tag{1}$$

[4] The Robinson Crusoe example emphasizes that the cognitive procedure is free of sentiments (such as envy), as Section 6 further argues.

where $C \in [0,1]$ is a constant representing the value of a "tie".

If $C = 0$ then $I(x)$ is simply the proportion of total objects smaller than x, or to the probability that a randomly encountered object is smaller. If $C = 1$, then $I(x)$ equals to the proportion of total objects which are not larger than x, or to a probability that a randomly encountered other object is no larger. If $C = 0.5$ one gets an expression similar to the definition of status in Robson [1992].

2.1 Finite Reference Set

Suppose first that the reference set X consists of N elements. For an element with magnitude x of the set X let us partition the set X into three subsets, containing elements which are smaller than x, equal to x and larger than x, which the number of elements in each subset to be $N^{smaller}$, N^{equal} and N^{larger}, respectively, so that $N^{smaller} + N^{equal} + N^{larger} = N$. Then, the evaluation of an element of magnitude x can be constructed simply as:

$$I(x) = 1 \cdot \frac{N^{smaller}}{N} + C \cdot \frac{N^{equal}}{N} + 0 \cdot \frac{N^{larger}}{N} \tag{2}$$

where $C \in [0,1]$. Using this procedure, one can assign evaluation value to all elements in the reference set X. Evidently, $I(x)$ is increasing in x. Notice that if, in addition, an individual possesses "more is better" preferences over magnitudes of the elements of X, these preferences can obviously be represented by $I(x)$ on X. Thus, $I(x)$ represents "more is better" preferences, yet $I(x)$ is cardinal, endogenous, and entirely determined by one's environment (i.e. reference set).

As the frequency processing tool allows for easy updating, once there is a change in the reference set, an evaluation $I(x)$ of a given element of magnitude x can be recursively updated. For example, suppose that the reference set increased with an addition of a new element with magnitude z. Then,

$$I_{updated}(x) = \frac{NI(x) + B}{N+1} \quad \text{where} \quad \begin{cases} B = 0 & \text{if} \quad x < z \\ B = C & \text{if} \quad x = z \\ B = 1 & \text{if} \quad x > z \end{cases}$$

Clearly, as the reference set changes, the evaluation of a particular element changes as well.

2.2 Continuous Reference Set

Suppose now that the reference set is a continuum, with $X = [x_{min}, x_{max}] \subseteq R$, and the objects' magnitudes are distributed with a continuous strictly increasing continuously differentiable cumulative density function, or "true" distribution, $F(\cdot)$ on X, with

$F(x_{min}) = 0, F(x_{max}) = 1$, and corresponding probability density function $f(\cdot) > 0$ on X.

Suppose an individual's ability to make ordinal comparisons is *perfect* - that is, she can tell a bigger from a smaller object even if the objects' magnitudes are only slightly different. In this case, the frequency with which the element of magnitude x ordinarily "outperforms" other elements (according to her perfect ordinal comparison tool) is given by the value of the distribution function $F(x)$. Thus, according to the above evaluation algorithm, an element of the magnitude x is evaluated as $I(x) = F(x)$. Notice also that the above result for the evaluation procedure can be interpreted as the object of a given size x can be evaluated by the probability of encountering smaller or equal magnitudes $\Pr[x \geq x']$, as it is equal to $F(x)$. In other words, an individual's evaluation can be described by a cumulative density function $F(\cdot)$. If, in addition, an individual possesses "more is better" preferences over magnitudes of the elements of X, these preferences can obviously be represented by $I(x)$ on X, and individual's (cardinal) utility function is equivalent to the distribution of magnitudes of the reference objects. In other words, a cognitive evaluation based on the two cognitive tools lead to the isomorphism of the utility function and a distribution function.[5]

The corresponding roles of the two cognitive tools can be seen more clearly if one writes the result of the cognitive evaluation procedure in the following way. Notice first that an outcome of a pairwise comparison can be described as:

$$\mathcal{D}_{perfect}(x, x') = \begin{cases} 1 & \text{whenever} \quad x > x' \\ C & \text{whenever} \quad x = x' \\ 0 & \text{whenever} \quad x < x' \end{cases} \tag{3}$$

where $C \in [0, 1]$ (in fact, for continuous strictly increasing distributions, the value of C is irrelevant). Throughout the paper, the function $\mathcal{D}(x, x')$ describes the outcome of a pairwise ordinal comparison of any two elements with magnitudes x and x' and thus models the workings of the ordinal tool. Here this function models the workings of the "perfect" ordinal tool as it as it, in essence, perfectly accurate in identifying the ordinal relationships between objects' magnitudes. Mathematically, the function $\mathcal{D}_{perfect}(x, x')$ here is a version of the Heaviside (step) function $H(x - x')$ with well-defined value C for the case when $x = x'$.

Next, one can evaluate an element of the magnitude x by the expected outcome of a

[5] This isomorphism of a utility function and a cumulative density function was first noticed by Van Praag [1968]. Kapteyn [1985 and references therein] took this observation further and proposed a "theory of preference formation", which is the closest existing theory to the one presented here. Both theories and the empirical work based on this so-called Leyden approach will be discussed further in Section 3.

pairwise comparison:[6]

$$I_{perfect}(x) = E\mathcal{D}_{perfect}(x) = \int_X \mathcal{D}_{perfect}(x, x')f(x')dx' = F(x) \tag{4}$$

Throughout the paper, the function $E\mathcal{D}(x)$ aggregates the outcomes of ordinal comparison of a fixed element with magnitude x against all the other elements x' in the reference set. Here, when the ordinal comparison tool is perfectly accurate, an individual's evaluation can be described by a cumulative density function $F(\cdot)$.

It is important to notice that the shape of the distribution of objects' magnitudes determines whether an individual evaluation exhibits increasing, decreasing, or constant marginal utility. Thus, if the distribution of magnitudes of the reference objects is of the power function form, individual evaluation is consistent with possession of a constant relative risk aversion (CRRA) utility function; and if the distribution is exponential, the evaluation is consistent with possession of a constant absolute risk aversion (CARA) utility function (see Castagnoli and Li Calzi [1996]).

The above model has interesting implications for cognitive evaluation of income. Given a fixed environment, it is increasing in one's income, as one would expect. That means that if self-reported happiness is correlated with income, a cross-sectional study would reveal self-reported happiness to be increasing with relative incomes - as, indeed, many studies, including Easterlin [1974] have shown. However, as other people's incomes change, so does satisfaction with one's own fixed income. Consider evaluations of a typical resident of, say, United States, if she evaluates her income using a perfect ordinal comparison tool on the basis of the country's empirical income distribution. As a typical income distribution is hump-shaped, the resulting evaluation will be S-shaped. Thus, the individual would experience increasing marginal evaluation when her income is below the modal country's income, and decreasing marginal evaluation when her income is above the modal country's income. It is tempting further to say that the individual's environment may determine her attitudes toward risk. In the case of the US resident that would mean that an individual will be risk loving when relatively poor (and, thus, buy lotteries) and risk averse when relatively rich (and, thus, buy insurance policies).

2.3 Comparative Statics

Let us now turn to another important question. Would Robinson Crusoe's evaluation of a given coconut change if he were on another island? Given the cognitive evaluation procedure, he would be happier with a 3 inch coconut on an island where an average coconut is 2 inch radius rathen than on another island where an average coconut is 5 inch radius - simply because coconuts smaller than 3 inch are more frequent on the first island rather

[6] As $\mathcal{D}_{perfect}(x, x') = H(x - x')$ is a degenerate distribution, on a formal level, the formulation (4) is a special case of the Castagnoli and LiCalzi [1996] (expected) probability model. See more on this theory later in Section 3.

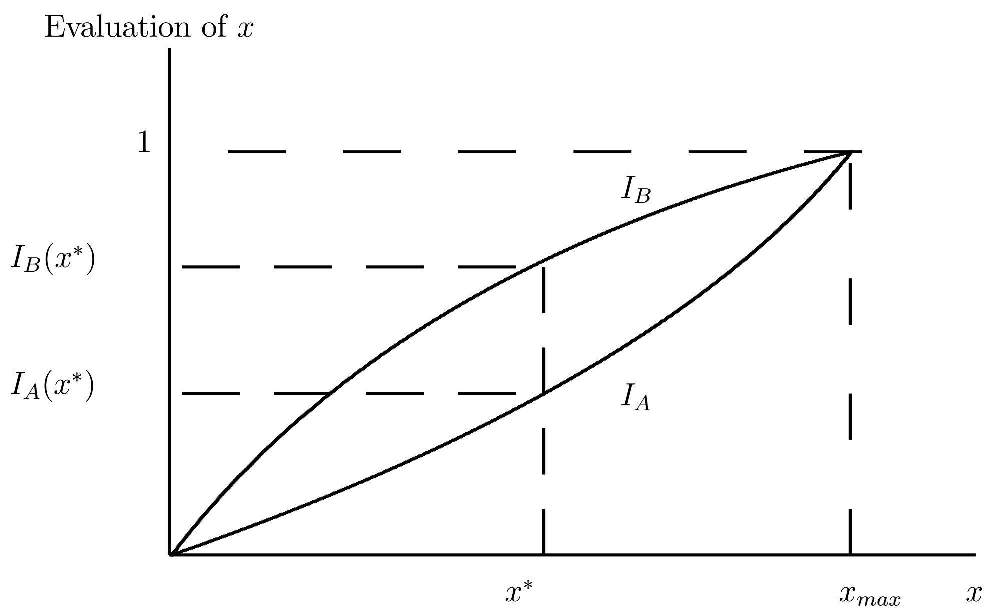


Figure 1: If $F_A \succeq_{FOSD} F_B$, any given object of magnitude x^* is evaluated higher against the reference set B, as smaller objects are more frequent there.

than on the second. Traditionally, such a phenomena would be explained by Crusoe's "lower aspirations" on the first island - yet, aspirations could be simply a psychological consequence of a cognitive evaluation process. The cognitive tools themselves are free of sentiments (this thesis is further expanded in Section 6).

