Expropriation Auctions: Predation Contests and Wallet Games *

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Abstract

A predation contest is an all-pay winner-take-all property reassignment game where each player bids to acquire the assets of all participants net of total bidding expenditures. It involves lower symmetric monotone equilibrium bidding in the ascending bid, first- and second-price auction formats than its winner-pay counterpart, the wallet game. When frictions are absent in these two types of expropriation auctions, the equilibrium bidders' payoff is identical to that in standard single-object independent private value auctions, extending the payoff equivalence result to this wider class of almost-common-value auctions.

Keywords: Conflict, arms race, expropriation, dissipation, all-pay auctions, wallet games, winner take all games, property reallocation, intra-specific competition, intra-specific predation, cannibalism.

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I Introduction

A fight to the death tends to generate fierce conflict, fueled not only by the prospect of being entirely wiped out, but also by the endogenous benefits of survival. If a firm pushes all the competitors out of the market, not only it will enjoy monopoly power, but also it will likely snap up the assets of the losing firms - whether those are capital goods or distribution networks. Importantly, the value of the losers' assets left for the winners to cannibalize is endogenous, reduced by the costs of whatever defensive and offensive activities the firms pursue to win the race for survival. So, when firms engage in (defensive) leveraged recapitalization and (offensive) leveraged buyouts, the equity left to the winning shark is lower than the "face" value of the acquired firm. Similarly, in a winner-take-all workplace investment contest where only one employee would keep the job and the rest of the group are fired, the quality of the investment output produced by the group might be lower as the members of the group will be fighting not only for their own survival but also to acquire the investment made by the fired members of the group.

Modelling such expropriative predatory contests is non-trivial because the precise value of the prizes to winning such contests is unknown - not only because the asset values are privately known by the current holders (as in a typical Bayesian game), but - importantly - because the value of the cannibalized assets could be endogenous. While incomplete-information contests tend to be modelled as all-pay auctions where the winner's prize belongs to a third party (a seller) (e.g. Krishna and Morgan, 1997), the existing models of expropriation, wars, and hostile take-overs, tend to focus on complete-information Tullock-type contests with exogenous prizes (e.g. Hirshleifer, 1995). To the best of my knowledge, incomplete-information expropriation contests with endogenous prizes have not been considered before.

This paper introduces a predation contest, which is an expropriative all-pay contest among N risk-neutral players who are endowed with assets with privately known objective values, independently and identically distributed. The winner of the deterministic contest is the one who spends the most on bidding activity, and collects whatever is left from the losers' asset endowments after they pay for their bidding expenditures. The symmetric monotone equilibrium bidding functions are derived here for an arbitrary distribution of asset values in each of the three *classic* auction formats, including ascending-price (English), second-price sealed-bid (Vickrey), and first-price sealed-bid formats. [1] In a frictionless benchmark predation contest, bidding is (weakly) more aggressive than in a *standard* independent private value (IPV) single-object winner-pay auction, and thus higher than in standard IPV all-pay counterparts.

The winner-pay counterpart of a predation contest is widely known as the wallet game,

[1] The descending-price (Dutch) auction format of a predation contest is not considered here, as it remains to be unclear how the losers' expenditures are determined in this case.

introduced by Bulow and Klemperer (1997), and which also involves expropriation of assets belonging to the contest participants. Both predation contests and wallet games thus belong to a general class of expropriation auctions, which are incomplete-information competitive property reassignment games where N players compete to keep one's own asset with privately known objective value and to acquire everyone else's assets. Klemperer (1999)'s symmetric equilibrium results for a uniform distribution of asset values are extended here to an arbitrary distribution, in the three classical formats and the descending-price (Dutch) format of the wallet game.

Importantly, in wallet games the winner is the only player who pays, and thus the net expected value of the expropriated assets is not affected by the rivals' arms race activities. In contrast, in predation contests all players pay their bids out of their assets and thus the amount left for the winner to collect is endogenous. As the net expected values of the expropriated assets (and thus the returns to arms race) in predation contests are dissipated by the losers' bidding activity, it is shown here that equilibrium bidding in predatory contests is less intense than in the corresponding wallet games, for an arbitrary distribution of values. From the loser's point of view, it does not matter whether or not she is paying her bid in predation contests, as she would get nothing anyway. However, in predation contests, her bid imposes a negative payoff externality on the winner, reducing the optimal bid.

Even though the total "face" value of all assets is purely common, each player has an independent private value for the asset which she would retain if she is a winner. Thus, it is shown here that the logic of the classical payoff equivalence theorem for independent private values extends to a wider class of almost-common-value auctions. Thus, the expected bidder's payoff and expected revenue are the same across the seven formats of expropriation auctions (i.e. in the three classic auction formats of both predation contests and wallet games and in the Dutch format of the wallet game). Moreover, in the frictionless benchmark case of expropriation auctions, the bidders' payoff is identical to that in the standard IPV single-object auctions, but the expected revenue is much higher – simply because there are more assets at stake.