To see that, consider two islands, A and B, which differ only in terms of size distributions of coconuts given by cumulative distributions F_A and F_B respectively. Suppose first that the size distribution of coconuts on the island A first order stochastically dominates the the size distribution of coconuts on the island B. That is, an average coconut on island A is bigger than on island B. The next statement says that this would lead to lower evaluation of almost every coconut on an island A. The argument follows directly from definition of the first order stochastic dominance (see Figure 1). [7]

Property 1 *If $F_A \succeq_{FOSD} F_B$, then $I_B(x) \geq I_A(x)$ for all x.*

In other words, Robinson Crusoe is less satisfied with a given coconut when the sizes of most other coconuts increase. This observation is important, as neoclassical theory would predict that one's satisfaction with a fixed object is not affected by one's environment.

[7] Figures 1 and 2 are drawn for a continuous reference set, however the results are valid for either finite or continuous reference set.

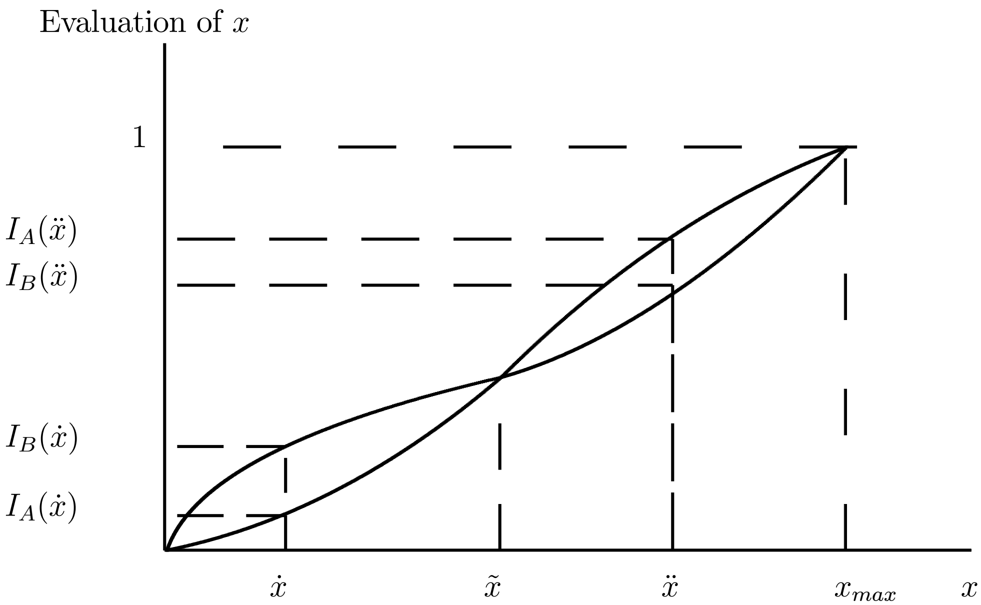


Figure 2: If $F_A \succeq_{SOSD} F_B$, a relatively small object $\dot{x}$ has a lower evaluation and a relatively large object $\ddot{x}$ has a higher evaluation when compared to a less dispersed reference set A.

Consider now an increase in the dispersion of coconuts' sizes in a sense of second order stochastic dominance, also known as generalized Lorenz dominance (which implies that generalized Lorenz curves do not intersect). The next statement says that this would lead to a relatively small coconut being evaluated lower on a relatively more "equal" island A, but a relatively large coconut being evaluated higher on the same island. Again, the property is obvious as it follows from the properties of second order stochastic dominance (see Figure 2).

Property 2 *If $F_A \succeq_{SOSD} F_B$, then $I_A(x) < I_B(x)$ for all $x < \tilde{x}$ and $I_A(x) > A_B(x)$ for all $x > \tilde{x}$, where $\tilde{x}$ is such that $F_A(\tilde{x}) = F_B(\tilde{x})$.*

What do these properties mean for cognitive evaluation of incomes? An increase in average income in a sense of first order stochastic dominance leads to a lower evaluation of each income level. In other words, an individual is less satisfied with her fixed income when everyone else's incomes increase. Also, an increase in income equality in a sense of second order stochastic dominance (or generalized Lorenz dominance) leads to lower evaluation of relatively low incomes, and higher evaluation of relatively large incomes. As Alesina, di Tella and MacCulloch [2003] report, in the US, the self-reported happiness of the poor is affected by inequality less than that of the rich - which may happen because greater equality means that, for those with relatively small incomes, there are fewer people with less or equal incomes.

Can one use this result to predict on which island Robinson Crusoe would prefer to live? It depends on the nature of objects we consider. The present model can suggest which island Robinson Crusoe might choose if he is destined to have the same coconut for life (as it would happen with height or intelligence), yet it is silent regarding his choice when other coconuts would become available to him (as in would happen in case of income). Yet, the experiments of Johansson-Stenman, Carlsson and Daruvala [2002] indicate that many subjects choose a hypothetical society where their imaginary grandchildren have higher relative, rather than absolute, income.

3 Related Literature

On the first glance, the above cognitive evaluation algorithm seems to offer the ultimate solution to, perhaps, the longest-standing problem in economics. That is, take a set of one-dimensional objects for which an individual is known to have "more is better" preferences (say, money), compute the distribution of magnitudes in the set - and one gets the utility function! Moreover, the resulting utility function is, by construction, cardinal, and it represents preferences which are endogenous (as it depends on the distribution of magnitudes of the reference objects), and may be interdependent (if the objects in the reference set belong to other people).

However, as the next subsection demonstrates, the approach of Section 2 is neither entirely new, nor it is entirely correct description of subjects' behavior of experiments. The subsection thus provides the motivation for the model of Section 4, while the remaining subsection surveys other related models.

3.1 Previous Theories and Empirical Observations

In economics, the oldest and most developed neo-cardinal methodology has been known as the Leyden approach. It stems out of an original model of the individual welfare function of income due to Van Praag [1968] based on the observation of isomorphism of utility and cumulative density functions. His model, however novel, is based on the complicated procedure involving infinite number of consumption groups, leading to a cardinal representation of individual utility with a cumulative density function of a lognormal distribution.

In his doctoral dissertation, published in 1977, Kapteyn took Van Praag's idea a step further and proposed a more accessible and, perhaps, the closest model to the cognitive evaluation procedure (see Kapteyn [1985 and references therein]). Kapteyn suggested that satisfaction with one's possessions is determined by the process of preference formation defined over "reference spaces". These reference spaces are based individual's environment and past observations and are constructed for each individual by weighting other observations with an individual-specific weight. In other words, according to Kapteyn, individuals evaluate their consumption by the rank of their consumption in the perceived distribution of consumption, just like in the above cognitive model, but this is explained by a social, and not cognitive, process. The implications of the Van Praag-Kapteyn model were later explored empirically (see, for example, Kapteyn, Wansbeek, and Buyze [1980], Kapteyn and Wansbeek [1982], Van de Stadt, Kapteyn, and Van de Geer [1985], also Van Praag and Frijters [1999]).[8] However, given the nature of the data collection process, these researchers could not control for the perceived distributions, and thus could not directly test the theory.

Interestingly, the experiments which are, in effect, a direct test of Kapteyn's theory in an abstract setting, were done in psychology about the same time as Van Praag noticed the isomorphism of utility function and distribution function. In the early 1960s a psychologist Parducci [1963, 1965, 1968, and references therein] conducted a series of subject-based experiments on subjective judgements of various stimuli (size, weight, numerousness and numerals). Parducci's initial hypothesis was that individual judgements should vary with the mean of stimuli presented (in accordance with the adaptation-level theory of Helson [1964]). Instead, Parducci [1963, 1965, 1968] reported that, when averaged over subjects, the category judgements of size, weight, numerousness and numerals are not so much affected by the mean stimuli, but instead follow the shape of the underlying distributions with a comparative statics behavior consistent with the Properties 1 and 2. In other words, individuals make evaluations based on the context, and for sizes, lengths, weights and so on, such context is provided by their distributions. However, while the experimentally observed evaluation functions have, in general, the same shape as the distribution of magnitudes in the reference set, they are nevertheless quite distinct from the "true" distributions - in particularly, the evaluation functions are flatter, less curved, than the true distributions.

It is tempting to speculate that it is this "flatness" of the evaluation function which was one of the reasons for Parducci [1963, 1965] to propose his celebrated range-frequency theory. According to this theory, individual judgements, averaged over subjects, can be better predicted by a weighted sum of two ranks - one in the actual distribution (frequency component) and the other in a uniform distribution (range component). In other words, individuals take into account both object's position relatively to all objects and its position relatively to the domain of the distribution.[9]

[8] For criticism of Leyden approach see Seidl [1994].

[9] Parducci's work thus introduced the idea that frequency of experiences may contribute to individual sense of well-being. This idea was further explored by Diener, Sandvik, and Pavot [1991]. A few more studies applied Parducci's range-frequency theory to self-reported satisfaction with income - including a laboratory study by Smith, Diener, and Wedell [1989], and an analysis of field data by Hagerty [2000], both with some success, though the researchers were not controlling the context of evaluation (i.e.

More recently, Morone, Seidl and Levati [2001] conducted an experimental study based on the Leyden group's income categorization methodology which allowed to control for the true distributions. It turned out that the same set of hypothetical incomes was evaluated differently once presented against a different distributions of incomes. Moreover, empirically observed evaluations, averaged over subjects, followed the general shape of each hypothetical distribution - that is, incomes presented against a concave distribution were evaluated by, in general, concave evaluation function, etc. These experimental results demonstrate that subjects's evaluations of income are influenced by the corresponding cumulative density functions, and are consistent with the Property 1 and 2 (for example, mean ratings for common stimuli is in general higher for positively skewed distribution rather for the negatively skewed one.) However, as in the original experiments of Parducci, the empirically observed evaluation functions, averaged over subjects, are less curved than the true distributions, and thus are more in the accordance with the range-frequency theory of Parducci.[10]

In other words, these experimental findings only partially support the Leyden approach of Kapteyn and Van Praag, or, indeed, any theory suggesting that evaluation functions are equal to the true distribution functions - including the theory of the cognitive evaluation of Section 2. However, as the Section 4 will show, a more realistic approach to human cognitive tools may in fact provide an alternative interpretation of existing experimental data.

3.2 Other Theories on Evaluation and Statistical Information

The Leyden approach of Van Praag and Kapteyn and the range-frequency theory of Parducci are not the only previous theories that relate human evaluations and statistical information.

Formally, the present model of cognitive evaluation is a special case of neo-cardinal approach of Castagnoli and LiCalzi [1996], yet these authors themselves admitted to be short of behavioral explanation for their model. Castagnoli and LiCalzi [1996] propose a simple procedure whereby a lottery is evaluated relatively to another lottery would result in "expected utility without utility", consistent with neoclassical expected utility function. They were the first to suggest that the distribution of a comparison lottery may determine attitudes toward risk. As such, when the underlying distribution is of the power function form, the corresponding utility function exhibits constant relative risk aversion (CRRA), and when it is exponential, it exhibits constant absolute risk aversion (CARA).

observable income distribution).

[10] The "flatness" of the mean evaluations can be observed from the data presented in Table 2 of Morone, Seidl and Levati [2001]. Both this paper and its follow-up, Seidl, Traub and Morone [2003], report the comparative static results based on the same experimental data.

Interestingly, both Leyden approach of Van Praag and Kapteyn and expected utility theory without utility of Castagnoli and LiCalzi have been applied to the behavioral phenomena considered by Kahneman and Tversky [1979]. Van Praag and Frijters [1999] suggested that the prospect theory's value function and Van Praag's IWF of income are very similar - both being S-shaped, exhibiting risk loving for relatively low incomes and risk aversion for relatively high incomes. LiCalzi with colleagues made similar observations regarding the relationship of Castagnoli and LiCalzi [1996] "expected utility without utility" and Kahneman and Tversky [1979] value function - see, LiCalzi [1999], Bordley and LiCalzi [2000], and DellaVigna and LiCalzi [2001]. Also see Harbaugh and Kornienko [2000] for similar work.

An ordinal rank in an observable distribution has been long used by economists as a measuring device, yet almost exclusively in the economics of relative position. Perhaps, Duesenberry [1949] was the first economist to notice that self-esteem depends on frequencies-of-occurrence of a particular outcomes of relative comparisons: "the drive [for self-esteem] operates through inferiority feelings aroused by unfavorable comparisons between living standards. The strength of such feelings suffered by one individual varies with the frequency with which he has to make an unfavorable comparison between the quality of goods he uses with those used by others". Luxuries and other observable goods happen to be particularly handy for assigning status, or relative position, to an individual (Hirsch [1976]). Since Frank [1985b] and, later, Robson [1992], status has been frequently operationalized as the ordinal rank, or the value of a cumulative density function of the distribution of positional goods, yet, this strand of research maintained that an individual derives utility from both absolute and relative values. Nevertheless, Frank's [1985b] model of status as a rank is a significant step forward from the first formal model of reference-based utility due to Duesenberry [1949] (developed further by Pollak [1976]), who suggested that individual satisfaction with their possessions is in part determined by comparison to a reference point - here the average possessions in one's community. Notice though that average, or mean, is only a single reference point, and much less informative than a collection of reference points aggregated into a distribution. The rank formulation of status has gain further popularity due to instrumental nature of status suggested by Postlewaite [1998, references therein].

It is important to mentioned that all the above mentioned works have a common theme of the use of frequencies and distributions for evaluations, and thus differ from other strands of literature on reference-dependent evaluations (see, for example, Gilboa and Schmeidler [2001]).

4 Cognitive Evaluation with an Imperfect Ordinal Tool

As it was shown in Section 2, if the ordinal comparison tool is perfectly accurate - that is, an individual is able to tell apart two objects even if their magnitudes are only slightly different - an object of a given magnitude can be evaluated by its rank in the true distribution of magnitudes of the reference objects. However, as Parducci [1963, 1965] and, more recently, Morone, Seidl and Levati [2001] found, the empirically observed category judgements of size, weight, numerousness and numerals, averaged over subjects, vary systematically with the true distributions, and conform to the comparative statics of Properties 1 and 2. However, the average judgements are distinct from the ranks in the true distributions, and, in particular, they are somewhere between the rank in the true distribution and the rank in a uniform distribution with the same support. These results would seem to be at odds with the model presented in Section 2 - or, indeed, any other work based on the equivalence of utility and a cumulative density function.

In contrast to previous neo-cardinal models described in Section 3, the cognitive approach may offer a novel explanation for this distinct behavior of the evaluation function. It is quite possible that the ordinal comparison tool may be imperfect in the sense that any two objects that are *perceived* to be similar may "tie" in a pairwise ordinal comparison. Consider, for example, the following set $A = \{10, 98, 98, 98, 99, 100\}$. According to the procedure described in Section 2.1, the cognitive evaluation of 99 in the above set A is the same as the evaluation of 99 in the following set $B = \{10, 10, 10, 10, 99, 100\}$ as 99 has exactly the same ordinal rank in both sets. Yet, some people may value 99 in set B higher than in set A. This may happen because some elements of the reference set may be *perceived* to be similar, so that, at least for evaluative purposes, an individual may instead perceive the sets to be as follows: $A' = \{10, 99, 99, 99, 99, 99\}$ and $B' = \{10, 10, 10, 10, 99, 99\}$. Now, using the algorithm of Section 2.1 with $C = 0.5$, $I_{A'}(99) = 3.5$ while $I_{B'}(99) = 5$.

As we will see below, such perceived similarities can, in fact, have a profound effect on the cognitive evaluation. The main point is the following. Suppose an individual is endowed with ordinal comparison and frequency processing tools only. If the ordinal comparison tool is imperfect in a sense that two distinct elements of a reference set may appear to be similar, then the resulting cognitive evaluation is only in part determined by the true distribution summarizing the magnitudes of reference elements $F(\cdot)$, and is distinct from the true distribution.

As there is no consensus on how to treat similarity relationships, below are a few possible treatments, all developed for the continuous case (yet the intuition applies for a finite reference set as well). Note that all of the ordinal comparison tools that are being considered throughout this paper satisfy the following conditions:

$$\frac{\partial \mathcal{D}(x,x')}{\partial x} \geq 0$$
$$\frac{\partial \mathcal{D}(x,x')}{\partial x'} \leq 0$$

That is, the result of a pairwise ordinal comparison of a given object with magnitude x with the magnitude x' of a single reference object is non-decreasing in the magnitude of a given object x but is non-increasing in the magnitude of each reference object x', just as previous theories have suggested (for a review see Rabin [1998]).

4.1 Rubinstein's Similarity Relation

It has long been known to psychologists that humans may not notice the difference between two magnitudes if it is less than a certain threshold - an observation widely known in psychology as Weber's law of just noticeable differences. This psychophysical law is the psychologist's counterpart of the law of diminishing marginal utility, and it says that the minimum amount by which stimulus intensity must be changed in order to produce a noticeable variation is proportional to the current level of the stimulus (for the exposition of various formal treatments of this phenomena see, for example, Ekman [1959] and Laming [1973]).

Clearly, when an individual fails to tell apart two distinct objects, the characteristics of the two objects are perceived to be *similar*. Thus, a reasonable treatment of perceived similarities should satisfy Weber's law. One of the more recently emerged similarity relationships, conforming to Weber's law, is due to Rubinstein [1988], and it implies that two distinct objects might be judged to be similar if the ratio of their magnitudes is neither large enough nor small enough. Formally, if the (imperfect) ordinal comparison tool is subject to Rubinstein's similarity relationship, an outcome of a pairwise comparison (3) is be written as:

$$\mathcal{D}_{Rubinstein}(x,x') = \begin{cases} 1 & \text{whenever} \quad x/x' > \lambda \\ C & \text{whenever} \quad \lambda^{-1} \leq x/x' \leq \lambda \\ 0 & \text{whenever} \quad x/x' < \lambda^{-1} \end{cases} \tag{5}$$

where $\lambda > 1$ is some constant. For any element of magnitude x, the outcomes of imperfect ordinal comparisons can be aggregated by using the frequency tool as follows:

$$I_{Rubinstein}(x) = E\mathcal{D}_{Rubinstein}(x) = \int_X \mathcal{D}_{Rubinstein}(x,x')f(x')dx' = (1-C)\cdot F(\lambda^{-1}x) + C\cdot F(\lambda x) \tag{6}$$

So, just as in the case of perfect ordinal comparisons, an element of the magnitude x can be evaluated by the expected outcome of a pairwise comparison against all the elements in the comparison set. What is important however is that the resulting evaluation