The above results are derived here for a general case which accommodates a possibility of frictions involved in the asset transfers between players. Even without taking bidding expenditures into account, the value of assets acquired by the winner might be less than their original, pre-fighting, "face" values due to taxes, conveyancing and other legal fees, physical damage or loss during transportation, depreciation, loss of value due to second-hand nature of the transferred asset, shared ownership/consumption with external parties, and so on. The winner might also pay a fee to retain her own asset, or her asset might depreciate or get damaged in fighting. Such transactional asset value erosion mimics Klemperer (1999)'s model of the wallet game, and allows for asymmetries in the frictions affecting the winner's retainment of own asset and acquisition of the losers' assets, driving the wedge between private and common value components as in Bulow and Klemperer (2002). It further underscores the payoff spillovers (inspired by the fee-shifting rules of Baye, Kovenock and deVries,

2005) as the core feature of expropriation auctions.

Note that in wallet games the losers' bids are virtual (or refunded), and thus require an auctioneer who collects the bids, and determines and obtains the winner's payment. In contrast, predation contests do not have to be conducted by an auctioneer, and could occur naturally without any specific institutions. The payoff equivalence implies that in expropriation auctions players do not care what happens to their bids – whether or not they are collected by an auctioneer, or "burned" in a wasteful fight, – and thus are indifferent between participating in a predation contest or in a wallet game. However, it matters for overall welfare whether there exists a governing institution which allows to collect the bids. When bids in an expropriation auction are not collected by an auctioneer (as could happen in naturally occurring predation contests), these expended resources end up as a conflictual waste, creating a deadweight loss.

While predatory contests occur among humans, they are particularly common among other living species. Polis (1981) surveys intra-specific predation as a density-dependent population size regulator. Such predatory competition may result in (direct or indirect) cannibalism of the loser and acquisition of exclusive use of territorial resources. The "territorial wars" are also well-documented in trees, as they tend to grow in height at the expense of expansion in girth in order to gain access to sunlight,[2] and this over-topping process is stronger in more densely populated stands (see, e.g. Falster and Westoby (2003), and references therein, as well as Farrior et al. (2016) for a more advanced treatment). However, the literature on over-topping and self-thinning in trees so far has ignored a possibility that by surviving the above-the-ground race, the tallest trees not only win the arms race for sunlight, but also are likely to enjoy an additional benefit in a form of below-the-ground nutrients from the decomposing biomass of the deceased rivals (see Asaeda et al., 2005). Note that the winner's "prize" value might be "transactionally" eroded as the winning tree might have to share the mineral nutrients left by the defeated rivals with the entire community of other locally-resident plant species.

These examples highlight the breadth of potential applications of the predation contest model, which is particularly suitable for analyzing institution-free, anarchic, environments, common in nature and sometimes occurring in distressed societies.

[2] Auctions, as rank-order perfectly discriminating contests, might be particularly relevant for plant competition, as a leaf placed just above the rival's leaf will block the sunlight.

II All-Pay Expropriation Auctions = Predation Contests

Suppose $N \geq 2$ risk-neutral players are endowed with assets which have common objective values (e.g. wallets containing cash). Player $i = \{1, \ldots, N\}$ is endowed with an asset with "face" value v_i, which does not depend on player's identity, but it is privately known only by player i. Each asset's "face" value v_i is independently drawn from commonly known distribution $F(v)$ on $[\underline{v}, \bar{v}]$, with $\underline{v} \geq 0$, which is continuous and strictly increasing, with $F'(v) = f(v) > 0$ and mean $\mu = \int_{\underline{v}}^{\bar{v}} v dF(v)$.

Players participate in a winner-take-all *expropriation auction* as follows. Each player i submits a "bid" $b_i \leq v_i$. The winner of the auction is the player who submits the highest bid, ties broken randomly. If i wins, she retains a portion α of own asset (losing a $1-\alpha$ share of it as a "retainment fee"), and acquires a portion β of each of the other $N-1$ assets (losing a $1-\beta$ share of those as a "transfer/delivery fee") *minus* the bids of all other $N-1$ players, so that the winner's *gross* total value of all assets is

$$V_i(v_i, v_{-i}) = \alpha v_i + \beta \sum_{j \neq i} v_j \tag{1}$$

The general form (1) mimics Klemperer (1999)'s formulation of wallet game, where parameters α and β captured private and common values, respectively. Here, instead, these parameters capture a possibility of transactional asset value erosion, so that the values of the assets retained and acquired by the winner could be eroded by asset transfer frictions, (possibly) collected by the *external* parties.[3] Here, $0 < \beta \leq \alpha \leq 1$ reflects that the frictions for acquiring other players' assets could be stronger than for retaining one's own asset. The *benchmark* case of $\alpha = \beta = 1$ represents the frictionless transfer situation.

The losers lose their entire assets, so $U_{i:b_i \neq b_{(1)}} = 0$. The winner's payment is determined by one of the three classic auction formats, ascending-price (English) auction, second-price sealed-bid (Vickrey) auction, or first-price sealed-bid auction. The winner's value of losers' expropriated assets is determined by the type of expropriation auction. A *predation contest* is an all-pay expropriation auction where all losers pay their entire bids, and the winner obtains the expropriated assets *net* of losers' bids, so that if i wins, she obtains the *net* total value of all assets, $\alpha v_i + \beta \sum_{j \neq i} v_j - \sum_{j \neq i} b_j = V_i(v_i, v_{-i}) - \sum_{j \neq i} b_j$. This is in contrast to its winner-pay counterpart known as the *wallet game* where only the winner pays, so that if i wins, she obtains the gross total asset value $V_i(v_i, v_{-i})$.

(i) Let us start with the ascending-price (English) predation contest.