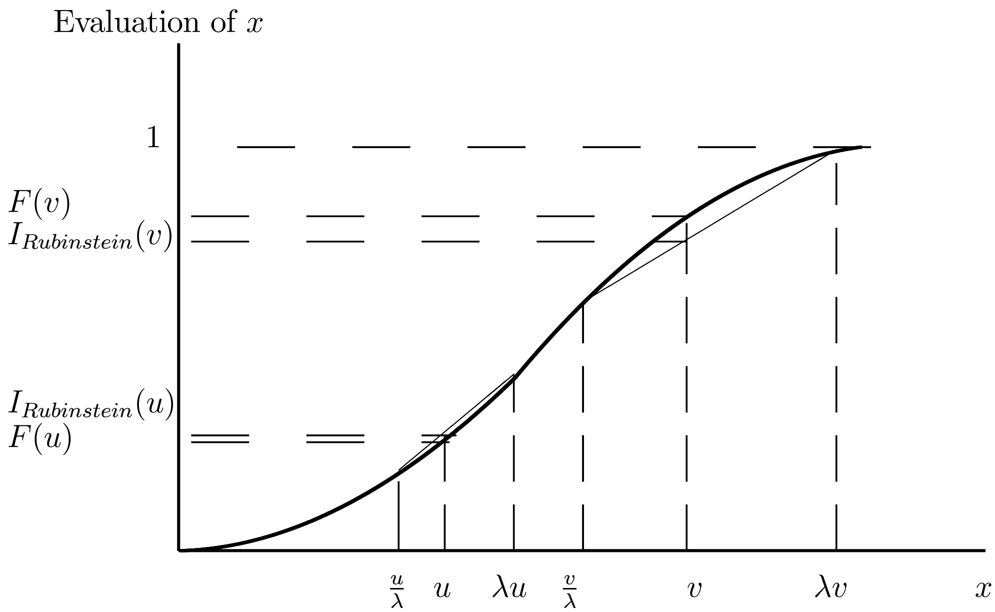


Figure 3: Imperfect ordinal comparison tool subject to Rubinstein's similarity condition $I_{Rubisntein}(x)$ may "moderate" the evaluation function, with objects on convex portions of true distribution $F(x)$ to be overvalued and on concave portions to be undervalued (for $\lambda = 1/C - 1, C < 0.5$).

$I_{Rubinstein}(x)$ is distinct from the true distribution of magnitudes $F(x)$, as it is a weighted sum of two distributions of the same type as the true distribution $F(x)$. In this case, a given object is evaluated by a weighted sum of the rank of an object relatively a bit below and of the rank of an object relatively far above. While the comparative statics of Properties 1 and 2 will in general be preserved, Rubinstein's similarity relation may lead to a complex distortion of the true distribution.

To see that, first notice that for $\lambda = 1/C - 1, C < 0.5$, the imperfect cognitive evaluation $I_{Rubinstein}(x)$ can be analyzed using Jensen's inequality, as in this case an element on a convex portion of a distribution is overvalued, and an element on a concave portion is undervalued (see Figure 3). Employing Jensen's inequality further, one can show that when $C < 0.5$ and $1 < \lambda \leq 1/C - 1$, an element on a concave portion is undervalued and when $C < 0.5$ and $\lambda \geq 1/C - 1$, or when $C \geq 0.5$ for all values of λ, an element on a convex portion is overvalued. In other words, an ordinal imperfection may have a moderating influence on magnitude evaluations as the imperfect evaluation function may be less curved than the "true" distribution.

Rubisntein's similarity condition, just like other forms of Weber's law, does not behave "well" on the boundaries of a distribution. One of the possible treatments of the elements with relatively small and relatively large magnitudes is as follows:

$$I^{general}_{Rubinstein}(x) = \begin{cases} CF(\lambda x) & \text{if } x < \lambda x_{min} \\ (1-C)F\left(\frac{x}{\lambda}\right) + CF(\lambda x) & \text{if } \lambda x_{min} < x < \frac{x_{max}}{\lambda} \\ C + (1-C)F\left(\frac{x}{\lambda}\right) & \text{if } x > \frac{x_{max}}{\lambda} \end{cases} \tag{7}$$

Thus, the imperfect ordinal comparison tool may lead to further distortions in the cognitive evaluation. Obviously, a different treatment of the lower and upper ends of the domain of the distribution will result in a different distortion.

4.2 Noisy Ordinal Comparison Tool

Suppose now that the ordinal comparison tool is noisy. That is, the imprecisions in the ordinal comparison tool are determined stochastically, by chance, rather than in any deterministic fashion. Let the error ϵ is distributed independently from the distribution of magnitudes, and has a distribution $G(\cdot)$. In this case, imperfect cognitive evaluation is as follows:[11]

$$I_{noise}(x) = E_{\mathcal{E}} E \mathcal{D}_{noise}(x \mid \epsilon) = \int_{\mathcal{E}} \int_{X'} D(x, x' \mid \epsilon) f(x') dx' g(\epsilon) d\epsilon \tag{8}$$

Thus, a cognitive evaluation based on a noisy ordinal comparison tool is determined not only by the true distribution of magnitudes $F(\cdot)$, but also by the error distribution, $G(\cdot)$. As the result, a noisy ordinal tool may result in a cognitive evaluation being functionally distinct from the original distribution - as the next two examples show.

Example: Multiplicative Noise

Let ordinal comparison tool be subject to a multiplicative noise ϵ as follows:

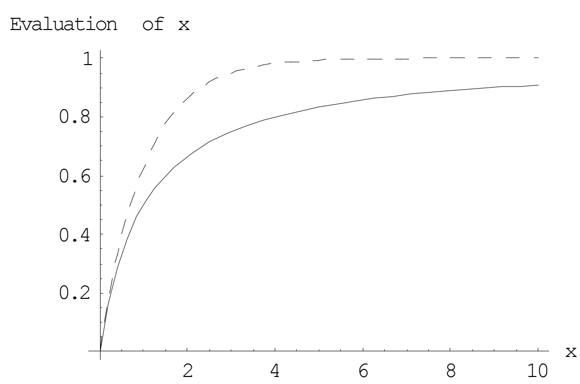
$$\mathcal{D}^{multiplicative}_{noise}(x, x' \mid \epsilon) = \begin{cases} 1 & \text{whenever} \quad x > \epsilon x' \\ C & \text{whenever} \quad x = \epsilon x' \\ 0 & \text{whenever} \quad x < \epsilon x' \end{cases} \tag{9}$$

so that $\mathcal{D}^{multiplicative}_{noise}(x \mid \epsilon) = F(\epsilon^{-1}x)$.

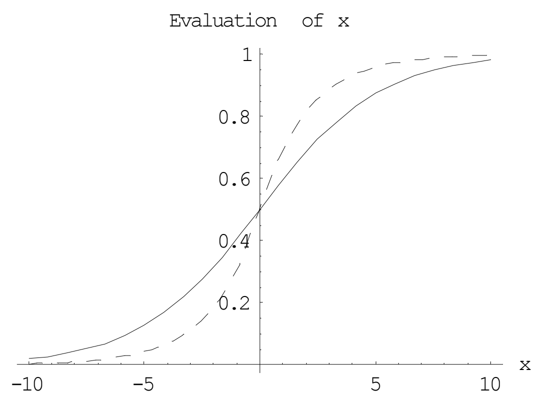
Let us further suppose that the true distribution of magnitudes is exponential, i.e. $F(x) = 1 - \exp(-\lambda x), x \in [0, \infty)$, while the reciprocal of noise $\epsilon^{-1} = \tau$ is Gamma-distributed, i.e. $g(\tau) = \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\beta\tau}, \tau \in [0, \infty)$. In this case, the noisy evaluation is given by:

$$I^{multiplicative}_{noise}(x) = \int_\tau F(\tau x) g(\tau) d\tau = \int_0^\infty (1 - \exp(-\lambda x \tau)) \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\beta\tau} d\tau = 1 - \left(\frac{\beta}{\lambda x + \beta}\right)^\alpha \tag{10}$$

[11] The noisy evaluation is based on the model of Castagnoli and Li Calzi [1996].



a) $I_{noise}^{multiplicative}(x)$ with $\lambda = 1, \alpha = 1, \beta = 1$;



b) $I_{logistic}(x)$ with $m = 0.5$;

Figure 4: Noisy cognitive evaluations (solid curves) may look very different from the true evaluations $F(x)$ (dashed curves).

which is a Pareto II, or Lomax, distribution, and quite distinct from the exponential distribution. In other words, an imperfect comparison tool subject to multiplicative noise may result in a cognitive evaluation functionally distinct from the true distribution of magnitudes (see Figure 4a).

Example: Additive Noise

Suppose now that an individual is able to estimate the difference in the magnitudes up to an error ϵ:

$$\mathcal{D}_{noise}^{additive}(x, x' \mid \epsilon) = \begin{cases} 1 & \text{whenever} \quad x > x' + \epsilon \\ C & \text{whenever} \quad x = x' + \epsilon \\ 0 & \text{whenever} \quad x < x' + \epsilon \end{cases}$$

Since the above noisy comparison tool can be expressed as the Heaviside step function $\mathcal{D}_{noise}^{additive}(x, x' \mid \epsilon) = H(x - x' - \epsilon)$, one can further write this noisy result of a pairwise comparison as follows:

$$\mathcal{D}_{noise}^{additive}(x, x') = \int_{\mathcal{E}} \mathcal{D}_{noise}^{additive}(x, x' \mid \epsilon) g(\epsilon) d\epsilon = G(x - x') \tag{11}$$

so that the evaluation can be written as

$$E\mathcal{D}_{noise}^{additive}(x) = \int_X \int_{\mathcal{E}} \mathcal{D}_{noise}^{additive}(x, x' \mid \epsilon) f(x') g(\epsilon) dx' d\epsilon = \int_X G(x - x') f(x') dx' \tag{12}$$

Let us further suppose that the noise ϵ is distributed with a logistic distribution function, i.e.

$$G(\epsilon) = \left(1 + e^{-m\epsilon}\right)^{-1}, \epsilon \in (-\infty, \infty)$$

Since $\epsilon = x - x'$, one can write the result of a pairwise comparison as a deterministic reverse logistic function:

$$\mathcal{D}_{logistic}(x, x') = \left(1 + e^{-m(x-x')}\right)^{-1}$$

In other words, the function $\mathcal{D}_{logistic}(x, x')$ is a smooth version of the step-function $\mathcal{D}_{perfect}(x, x')$. As the result, the evaluation can be written as

$$I_{logistic}(x) = \int_X \frac{f(x')}{1 + e^{-m(x-x')}} dx' \tag{13}$$

Suppose further that the true distribution is Laplace, i.e. $f(x) = 0.5me^{-|mx|}, x \in (-\infty, \infty)$, so that the logistic evaluation is:

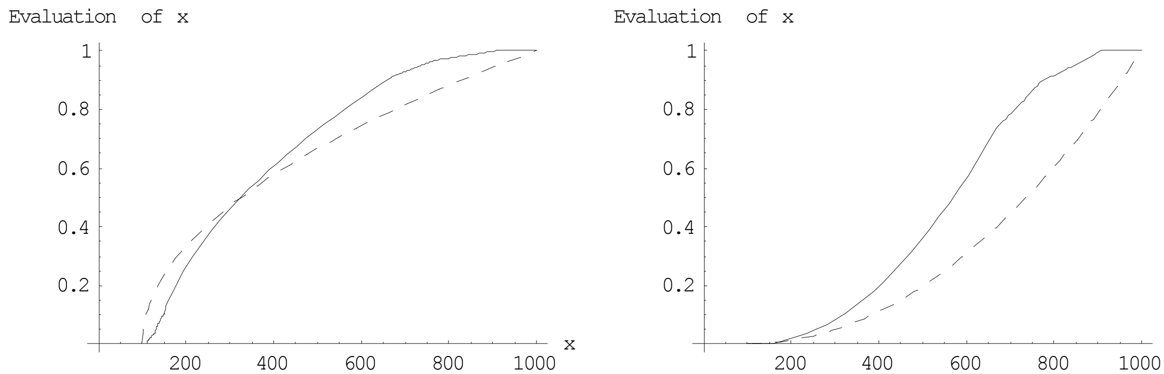
$$I_{logistic}(x) = \int_X \frac{0.5me^{-|mx|}}{1 + e^{-m(x-x')}} dx' = 0.5\left(1 + (e^{mx} - e^{-mx})\log(1 + e^{-mx}) - mxe^{-mx}\right)$$

Again, an imperfect comparison tool subject to additive logistic noise may result in a cognitive evaluation functionally distinct from the true distribution of magnitudes (see Figure 4b).

4.3 Imperfect Ordinal Comparisons and Experimental Data

Notice that the present cognitive evaluation model goes beyond providing a psychological rationale for the above neo-cardinal models, by offering a framework which may accomodate both the theories of the Leyden group and Parducci's experimental findings. If the ordinal tool prefect, the model suggests that the cognitive evaluation function is identical to the cumulative density function, thus consistent with Van Praag [1968] and Kapteyn [1985, references therein]. Yet it provides a possible explanation for why Leyden approach has not been confirmed in a laboratory, and , instead, the recent experimental findings on income categorization by Morone, Seidl and Levati [2001] have been so far more successfully explained by range-frequency theory of Parducci [1963, 1965, 1968].

The key could be in the fact that human ordinal comparison tool might not be perfectly accurate. If the ordinal tool is imperfect (as described above), the shape of the evaluation function varies systematically with the shape of the true distribution but moderates the evaluations. To see that, note first that in the experimental studies on subjective evaluations of magnitudes of Parducci [1963, 1965] and Morone, Seidl and Levati [2001], the subjective evaluations were aggregated over participating subjects. Suppose now that subjects' ordinal comparison tools belong to the same class - say, they are subject to the Rubinstein's similarity condition and are evaluated according to the evaluation function given by (7), they still may have individual-specific parameters. Consider, for example, a group of three subjects, say, with the same value of ties C, but with different λ_1, λ_2 and λ_3. Suppose further that the true distribution of objects



a) $\sum I_i(x)$ for a positively skewed distribution; b) $\sum I_i(x)$ for a negatively skewed distribution;

Figure 5: Aggregated imperfect cognitive evaluations $\sum I_{Rubinstein}(x, \lambda_i)$ (solid curves) may look very different from the true distributions $F(x)$ (dashed curves).

is of the power function form: $F(x) = ((x - a)/(b - a))^{\alpha}$. Figure 5 demonstrates a possible cognitive evaluation, averaged over the subjects, for positively and negatively skewed distributions. It is important to stress that, in the above Figure, the pronounced distortions at the upper end of the distribution are generated here by the particular treatment of the boundaries given in (7), and will change with the different treatment of the boundaries.

Alternatively, subjects' ordinal comparison tools may be subject to noise as described by the equation (8). As all of the above examples of an imperfect ordinal comparison tool demonstrate, the cognitive evaluation, while having the shape similar to the true distribution, may nevertheless be markedly different from it. In other words, imperfections in the ordinal comparison tools distort the "true" evaluation, and when subjects vary in their imperfections in the ordinal assessments, the aggregated evaluation will be further away from the true evaluation $F(x)$.

However, as individual ordinal comparison tools are still the object of intensive current research in cognitive psychology (see, for example, Xu [2003]), it remains to be determined which imperfect ordinal tool is the best to describe subjects' evaluations.

5 Interdependent Evaluations and Implications for Welfare

Is there any difference between the way Robinson Crusoe evaluates a coconut and the way Mrs. Brown evaluates her house? As Section 6 further argues, there is little difference in the two cognitive processes as a house is evaluated the same way as a coconut - that is, against a reference set consisting of other houses, using two primitive cognitive tools. Yet, there is an important feature of cognitive evaluation of a house - the reference set here consists of houses that may *belong to other people*. In other words, in a private ownership economy most (if not all) ordinal comparisons are necessarily done interpersonally. This is even more pronounced for individual characteristics (height, intelligence, beauty, etc.) as well as salaries, incomes, bonuses, and wealth, as the reference set would necessarily involve what *other people have*. The sentiments arising from relative comparisons have been known since Adam Smith [1759] and Thorstein Veblen [1899], and have been traditionally explained by envy. Yet envy may just be a social consequence of the evaluation process, while the cognitive tools themselves are free of sentiments (see more on this in Section 6).

These interpersonal comparisons have important consequence as a change in what Mr. Jones has changes how Mrs. Brown evaluates what she has. A boy starts to evaluate his height once his parents place him back to back against another child and judge who of the two is taller. Each back-to-back ordinal comparison provides some information about boy's height. The more often the boy comes out being taller, the happier he would be with his height, and otherwise - the more often he comes out shorter, the less satisfied with his height he would be, and more urgent would be his desire to grow up. Now suppose that a boy significantly grows up during summer months. Given his peers' height at the end of spring, he (and his parents) might feel quite happy with his height. Yet, as he grew up, his peers grew up, too, as the boy would discover upon his return back to school. To keep his satisfaction with height at his pre-summer level, the child has to grow at least as fast as everybody else "to keep in the same place", and to be happier with his height, he has to grow "twice as fast". This "Red Queen" effect is particularly important for cognitive evaluation of income as it involves the reference set consisting entirely of *other people's* incomes. That is, given a fixed environment, the evaluation is increasing in one's income, but as some people's incomes change, the evaluation of fixed income changes as well, leading to the desire to "Keep up with the Joneses".

What would this imply for societal welfare? Let us consider a society where each individual's satisfaction is entirely determined by the cognitive evaluation of what she has against what other people have (this could be height, intelligence, income, happiness, and so on). Let us further concentrate on a private ownership economy with a continuum of individuals, where all individuals encounter the same reference set of objects X. Each of these reference objects belongs to an individual in this economy, and the magnitudes of objects are distributed with a cumulative density function $F(\cdot)$. To ensure interpersonal comparability, let us assume that all individuals evaluate their possessions x relatively to these encountered objects from the same set X, and possess the same cognitive tools. In particular, all individuals have the same ordinal comparison tool[12] subject to the following "zero-sum property":

$$\mathcal{D}(x, x') + \mathcal{D}(x', x) = 1 \qquad (14)$$

[12] When other objects belong to other people, the ordinal comparisons may be especially inaccurate and thus the imperfect cognitive evaluations of Section 4 are particularly important here.

In other words, a pairwise comparison has opposite outcomes for any two individuals, i.e. a favorable outcome for one individual means an unfavorable outcome for the other. Notice that the specifications of the ordinal comparison tool, considered in Sections 2 and 4, namely, $\mathcal{D}_{perfect}(x, x')$, $\mathcal{D}_{Rubinstein}(x, x')$ (both with $C = 0.5$), and $\mathcal{D}_{logistic}(x, x')$, satisfy the zero-sum property. The zero-sum property leads to an important result.