[3] For example, parameter α could represent asset depreciation, while parameter β could capture that, in intra-species tree competition, there could be other plants which join the winner in extracting the resources from the deceased biomass of the defeated rival trees.

Proposition 1. *In a symmetric equilibrium of an ascending-price predation contest with N players, the first, the i-th, and the final quits happen at the following prices:*

$$b_{(N)} = \frac{\alpha + \beta(N-1)}{N} v_{(N)} \tag{2}$$

$$b_{(i)} = \frac{\alpha + \beta(i-1)}{i} v_{(i)} - (\alpha - \beta) \sum_{j=i+1}^{N} \frac{v_{(j)}}{j(j-1)}, i \in \{2, \dots, N-1\} \tag{3}$$

$$b_{(2)} = \frac{\alpha + \beta}{2} v_{(2)} - (\alpha - \beta) \sum_{j=3}^{N} \frac{v_{(j)}}{j(j-1)} \tag{4}$$

Importantly, in the asymmetric case of $\beta < \alpha$, a player i's own asset with value v_i is more valuable to her if she wins than to some other player $j \neq i$ - simply because i gets αv_i if she wins, while j gets $\beta v_i < \alpha v_i$ if he wins. As the result, when i bids b_i, player j's net value of i's asset is $\delta_i = \beta v_i - b_i < \alpha v_i - b_i$. So given that player i bids so that she is indifferent to finding herself a winner, player j's net value δ_i of i's asset is negative. Thus, to get a non-negative payoff when he wins, j sets aside some positive amount of own consumable remainder $\alpha v_j - b_j$ to cover the negative net value of i's expropriated asset. Similarly, the player with asset value $v_{(N)}$ does not bid her entire asset value, or $b_{(N)} < \alpha v_{(N)} \leq v_{(N)}$, despite she would lose her asset anyway. This asymmetry is the key, as when $\alpha = \beta$, the net value of an expropriated asset is zero, and thus in equilibrium player i would bid up to αv_i, which is the value of her own asset to herself.

Corollary 1. *In the frictionless benchmark case of $\alpha = \beta = 1$, in an ascending-price (English) auction, all players bid $b(v) = v$, as in the standard single-object winner-pay auction.*

In equilibrium, the least-endowed player is indifferent between biding zero and any amount up to $b(\underline{v})$ and thus randomizes on $[0, b(\underline{v})]$. However, to avoid being predated upon by the least-endowed player, the player just above her bids:

$$\lim_{v \to \underline{v}} b(\underline{v}) = \frac{\alpha + \beta(N-1)}{N} \underline{v} \tag{5}$$

which holds in the three classical auction formats of predation contest.

(ii) In the second-price sealed-bid (Vickrey) predation contest, for each player i, the outcomes are determined as follows:

$$U_i(b_i, v_i, b_j, v_j) = \begin{cases} \alpha v_i - \max_{j \neq i} b_j + \sum_{j=1}^{j=N-1} (\beta v_j - b_j) & \text{if } b_i > \max_{j \neq i} b_j \quad \text{(i is a unique winner)} \\ \alpha v_i - b_i + \frac{1}{K} \sum_{j=1}^{j=N-K} (\beta v_j - b_j) & \text{if } b_i = \max_{j \neq i} b_j \quad \text{(i is tied for a winner)} \\ 0 & \text{if } b_i < \max_{j \neq i} b_j \quad \text{(i loses)} \end{cases}$$

where $K = \#\{k : b_k = b_i\}$ is the number of players who are tied for a winner. Since the winner's payoff here directly depends on the losers' bids, the standard dominance argument does not work, even in the frictionless benchmark case of $\alpha = \beta = 1$ despite there players still bid their entire values in equilibrium, exactly as in the standard IPV single-object counterpart.

Proposition 2. *In a symmetric strictly monotone equilibrium of a second-price sealed-bid (Vickrey) predation contest, the equilibrium bidding strategy is given by (5) and*

$$b(v) = \frac{\alpha + \beta}{2} v - \frac{\alpha - \beta}{2} \frac{N-2}{N} \frac{\int_{\underline{v}}^{v} t dF(t)^{\frac{N}{2}}}{F(v)^{\frac{N}{2}}} \tag{6}$$

Corollary 2. *In the frictionless benchmark case of $\alpha = \beta = 1$, in second-price sealed-bid (Vickrey) auction, all players bid $b(v) = v$, as in the standard IPV single-object winner-pay auction.*

(*iii*) In the first-price sealed-bid predation contest, for each player i, the outcomes are determined as follows:

$$U_i(b_i, v_i, b_j, v_j) = \begin{cases} \alpha v_i - b_i + \sum_{j=1}^{j=N-1}(\beta v_j - b_j) & \text{if } b_i > \max_{j \neq i} b_j \quad \text{(i is a unique winner)} \\ \alpha v_i - b_i + \frac{1}{K}\sum_{j=1}^{j=N-K}(\beta v_j - b_j) & \text{if } b_i = \max_{j \neq i} b_j \quad \text{(i is tied for a winner)} \\ 0 & \text{if } b_i < \max_{j \neq i} b_j \quad \text{(i loses)} \end{cases}$$

where $K = \#\{k : b_k = b_i\}$ is the number of players who are tied for a winner.