Property 3 *In a private ownership economy, if all individuals have the same reference set consisting of objects belonging to all other individuals in the economy and have identical ordinal comparison tool subject to the zero-sum property (14), the utilitarian measure of the total welfare $W = \int_X I(x)f(x)dx$ is independent of the distribution of a good and equals to 0.5.*

Proof: Rewrite the total welfare as:

$$W = \int_X I(x)f(x)dx = \int_X E\mathcal{D}(x)f(x)dx = \int_X \int_X \mathcal{D}(x, x')dF(x)dF(x')$$

Since $\mathcal{D}(x, x')$ is defined for any pair of x and x', and, using the zero-sum property (14), one can write W as

$$W = \int_X E\mathcal{D}(x')f(x')dx' = \int_X \int_X \mathcal{D}(x', x)dF(x)dF(x') = \int_X \int_X (1-\mathcal{D}(x, x'))dF(x)dF(x')$$

so that

$$W = \int_X \int_X \mathcal{D}(x, x')dF(x)dF(x') = 0.5 \quad \blacksquare \tag{15}$$

Thus, it is possible that the economic processes and public policies that affect the distribution of privately owned reference objects, may *not* affect the society's total welfare, or, for that matter, the *average* satisfaction - even though they are able to affect the satisfaction of a single individual. This is because, by changing the possessions of each individual, such policies change the distribution of these possessions in the economy. And since individuals adapt their evaluations to the new human environment, a change in a distribution of possessions not only changes how much each individual has, but also how each individual evaluates her own lot, so that the resulting welfare may be independent of the distribution. This in turn implies that purely redistributive policies could be welfare-neutral.

This result can be used to explain recent empirical puzzles. It is widely assumed that an individual's welfare, or happiness, may, at least in part, depend on individual's evaluation of her income. However, Easterlin [1974] pointed out that average self-reported happiness in USA stayed practically unchanged in the post-war period. From neoclassical point of view, this is highly surprising given that real incomes increased nearly doubled during this time.[13] However, if instead every individual faces the same distrib-

[13] Subsequent studies (see Easterlin [1995] and references therein) have mostly confirmed this initial finding - and even when a positive relationship between happiness and absolute incomes was found, it was quite small. For more information on the relationship between income and happiness see, for example, Oswald [1997] and Easterlin [2001]. For other possible factors affecting individual well-being see, for example, Kahneman [1999].

ution (which might well be possible thanks to the media), and assesses own well-being using the same cognitive tools subject to the zero-sum condition, then the *average* economic well-being be unaffected by economic growth and income redistribution.

6 Cognitive Evaluation and Concern with Status

For most goods of interest to economists, such as income, consumption, intelligence, and so on, cognitive evaluation may look like a concern with status. This is because, as part of the cognitive process of evaluation, individuals compare their possessions with other objects observable - which coincidentally may belong to other people. To evaluate an apple, an individual compares it against other apples present - whether these other apples are on a tree, on a store display, or in hands of other people. Comparing *an* apple with *other* apples and comparing *her* apple with *other people's* apples involve the same cognitive processes, and regardless of the ownership structure, other apples serve as a *yardstick*. (Note, however, footnote 12.)

While *cognition* may be blind with respect to ownership structure, *choice* is not. Ownership structure generally imposes constraints on choice: picking the largest apple from a tree branch is different from taking the largest apple out of someone's hands. Human physique imposes more formidable constraint on choice: while an individual may wish to be a few inches taller, to have longer fingers or a smaller nose, some choices are still prohibitively expensive. However, even when the choice is irrelevant, as in the case of height, an individual may still be able to make evaluations and experience emotions as consequences of these evaluations.

A child may value her height as much as it enables her to get a candy from a tabletop or to reach for a light switch, - until she meets other children and starts evaluating her height relatively to them. In the first instance her yardstick is objects in her house, in the second - her playmates. What other people have often becomes a natural point of comparison - especially when the objects are inseparable from their owners. Height, strength, beauty, intelligence, happiness and other human characteristics are unthinkable without individuals who possess them. In a private ownership economy the majority of randomly encountered goods are owned by someone. And, as consumption expenditures, incomes, wealth, and money are commonly attributed to individuals, most of items of interest to economists may well have been evaluated interpersonally.

Importantly, the inseparability of goods and their owners in a private ownership economy implies that the same cognitive mechanism might be applied two ways: to evaluate objects and to evaluate individuals by the objects they own. In other words, cognition have endowed humans with a "measuring tape", a yardstick, of two sides - one side to measure individuals by the objects they own, the other to measure objects using the individuals who own them.

While Adam Smith [1759] talked about "the poor man's son" fancying a lodging in a palace (that is, a beautiful and spacious house rather than a place where a specific other individual lives), the Darwinian revolution shifted the emphasis from interpersonal comparisons to interpersonal competition. As both natural and sexual selection are relative, rather than absolute, phenomenon, humans may have evolved to care about their relative fitness with respect to other people. Differences in relative fitness established a basis for social differentiation with its virtuous cycle of enhancing the survival and mating opportunities for those with superior physical, intellectual and emotional prowess. In ancient hunter-gather societies, being taller, stronger, and smarter than others would increase one's chances of survival and successful reproduction. At the later stages of social development, length of a string with shells or enemies' teeth, weight of a sack with money, or size of a house have evolved into a relatively costless indicator of an owner's unobserved physical, intellectual, and emotional prowess, of his relative chance of successful selection - if not "natural" but at least social (and thus reproductive).

It was the institution of private ownership that established a mapping between individuals and their possessions, making possible to evaluate individuals by what they own, and it was Veblen [1899] who was first to notice the relationship between property rights and inseparability of goods and their owners. Interestingly, Veblen's [1899] assertion that people evolved to treat relative success as "the conventional end of action" is consistent with the Darwinian view of natural selection. He also pointed out that humans seem to perceive relative consumption and wealth as representing relative fitness. Henceforth, the attention has been centered on individual's *status* as a measure of relative fitness with respect to a group of individuals, and on evaluation of individuals by means of their possessions. Hirsch [1976] made it explicit by introducing a term *positional goods* to denote those observable goods that can be used to compare an individual to other individuals. While Duesenberry [1949] introduced relative comparisons with respect to the average possessions in one's community, since Frank [1985b] status has been frequently operationalized as an ordinal rank, or the value of cumulative density function of the distribution of positional goods, as it represents a proportion of people with worse relative fitness.

Yet, from the cognitive point of view, there is an observational equivalence of assessing goods owned by other individuals and assessing individuals by the goods they own. This duality between the goods and their owners has been long overlooked, and the interpersonal competition approach has dominated the economics of relative concerns - until quite recently Samuelson [2004] came up with an ingenious hypothesis. Nature, Samuelson [2004] suggests, faces the problem of making sure that individuals respond adequately to a changing environment when the ability to process information is costly. He presented an evolutionary model in which individuals observe consumption choices of others though a filter of survival and use this information to make own choices in an uncertain environment. That is, Nature does not give individuals utility functions, but rather the tools by which to acquire them. Furthermore, if complexity is costly, the appropriate tool should be very simple. Perhaps the simplest method is to rely on relative comparison alone. Samuelson [2004] goes on to show that using other people's consumption for informational purposes, and using own relative consumption for the interpersonal competition purposes can lead to opposite comparative statics predictions.

Remarkably, Samuelson's idea is well in line with arguments recently put forward by a number of evolutionary psychologists and anthropologists. Humans may have evolved to adapt the existing cognitive tools to a variety of tasks. There seems to be just a few of these (Duchaine, Cosmides and Tooby [2001]), the most prominent being *social inference* tools. That is, human brain seems to be better at processing human rather than non-human information [Dunbar, 1995]. Thus, it is logical to surmise, as Samuelson [2004] does, that humans may have evolved to adapt our (possibly, superior) social cognition tools to deal with a variety of tasks. And, while the list of available tools is quite short, the number of situations they can be of use is vast.

This paper is agnostic about whether one side of the yardstick (or the "measuring tape") have evolved earlier than the other. Humans might have developed the skills to compare the objects first, and then applied these skills to compare individuals. Alternatively, following the line of arguments of Frank [1985a], it is also possible that humans are hard-wired with relative concerns, so they have developed the skills to compare individuals first, and then applied these skills to compare objects.

All said, interpersonal comparisons may have nothing to do with interpersonal competition, or with status concerns. Instead, other people's possessions may provide useful information about uncertain environment. While people may compare each other based on their salaries, and job applicants may be compared by perspective employers based on their curriculum vita, it may also be true that a relative's salary may provide information about current economic conditions, and colleague's publication list may indicate the state of academic affairs. Simply put, a neighbor's house, a friend's car, a classmate's salary, or a colleague's publication list, may serve as a yardstick, a measuring tape.

7 Conclusions

This paper is written in an attempt to combine insights from evolutionary and cognitive sciences to explain some empirical phenomena of recent interest to economists and psychologists. This work has five main contributions to the literature.

First, it suggests a simple parsimonious model of how an object can be evaluated against an observable reference set - which is provided by one's environment. A minimal set of primitive cognitive tools is enough to construct an evaluation which, coupled with "more is better" preferences, is consistent with a possession of a utility function. Given that the cognitive tools assumed are compatible with well-documented cognitive tools - ordinal comparisons and frequency processing, and are hard-wired by evolution (thus in compliance with the arguments of Robson [2001, 2002]), this model provides a new justification for use of a utility function. Yet, the resulting evaluation function is different from a neoclassical utility function as it is cardinal, is determined by one's environment, and - for most objects of interest to economists - exhibits interdependency.

Second, the evaluation process suggested is inherently reference-dependent. If a given object is larger than a reference object, the object's evaluation tends to increase, and if the object is smaller, its evaluation tends to decrease - just as previous research has suggested (for the review see Rabin [1998]). Yet, here an object is evaluated relatively to a collection of reference points rather than a single point such as average or modal value. As the result, the model provides significant restrictions on the possible form of evaluation functions an individual might have in a given environment, as her evaluation of objects depends on her environment. Thus, as the environment changes, the evaluation of an object changes, too. Take evaluation of income. Given a fixed environment, it is increasing in one's income, as one would expect. That means that if self-reported happiness is correlated with income, a cross-sectional study would reveal self-reported happiness to be increasing with relative incomes - as, indeed, many studies, including Easterlin [1974] have shown. However, as other people's incomes change, so does satisfaction with one's own fixed income. Thus, a change in a distribution of incomes not only changes how much each individual has, but also how each individual evaluates his own lot. An increase in average income in a sense of first order stochastic dominance leads to lower evaluation of each income level. In other words, an individual is less satisfied with her fixed income when everyone else's incomes increase. Also, an increase in income equality in a sense of second order stochastic dominance (or generalized Lorenz dominance) leads to lower evaluation of relatively low incomes, and higher evaluation of relatively large incomes. As Alesina, di Tella and MacCulloch [2003] report, in the US, the self-reported happiness of the poor is affected by inequality less than that of the rich - which may happen because greater equality means that, for those with relatively small incomes, there are fewer people with less or equal incomes.