Proposition 3. *In a symmetric strictly monotone equilibrium of a first-price sealed-bid predation contest, the equilibrium bidding strategy is given by (5) and*

$$b(v) = \frac{N-1}{N}(\beta + \alpha(N-1)) \frac{\int_{\underline{v}}^{v} t dF(t)^N}{F(v)^N} - \alpha(N-2) \frac{\int_{\underline{v}}^{v} t dF(t)^{N-1}}{F(v)^{N-1}} \tag{7}$$

Corollary 3. *In the frictionless benchmark case of $\alpha = \beta = 1$, the bidding in first-price predation contest (1PB) is given by*

$$b_{1PB}(v) = (N-1) \frac{\int_{\underline{v}}^{v} t dF(t)^N}{F(v)^N} - (N-2) \frac{\int_{\underline{v}}^{v} t dF(t)^{N-1}}{F(v)^{N-1}} \tag{8}$$

which is higher than in the corresponding standard IPV first-price winner-pay (1WP) and all-pay (1AP) auctions:

$$b_{1PB}(v) > b_{1WP}(v) \geq b_{1AP}(v)$$

For example, in frictionless benchmark case with $N = 2$, players bid as in the standard single-object first-price winner-pay auction but with 3 players.

III Winner-Pay Expropriation Auctions = Wallet Games

Consider now the winner-pay expropriation auctions, known as wallet games. The primitives of the model are the same as described in Section II, except only the winner pays. In contrast to the predation contests, the descending-price (Dutch) auction format of wallet game remains to be strategically equivalent to the first-price sealed-bid auction format.

(*i*) In the symmetric equilibrium of an ascending-price (English) auction, Klemperer (1999)'s original results hold for an arbitrary distribution of values.

Proposition 4. *(Klemperer 1999) In a symmetric equilibrium of an ascending-price (English) wallet game with N players, the first, the i-th, and the final quits happen at the following prices:*

$$b_{(N)} = (\alpha + \beta(N-1))v_{(N)} \tag{9}$$

$$b_{(i)} = (\alpha + \beta(i-1))v_{(i)} + \beta \sum_{j=i+1}^{N} v_{(j)}, i \in \{2, \ldots, N-1\} \tag{10}$$

$$b_{(2)} = (\alpha + \beta)v_{(2)} + \beta \sum_{j=3}^{N} v_{(j)} \tag{11}$$

In the three classic auction formats as well as in the descending-price (Dutch) auction format of wallet game, the player just above the one with the lowest possible value bids N times more than in predation contests:

$$\lim_{v \to \underline{v}} b(\underline{v}) = (\alpha + \beta(N-1))\, \underline{v} \tag{12}$$

(except in both cases $b(\underline{v}) = 0$ if $\underline{v} = 0$).

(*ii*) In the second-price sealed-bid (Vickrey) auction, the winner pays the second-highest bid. The following generalizes Klemperer (1999)'s results for a uniform distribution to an arbitrary distribution $F(\cdot)$.

Proposition 5. *In a symmetric strictly monotone equilibrium of a second-price sealed-bid (Vickrey) wallet game, the equilibrium bidding strategy is given by (12) and*

$$b(v) = (\alpha + \beta)v + \beta(N-2)\frac{\int_{\underline{v}}^{v} t dF(t)}{F(v)} \tag{13}$$

(*iii*) Finally, consider the first-price sealed-bid and the descending-price (Dutch) wallet games, where the winner pays own bid. Klemperer (1999) used the revenue equivalence to derive the symmetric equilibrium bidding function in the first-price sealed-bid wallet game for a uniform distribution. The following derives the symmetric equilibrium bidding function for an arbitrary distribution.

Proposition 6. *In a symmetric strictly monotone equilibrium of a first-price sealed-bid and descending-price (Dutch) wallet games, the equilibrium bidding strategy is given by (12) and*

$$b(v) = \alpha \frac{\int_{\underline{v}}^{v} t dF(t)^{N-1}}{F(v)^{N-1}} + \beta(N-1) \frac{\int_{\underline{v}}^{v} t dF(t)}{F(v)} \tag{14}$$

(*iv*) Similarly to the finding of Krishna and Morgan (1997) that winner-pay single-object auctions entail more intense bidding than their all-pay counterparts, equilibrium bidding in wallet games is higher than in the corresponding predation contest counterparts.

Proposition 7. *In each of the three classic auction formats, the symmetric equilibrium bidding is higher in wallet games than in the corresponding predation contests, i.e that* $\Delta(v) = b(v)^{wallet} - b(v)^{predation} > 0$ *for all* $v \in [\underline{v}, \bar{v}]$ *(except* $\Delta(\underline{v}) = 0$ *if* $\underline{v} = 0$*).*

IV Expected Payoff and Expected Revenue in Expropriation Auctions

Let us turn to the welfare aspects of expropriation auctions. Here, N assets are redistributed from N players to one, so the expected "face" value of the total "pie" is

$$N\mu = \sum_{j=1}^{N} Ev_{(j,N)} \tag{15}$$

where $Ev_{(j,N)}$ denotes the expected value of j-th order statistics out of N samples. Unless expropriation is frictionless (as in the benchmark case of $\alpha = \beta = 1$), the total expected asset value EL lost to transactional erosion (or paid to the external parties) is independent of the bidding expenditures:

$$EL = (1-\alpha)Ev_{(1,N)} + (1-\beta)(N\mu - Ev_{(1,N)}) \tag{16}$$

The players (with total welfare EW) and the auctioneer divide the remaining asset "pie":

$$EW + ER = N\mu - EL = \beta N\mu + (\alpha - \beta)Ev_{(1,N)} \tag{17}$$

where the expected revenue ER combines the expected winner's payment and the expected losers' bids (and might be wasted in a "flame of conflict" when institutions are absent).