Third, the model suggests that the cognitive evaluation of an object of a given magnitude is determined by its relative position in a distribution of objects of the same type. However, the present cognitive model goes beyond a mere provision of a psychological rationale for a model implying that a cardinal evaluation isomorphic to a cumulative density function. As the present approach can also accommodate cognitive imperfections, synthesizing some existing theories found in economics and psychology. When the cognitive tools are perfect, the evaluation function coincides with cumulative density function of an observable distribution of magnitudes of reference objects, consistent with Van Praag [1968] and Kapteyn [1985]. Yet it provides a possible explanation for why the Leyden approach has been only partially confirmed in a laboratory. The key is that human ordinal comparison tools might not be perfectly accurate. If the ordinal tool is imperfect (e.g. if it is subject to Rubinstein's [1988] similarity relation, which is, in essence, a form of Weber's law of just noticeable differences, or it is noisy), the shape of the evaluation function varies systematically with the shape of the true distribution, but it is moderated, less curved, and thus more in line with the recent experimental findings on income categorization, which so far has been more successfully explained by range-frequency theory of Parducci [1963, 1965, 1968].

Fourth, when reference sets consists of what other people have (as happens for objects unthinkable without their owners - such as income, consumption, intelligence, height, happiness, and so on) the distributions of such objects may be welfare-neutral. This is because each individual evaluates her possessions against a reference set of other people's possessions, but her possessions at the same time are part of other people's reference set. As the result of this interdependency of evaluations, when one individual's possessions change, other people's evaluations of own possessions change as well. And when all individuals in the economy have identical reference sets (consisting of what other people have) and employ identical cognitive tools with the ordinal comparison tool subject to the zero-sum property, utilitarian measure of welfare may be identical for all distributions of possessions. This welfare neutrality of distribution could shed light on the puzzle first posed by Easterlin [1974], who noticed that average self-reported happiness in the post-war period USA was largely unchanged across time. Moreover, there was little difference in average self-reported happiness across countries. This is remarkable given the significant rise in absolute incomes in the post-war USA, and vast differences in aggregate income and income inequality across countries.

Fifth, for most goods of interest to economists, such as income, consumption, intelligence, and so on, cognitive evaluation may look like a concern with status. This is because, as part of the cognitive process of evaluation, individuals compare their possessions with other objects observable - which coincidentally may belong to other people, and, regardless of the ownership structure, other observable objects serve as a measuring tape, a yardstick. In a private ownership economy where all objects belong to individuals, this measuring tape is of two sides - one to measure objects, and the other to measure individuals. It is this double image that may result in feelings of inferiority and lead to envy.

There are a few limitations of the use of the cognitive evaluation model. First, as the workings of the basic cognitive tools used in the evaluation algorithm are still under investigation, a direct test of the predictions of the model could be some time away. In other words, until our understanding of the human ordinal comparison as well as frequency processing tools is complete, the direct test of the present model may not be fruitful. Second, the model describes the evaluation of objects that differ only in one dimension. In order to construct cognitive evaluations of multidimensional objects, further research needs to be undertaken in order to identify the cognitive tools used in comparing apples and oranges.

Yet, despite its shortcomings, the present paper identifies the key areas where the research needed to be done prior to any meaningful construction of a cardinal utility - namely, the workings of our cognitive tools.

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Appendix A: Other Related Issues

This appendix contains some speculation on related issues.

Habit Formation

More sophisticated cognitive tools may allow an individual to acquire path- or history-dependent evaluations. That is, besides presently available observations, memory of past observations could affect one's evaluation as well. When the other objects of the same type are not presently available for comparison, people may use their memory to build up their evaluations of currently present objects by comparing them to those in the past. An evidence of such evaluations is abound: people often verbalize their evaluations as "This is the saltiest soup I have ever tasted", "This lettuce is fresher than the one I bought yesterday", etc. Such evaluations are possible when own accumulated experience is adequate for meaningful discrimination (obviously, the phenomena of forgetting is important here, especially if the reference set is accumulated over time). But when it comes down to novel things - such as when a traveller is offered ethnic food in a new country, or a toddler is offered a banana - own experiences may not be so useful, and an individual may build own evaluation of a good based on how much of the good other people have in their hands.

When own experience is sufficient, one may compare own present to own past. But when own experience is not enough, one may extract information from other people's present and/or past experiences to assess own present, or

$$I(x(i^*, t^*)) = \int^{i^*} \int^{t^*} f(i,t) di dt$$

where i is an index of individuals, t is time index, and $f(i,t)$ is a joint density of a two-dimensional distribution. In other words, this model, just as the model of Kapteyn [1985, references therein], generalizes two observations due to Duesenberry [1949] - that people's past consumption experiences as well as interpersonal comparisons are relevant to individual satisfaction with her present consumption. By conditioning observations on a particular time, one can get a version of "keeping up with the Joneses" formulation, or

$$I(x(i^*, t^*)) = \int^{i^*} f(i, t^*) di$$

By conditioning observations on a particular individual, one can get a version of "habit formation" formulation, or

$$I(x(i^*, t^*)) = \int^{t^*} f(i^*, t) dt$$

Such a generalization is possible because individuals may use the same subset of cognitive tools irrespective of whether the information comes from their own experience, or from other people.

“Global” vs. “Local” Environment

Notice that the nature of the algorithm used to construct the cognitive evaluation opens up the possibility that individual evaluations may differ across people. This is because people’s immediate, *local* environment may be different even though they live in the same community, facing the same *global* environment. Let each individual’s perceived distribution is given by possessions of individuals in the *reference group*. Each individual’s reference group is constructed as sample of overall population as follows:

$$E\mathcal{D}(x) = \int_X \mathcal{D}(x, x')g(x, x')f(x')dx' \qquad (16)$$

where $g(x, x')$ is a function specifying how an individual with an object of the magnitude x weighs the outcome of a pairwise comparison of the magnitude of possessions x' of a some other individual (as in Kapteyn [1985, references therein]). Alternatively, the weighting function $g(x, x')$ can be though as a matching function, prescribing the likelihood of meeting other individuals from various social strata. The social welfare is not affected by the local specification if all individuals have an equal chance to meet other individuals from either tail of the distribution, i.e. the weighting function $g(x, x')$ is constant. Yet, if the reference group is different for each individual (for example, if individuals are more likely to compare their position with other individuals of similar characteristics), the societal welfare will depend not only on the “global” distribution, but also on how each individual constructs her own “local” reference group.

“Goods” vs. “Bads”

Another important feature of the present model is that it implicitly assumes that an individual is endowed with an independently formed knowledge that an object in question is a “good”, not a “bad”. Strictly speaking, the mechanism presented in this paper is a mechanism of evaluation, not of preference. It provides an evaluation of an *absolute* value of marginal utility, but is silent about the *sign* of marginal utility, as this requires an independent judgement of whether an object is a “good” or a “bad”. To know what is “good” and what is “bad” one needs an extra tool - an ability to build causal links such as “Higher consumption means higher probability of survival”. An individual may acquire such causal links through own trial-and-error experience, or by observing what other people have - or what they do not have. Despite learning processes are outside of the scope of the present paper, I would like to speculate on how social inference skills may enable an individual to figure out whether more is better or worse.

Notice first that in a relatively stable environment, observing what other people have through the filter of their physical or social survival, an individual may figure out what is “good” and what is “bad”. In Samuelson [2004], higher consumption can be “good” in one environment but may be “bad” in the other, and observing others’ consumption through a filter of *physical* survival allows one to figure out whether more is better or worse in a relatively stable environment. Similarly, observing other people's possessions through the filter of their *social* survival, an individual may figure out what is "good" and what is "bad". Humans tend to behave as if they think that high-status individuals are better endowed - if not with physical goods, then with information. Even Adam Smith [1759] noticed the role of society and the rich in particular, in defining one's preferences. "To one who was to live alone in a desolate island it might be a matter of doubt, perhaps, whether a palace [...] would contribute most to his happiness and enjoyment. [...] He does not even imagine that they [the rich and the great] are really happier than other people: but he imagines that they possess more means of happiness".

Interestingly, in a private ownership economy, relative abundance of "unowned" goods may signal to a sophisticated individual which object is a "good", and which one is a "bad". Take two adjacent berry bushes in a public park of a big city, one full of ripe berries, the other almost fruitless, bearing traces of recent harvesting. An old enough child would be able to infer which berries are "bad" (such as poisonous yew berries), and which ones are "good" from the relative abundance of the berries. Plenty of empty bottles on a sidewalk and hardly any golden rings indicate which object is rubbish and which one is not. And relative abundance may reveal the state of environment - as hundreds of coins on the sidewalks of Rio de Janeiro in 1980s inform an unsuspecting foreign visitor of the poor state of the country's monetary system.

Appendix B: Ordinal Comparison Tool Subject to Weber's Law

Humans may not notice the difference between two magnitudes if it is less than a certain threshold - an observation widely known in psychology as Weber's law of just noticeable differences. This psychophysical law is the psychologist's counterpart to the law of diminishing marginal utility, and it says that the minimum amount by which stimulus intensity must be changed in order to produce a noticeable variation is proportional to the current level of the stimulus (for the exposition of various formal treatments of this phenomena see, for example, Ekman [1959] and Laming [1973]).