Drawing on Myerson (1981) and Klemperer (1999, Appendix A), the result below establishes payoff and revenue equivalence for expropriation auctions. What matters here is that for each player there exists an asset whose independent objective value is the player's private information, and the player will end up with that asset only if she is a winner. For payoff equivalence it does not matter that she also gets other assets if she wins, as those do not depend on her type.

Proposition 8. *Suppose the "face" value v of each of $N \geq 2$ assets is a common objective value independently drawn from a commonly known continuous and strictly increasing distribution $F(v)$ on $[\underline{v}, \bar{v}]$, with $\underline{v} \geq 0$. For each of these assets, there is a single risk-neutral player who is privately informed only about the face value of that asset. Then any asset reassignment mechanism in which (a) all N assets are always assigned to a single player who holds the asset with the highest "face" value, and (b) any player with the lowest possible value $\underline{v}$ expects zero payoff, yields*

(i) *the same expected payoff to a player privately informed about an asset of value v:*

$$EU(v) \quad = \quad \alpha \int_{\underline{v}}^{v} F(t)^{N-1} dt \qquad \textit{and} \tag{18}$$

(ii) *the same expected revenue:*

$$ER = \beta N \mu - \beta E v_{(1,N)} + \alpha E v_{(2,N)} \tag{19}$$

Klemperer (1999) pointed out that the three classic auction formats and the descending-price (Dutch) format of wallet games generate the same expected revenue, and derived it for a uniform distribution. One can verify (19) using (tedious) direct derivations in symmetric strictly monotone equilibria of the seven formats of expropriation auctions for an arbitrary distribution (see Online Appendix).

In the frictionless benchmark case (when $\alpha = \beta = 1$), the classical payoff equivalence result for independent private values extends to the general class of almost-common-value expropriatory auctions. But the expected revenue is higher, as in the English and Vickrey frictionless benchmark predation contests players bid their value as in the standard winner-pay single-object counterparts (see Corollaries 1 and 2), and almost everyone pays own bid. And, compared to the standard single-object first-price all-pay auction, where all players pay their bids, the bidding war is fiercer in benchmark predation contest (see Corollary 3). So relatively to the standard single-object auctions, benchmark expropriation auctions raise much higher revenue - but this is simply because there are more assets at stake. Here each player "defends" a single asset, which she retains only if she wins, when she also acquires $N-1$ assets (though bidding in predation contests dissipates values of these acquired assets).

Corollary 4. *In the frictionless benchmark case of $\alpha = \beta = 1$, the player's expected payoff in an expropriation auction is identical to that in the standard single-object IPV auctions:*

$$EU(v) \quad = \quad \int_{\underline{v}}^{v} F(t)^{N-1} dt \tag{20}$$

but the expected revenue is higher:

$$ER = N\mu - E v_{(1)} + E v_{(2)} > E v_{(2)} \tag{21}$$

However, in expropriation auctions, transactional frictions are important, as for $\alpha < 1$, the player's expected payoff EU and the players' total welfare EW are lower than in the corresponding standard auctions. The transactional value loss EL could even exceed the expected revenue ER (for example, when $\alpha = \beta \leq \frac{1}{2}$), underscoring the potentially non-negligible roles of value erosion and the external parties in expropriatory conflicts.

V Discussion

Expropriation is a common phenomenon in the living world. Importantly, the above results on bidding behavior are derived assuming individual rationality *once players enter an expropriation auction.* However, it is not individually rational to do so voluntarily (just like players would not voluntarily participate in a typical Prisoners' Dilemma game). This is because in every expropriation auction, the player's expected payoff is lower than the value of her asset v (as $EU(v) = \alpha \int_{\underline{v}}^{v} F(t)^{N-1} dt < \alpha v F(v)^{N-1} \leq v$). But in the absence of property rights (and their protection), a holder of desirable resources might become a pray against their will. Faced with unfettered expropriation, one would try to fight the rivals off, as otherwise one would lose own assets with certainty (see Kornienko, 2019).

Due to involuntary nature of participation in expropriation auctions, these mechanisms should not be seen as a way to raise revenue. Instead, they highlight the welfare aspects of institution-free, anarchic environments. When bidding expenditures are not collected by anyone, expropriation auctions generate the greatest loss to the society. In contrast, whoever is capable of appropriating the bids as revenues (for example, arms dealers) could amass a significant wealth, becoming the true winner of expropriation.

VI Proofs

VI.1 Proof of Proposition 1

In the symmetric equilibrium, players quit when they are indifferent to finding themselves a winner. The first quit happens where all bidders are tied at the lowest value:

$$b_{(N)} = \alpha v_{(N)} + \beta(N-1)v_{(N)} - (N-1)b_{(N)} \Rightarrow b_{(N)} = \beta v_{(N)} + \frac{\alpha - \beta}{N} v_{(N)}$$

Thus the net value $\Delta_{(N)}$ which the winner gets from the lowest value player is

$$\Delta_{(N)} \;\; = \;\; \beta v_{(N)} - b_{(N)} = -(\alpha - \beta)\frac{v_{(N)}}{N} \leq 0$$

The next quit will be at

$$\begin{aligned} b_{(N-1)} &= \alpha v_{(N-1)} + \beta(N-2)v_{(N-1)} - (N-2)b_{(N-1)} + \Delta_{(N)} \Rightarrow \\ b_{(N-1)} &= \beta v_{(N-1)} + (\alpha-\beta)\left(\frac{v_{(N-1)}}{N-1} - \frac{1}{N-1}\frac{v_{(N)}}{N}\right) \end{aligned}$$

Since $v_{(N-1)} > v_{(N)}$, the net value that the winner gets from this player is non-positive:

$$\Delta_{(N-1)} = \beta v_{(N-1)} - b_{(N-1)} = -(\alpha-\beta)\left(\frac{v_{(N-1)}}{N-1} - \frac{1}{N-1}\frac{v_{(N)}}{N}\right) \leq 0$$

The next quit will be at

$$\begin{aligned} b_{(N-2)} &= \beta v_{(N-2)} + (\alpha-\beta)\frac{v_{(N-2)}}{N-2} + \frac{\Delta_{(N-1)} + \Delta_{(N)}}{N-2} = \\ &= \beta v_{(N-2)} + (\alpha-\beta)\left[\frac{v_{(N-2)}}{N-2} - \frac{1}{N-2}\frac{v_{(N-1)}}{N-1} - \frac{1}{N-1}\frac{v_{(N)}}{N}\right] \end{aligned}$$

And so on. The penultimate quit happens at the price

$$b_{(3)} = \beta v_{(3)} + (\alpha-\beta)\left[\frac{v_{(3)}}{3} - \frac{1}{3}\frac{v_{(4)}}{4} - \frac{1}{4}\frac{v_{(5)}}{5} - \ldots - \frac{1}{N-1}\frac{v_{(N)}}{N}\right]$$

The final quit, and thus the price paid by the winner, is at the price

$$b_{(2)} = \beta v_{(2)} + (\alpha-\beta)\left[\frac{v_{(2)}}{2} - \frac{1}{2}\frac{v_{(3)}}{3} - \frac{1}{3}\frac{v_{(4)}}{4} - \ldots - \frac{1}{N-1}\frac{v_{(N)}}{N}\right]$$

Overall, the i-th quit by the player with value $v_{(i)}$ is at the price:

$$b_{(i)} = \frac{\alpha + \beta(i-1)}{i} v_{(i)} - (\alpha-\beta)\sum_{j=i+1}^{N} \frac{v_{(j)}}{j(j-1)}, i \in \{2, \ldots, N-1\}$$

E.g., the final quit by the player with value $v_{(2)}$ is at the price (4). □

VI.2 Proof of Proposition 2

Suppose players simultaneously choose their bids according to symmetric strictly increasing and differentiable function $b = b(v)$. Continuity of the distribution $F(\cdot)$ ensures no ties in the equilibrium. Since the winner pays the second-highest bid, her expected payment is the expected highest bid among the other $N-1$ players conditional on all of them bidding less (and, given the symmetric strategy, that all of them have lower values v). Thus, player chooses value $\tilde{v}$ to maximize:

$$EU(v, \tilde{v}) = \left(\alpha v - \frac{\int_{\underline{v}}^{\tilde{v}} b(t) dF(t)^{N-1}}{F(\tilde{v})^{N-1}} + \beta(N-1)\frac{\int_{\underline{v}}^{\tilde{v}} t dF(t)}{F(\tilde{v})} - (N-1)\frac{\int_{\underline{v}}^{\tilde{v}} b(t) dF(t)}{F(\tilde{v})}\right) F(\tilde{v})^{N-1}$$

where the first term is the winner's post-transfer value of her own asset, the second term is the expected winner's payment, the third and final terms are the total expected post-transfer values of losers' assets and their total expected bids, both conditional on the losers having lower asset values. First-order condition:

$$\frac{d\left[\int_{\underline{v}}^{\tilde{v}} b(t)dF(t)^{N-1} + (N-1)F(\tilde{v})^{N-2}\int_{\underline{v}}^{\tilde{v}} b(t)dF(t)\right]}{d\tilde{v}} = \alpha(N-1)vF(\tilde{v})^{N-2}f(\tilde{v}) + \frac{d\left[\beta(N-1)F(\tilde{v})^{N-2}\int_{\underline{v}}^{\tilde{v}} tdF(t)\right]}{d\tilde{v}}$$

In the symmetric equilibrium it must be that $\tilde{v} = v$. Integrating both sides, and after manipulations:

$$\int_{\underline{v}}^{v} b(t)dF(t)^{N-1} + (N-1)F(v)^{N-2}\int_{\underline{v}}^{v} b(t)dF(t) = \alpha\int_{\underline{v}}^{v} tdF(t)^{N-1} + \beta(N-1)F(v)^{N-2}\int_{\underline{v}}^{v} tdF(t) + Const_1$$

Differentiating both sides, and after manipulations:

$$\frac{d\left[F(v)^{\frac{N-2}{2}}\int_{\underline{v}}^{v} b(t)dF(t)\right]}{dv} = \frac{\alpha-\beta}{2}vF(v)^{\frac{N-2}{2}}f(v) + \beta\frac{d\left[F(v)^{\frac{N-2}{2}}\int_{\underline{v}}^{v} tdF(t)\right]}{dv}$$