Now consider an ordinal comparison tool which is subject to Weber's law in its original formulation, so that an outcome of a pairwise comparison (3) is be written as:

$$\mathcal{D}_{Weber}(x, x') = \begin{cases} 1 & \text{whenever} \quad x - x' > ax \\ C & \text{whenever} \quad |x - x'| < ax \\ 0 & \text{whenever} \quad x' - x > ax \end{cases} \tag{17}$$

where the threshold parameter $a \in (0, 1)$ is a constant called Weber's fraction. So that an element of magnitude x is evaluated by the expected outcome of a pairwise comparison

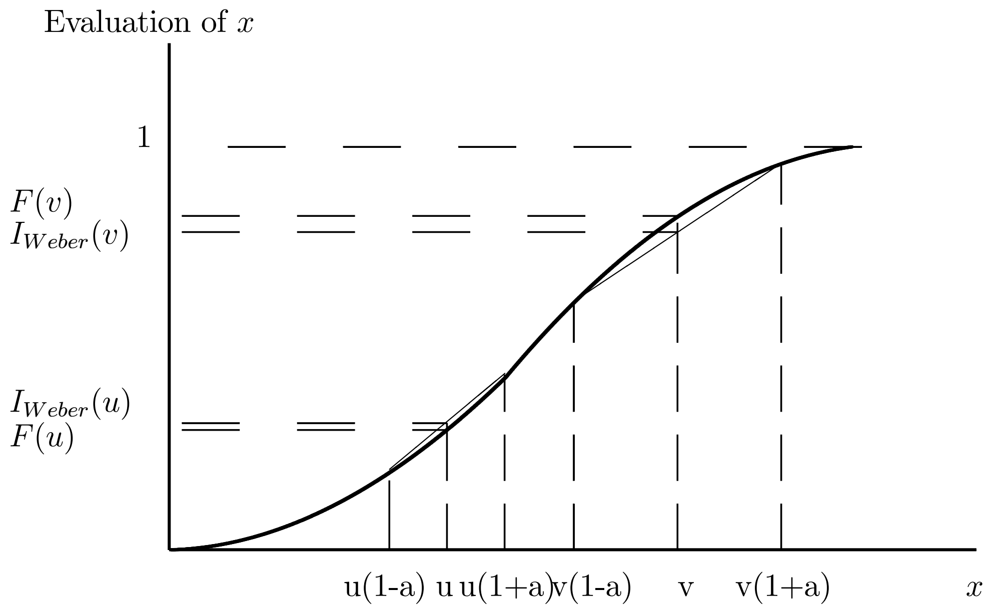


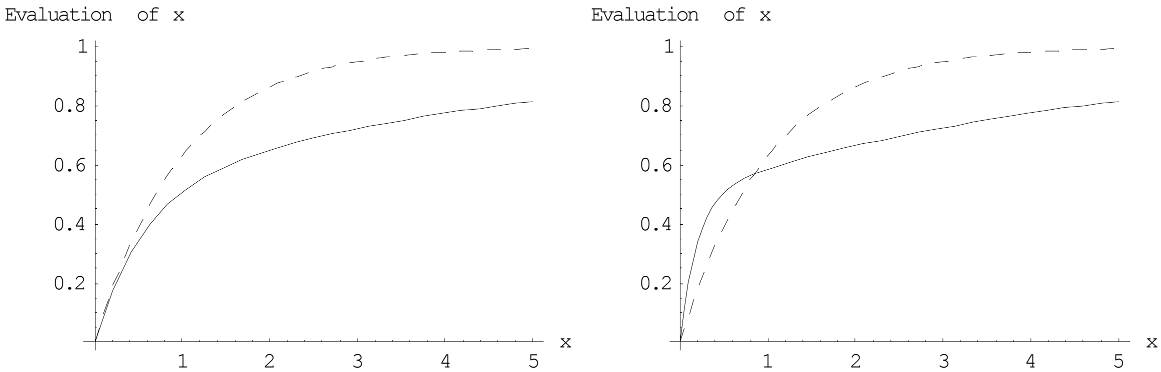
Figure 6: Imperfect ordinal comparison tool subject to Weber's Law $I_{Weber}(x)$ may "moderate" the evaluation function, with objects on convex parts to be overvalued and on concave parts to be undervalued (for $C = 0.5$).

as:

$$I_{Weber}(x) = E\mathcal{D}_{Weber}(x) = \int_X \mathcal{D}_{Weber}(x, x')f(x')dx' = (1-C)\cdot F((1-a)x)+C\cdot F((1+a)x) \tag{18}$$

In other words, an individual's evaluation can be described by an average of the ranks of an item a bit below and of an item a bit above. Obviously, the closer a is to zero, the closer is the evaluation to the true rank. Notice that $F(ax)$ and $F((1-a)x)$ are distributions of the same type as the true distribution $F(x)$. Thus, the imperfect evaluation $I_{Weber}(x)$ traces the "true" evaluation $F(x)$, so the comparative static properties hold. As Figure 6 demonstrates, for $C = 0.5$, Weber's law may lead to overvaluation of an item on a convex part of a distribution, and to undervaluation of an item on a concave part (one can see this using Jensen's inequality). In other words, Weber's law may have a moderating influence on magnitude evaluations as the evaluation function $I_{Weber}(x)$ is less curved than the "true" evaluation $F(x)$.

While Rubinstein's similarity condition is, in essence, a form of Weber's law, the two create different distortions of the evaluation function. For example, Figure 7 shows the two cognitive evaluations for an exponential distribution $F(x) = 1 - exp(-x)$ with $C = 0.5$. Notice that the two approaches are not directly comparable, so $\lambda = 1/(a-1)$ so that $F((1-a)x) = F(\lambda^{-1}x)$ here.



a) $I_{Weber}(x)$ with $a = 0.8$; b) $I_{Rubinstein}(x)$ with $\lambda = 5$.

Figure 7: The imperfect cognitive evaluations (solid curves) may look very different from the true evaluations $F(x)$ (dashed curves).

Notice that the precise formulation of the ordinal comparison tool is important as the welfare neutrality result (15) does not hold when ordinal comparisons do not satisfy the zero-sum property. Thus, if an ordinal comparison tool is subject to Weber's law, the total welfare is given by:

$$W_{Weber} = \int_X \int_X \mathcal{D}_{Weber}(x, x')dF(x)dF(x') = \int_X (1-C)\cdot F((1-a)x) + C\cdot F((1+a)x))dF(x) \tag{19}$$

The above expression depends on Weber's fraction (threshold parameter) a, and, the higher is a, the greater is the deviation from welfare-neutrality, i.e. from $W = 0.5$. For example, for an exponential distribution $F(t) = 1 - \exp(-t)$ and $C = 0.5$, the value of social welfare is equal to $1 - \frac{2}{4-a^2} < 0.5$. For most distributions $F(x)$, W_{Weber} depends not only on the Weber's fraction, but also on a number of parameters of the distribution (e.g. numerical simulations show that for a lognormal distribution, W_{Weber} depends also on the variance σ^2 but not on mean μ).

While the above formulation of Weber's law does lead to a moderated evaluation function, the effect is not very pronounced. The distortion of the evaluation function becomes even more significant if one takes a general form of Weber's law:

$$\mathcal{D}_{Weber}^{general}(x, x') = \begin{cases} 1 & \text{whenever} \quad x - x' > ax + b \\ C & \text{whenever} \quad |x - x'| < ax + b \\ 0 & \text{whenever} \quad x' - x > ax + b \end{cases} \tag{20}$$

For expositional simplicity, set $C = 0.5$. When the support of the distribution is finite, i.e. $X = [x_{min}, x_{max}] \subseteq [0, \infty)$, one has to allow for treatment of extreme values. In this case, an element of the magnitude x is evaluated by the expected outcome of a pairwise comparison as:

$$I_{Weber}^{general}(x) = \begin{cases} 0.5F\left(\frac{x_{min}+b}{1-a}\right) & \text{if } \ x < \frac{x_{min}+b}{1-a} \\ 0.5[F((1-a)x-b)+F((1+a)x+b)] & \text{if } \ \frac{x_{min}+b}{1-a} < x < \frac{x_{max}-b}{1+a} \\ 0.5\left(1+F\left(\frac{x_{max}-b}{1+a}\right)\right) & \text{if } \ x > \frac{x_{max}-b}{1+a} \end{cases}$$

Weber's law in its general form may also result in the breakdown of the welfare-neutrality of distribution:

$$\begin{aligned} W_{Weber}^{general} &= 0.5\left(\left(F\left(\frac{x_{min}+b}{1-a}\right)\right)^2 + 1 - \left(F\left(\frac{x_{max}+b}{1+a}\right)\right)^2\right) \\ &+ 0.5\int_{\frac{x_{min}+b}{1-a}}^{\frac{x_{max}-b}{1+a}} [F((1-a)x-b)+F((1+a)x+b)]dF(x) \end{aligned}$$

For example, if income is distributed as a power function distribution $F(t) = t^{\lambda}, \lambda > 0$ on $[0,1]$, and, for simiplicity, let $b = 0$. Then total welfare is:

$$W_{Weber}^{general} = \frac{1}{2} + \frac{(1-a)^{\lambda} + (1+a)^{\lambda} - 2}{4(1+a)^{2\lambda}}$$

which is more than the welfare-neutral value 0.5 when $\lambda > 1$ (i.e. for higher-mean, negatively skewed distributions) and less than 0.5 when $\lambda < 1$ (i.e. for lower-mean, positively skewed distributions).