Integrating both sides:

$$F(v)^{\frac{N-2}{2}}\int_{\underline{v}}^{v} b(t)dF(t) = \frac{\alpha-\beta}{N}\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}} + F(v)^{\frac{N-2}{2}}\int_{\underline{v}}^{v} \beta tdF(t) + Const_2$$

Clearly, $Const_2 = 0$. After manipulations:

$$\int_{\underline{v}}^{v} (b(t)-\beta t)dF(t) = \frac{\alpha-\beta}{N}F(v)^{-\frac{N-2}{2}}\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}}$$

Differentiating both sides, and after manipulations:

$$(b(v)-\beta v)f(v) = \frac{\alpha-\beta}{2}vf(v) - \frac{\alpha-\beta}{N}\frac{N-2}{2}F(v)^{-\frac{N}{2}}f(v)\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}}$$

After manipulations, one gets (6). □

VI.3 Proof of Proposition 3

Conditional on all other players following equilibrium strategy $b(\cdot)$, a player chooses value $\tilde{v}$ to maximize

$$EU(v,\tilde{v}) = \left(\alpha v - b(\tilde{v}) + (N-1)\beta\frac{\int_{\underline{v}}^{\tilde{v}} tdF(t)}{F(\tilde{v})} - (N-1)\frac{\int_{\underline{v}}^{\tilde{v}} b(t)dF(t)}{F(\tilde{v})}\right)F(\tilde{v})^{N-1}$$

First-order condition:

$$\alpha v \frac{dF(\tilde{v})^{N-1}}{d\tilde{v}} + \beta(N-1)F(\tilde{v})^{N-2}\tilde{v}f(\tilde{v}) + \beta(N-1)\frac{dF(\tilde{v})^{N-2}}{d\tilde{v}}\int_{\underline{v}}^{\tilde{v}} t dF(t) -$$

$$- \left[\frac{d\left(b(\tilde{v})F(\tilde{v})^{N-1}\right)}{d\tilde{v}} + \frac{d\left((N-1)F(\tilde{v})^{N-2}\int_{\underline{v}}^{\tilde{v}} b(t)dF(t)\right)}{d\tilde{v}}\right] = 0$$

In equilibrium, $v = \tilde{v}$. After manipulations:

$$\frac{d\left(b(v)F(v)^{N-1}\right)}{dv} + \frac{d\left((N-1)F(v)^{N-2}\int_{\underline{v}}^{v} b(t)dF(t)\right)}{dv} =$$

$$= (\alpha+\beta)v\frac{dF(v)^{N-1}}{dv} + \beta(N-1)\frac{dF(v)^{N-2}}{dv}\int_{\underline{v}}^{v} t dF(t)$$

Integrating both sides, and after integrating by parts and manipulations:

$$b(v)F(v)^{N-1} + (N-1)F(v)^{N-2}\int_{\underline{v}}^{v} b(t)dF(t) =$$

$$= \alpha\int_{\underline{v}}^{v} t dF(t)^{N-1} + \beta(N-1)F(v)^{N-2}\int_{\underline{v}}^{v} t dF(t) + Const_1$$

Clearly, $Const_1 = 0$. After manipulations:

$$\frac{d\left[F(v)^{N-1}\int_{\underline{v}}^{v} b(t)dF(t)\right]}{dv} = \alpha\frac{dF(v)}{dv}\int_{\underline{v}}^{v} t dF(t)^{N-1} + \beta\frac{dF(v)^{N-1}}{dv}\int_{\underline{v}}^{v} t dF(t)$$

Integrating both sides:

$$F(v)^{N-1}\int_{\underline{v}}^{v} b(t)dF(t) = \alpha\int_{\underline{v}}^{v} dF(t)\int_{\underline{v}}^{t}\omega dF(\omega)^{N-1} + \beta\int_{\underline{v}}^{v} dF(t)^{N-1}\int_{\underline{v}}^{t}\omega dF(\omega) + Const_2$$

Again, $Const_2 = 0$. After manipulations:

$$\int_{\underline{v}}^{v} b(t)dF(t) = \alpha F(v)^{-(N-1)}\int_{\underline{v}}^{v} dF(t)\int_{\underline{v}}^{t}\omega dF(\omega)^{N-1} + \beta F(v)^{-(N-1)}\int_{\underline{v}}^{v} dF(t)^{N-1}\int_{\underline{v}}^{t}\omega dF(\omega)$$

Differentiating both sides:

$$\begin{aligned} b(v)f(v) \;\; = \;\; & -\alpha(N-1)F(v)^{-N}f(v)\int_{\underline{v}}^{v} dF(t)\int_{\underline{v}}^{t}\omega dF(\omega)^{N-1} + \alpha F(v)^{-(N-1)}f(v)\int_{\underline{v}}^{v} t dF(t)^{N-1} - \\ & -\beta(N-1)F(v)^{-N}f(v)\int_{\underline{v}}^{v} dF(t)^{N-1}\int_{\underline{v}}^{t}\omega dF(\omega) + \beta(N-1)F(v)^{-(N-1)+N-2}f(v)\int_{\underline{v}}^{v} t dF(t) \end{aligned}$$

After integrating by parts and manipulations, one gets (7). □

VI.4 Proof of Corollary 3

Rewrite (8) as $b(v)_{1PB} = b_{1WP,N}(v) + (N-1)\left[b_{1WP,N+1}(v) - b_{1WP,N}(v)\right]$, where $b_{1WP,M}(v)$ is the symmetric equilibrium bidding function in first-price winner-pay auction with M players. Since $b_{1WP,M}(v)$ increases in M, $b_{1PB}(v) > b_{1WP,N}(v)$ and, thus, $b_{1PB}(v) > b_{1AP,N}(v)$. $\square$

VI.5 Proof of Proposition 5

Given the symmetric monotone strategy, player i chooses value $\tilde{v}$ to maximize:

$$EU(v,\tilde{v}) \quad = \quad \left(\alpha v + \beta(n-1)\frac{\int_{\underline{v}}^{\tilde{v}} t dF(t)}{F(\tilde{v})} - \hat{b}(\tilde{v})\right) F(\tilde{v})^{n-1}$$

First-order condition:

$$\alpha(N-1)vF(\tilde{v})^{N-2}f(\tilde{v}) + \beta(N-1)(N-2)F(\tilde{v})^{N-3}f(\tilde{v})\int_{\underline{v}}^{\tilde{v}} t dF(t) + \beta(N-1)\tilde{v}F(\tilde{v})^{n-2}f(\tilde{v}) -$$
$$-(N-1)b(\tilde{v})F(\tilde{v})^{N-2}f(\tilde{v}) = 0$$

In equilibrium, $v = \tilde{v}$. After manipulations, one gets (13). $\square$

VI.6 Proof of Proposition 6

Given the symmetric monotone strategy, player i chooses value $\tilde{v}$ to maximize:

$$EU(v,\tilde{v}) \quad = \quad \left(\alpha v + \beta(N-1)\frac{\int_{\underline{v}}^{\tilde{v}} t dF(t)}{F(\tilde{v})} - b(\tilde{v})\right) F(\tilde{v})^{N-1}$$

First-order condition:

$$\alpha v\frac{dF(\tilde{v})^{N-1}}{d\tilde{v}} + \beta(N-1)F(\tilde{v})^{N-2}\tilde{v}\frac{dF(\tilde{v})}{d\tilde{v}} + \beta(N-1)\frac{dF(\tilde{v})^{N-2}}{d\tilde{v}}\int_{\underline{v}}^{\tilde{v}} t dF(t) - \frac{d\left[b(\tilde{v})F(\tilde{v})^{N-1}\right]}{d\tilde{v}} = 0$$

In the equilibrium $\tilde{v} = v$. After manipulations:

$$\frac{d\left[b(v)F(v)^{N-1}\right]}{dv} \quad = \quad (\alpha+\beta)v\frac{dF(v)^{N-1}}{dv} + \beta(N-1)\frac{dF(v)^{N-2}}{dv}\int_{\underline{v}}^{v} t dF(t)$$

Integrating both sides, and after integrating by parts and manipulations:

$$b(v)F(v)^{N-1} \quad = \quad \alpha\int_{\underline{v}}^{v} t dF(t)^{N-1} + \beta(N-1)F(v)^{N-2}\int_{\underline{v}}^{v} t dF(t) + Const$$

Clearly, $Const = 0$. After manipulations, one gets (14). $\square$

VI.7 Proof of Proposition 7

(i) In the ascending-price (English) auctions, the first quit in the wallet game happens at price $b^{wallet}_{(N)} = (\alpha + (N-1)\beta)v_{(N)}$ which is N times higher than $b^{predation}_{(N)} = \frac{\alpha+(N-1)\beta}{N}v_{(N)}$ in the predation contest. The difference in the subsequent quits (3) and (10) (up to the final quit of player $i = 2$) is

$$\begin{aligned}\Delta(v) &= \left[(\alpha + \beta(i-1))v_{(i)} + \beta\sum_{j=i+1}^{N} v_{(j)}\right] - \left[\frac{\alpha + \beta(i-1)}{i}v_{(i)} - (\alpha - \beta)\sum_{j=i+1}^{N}\frac{v_{(j)}}{j(j-1)}\right] = \\ &= (\alpha + \beta(i-1))\frac{i-1}{i}v_{(i)} + \sum_{j=i+1}^{N}\left(\beta + \frac{\alpha - \beta}{j(j-1)}\right)v_{(j)} > 0\end{aligned}$$

(ii) In the second-price sealed-bid (Vickrey) auctions, the difference in the bids (6) and (13) is

$$\begin{aligned}\Delta(v) &= \left[(\alpha + \beta)v + \beta(N-2)\frac{\int_{\underline{v}}^{v} tdF(t)}{F(v)}\right] - \left[\frac{\alpha + \beta}{2}v - \frac{\alpha - \beta}{2}\frac{N-2}{N}\frac{\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}}}{F(v)^{\frac{N}{2}}}\right] = \\ &= \frac{\alpha + \beta}{2}v + \beta(N-2)\frac{\int_{\underline{v}}^{v} tdF(t)}{F(v)} + \frac{\alpha - \beta}{2}\frac{N-2}{N}\frac{\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}}}{F(v)^{\frac{N}{2}}} > 0\end{aligned}$$

(iii) In the first-price sealed-bid auctions, the difference in the bids (7) and (14) is

$$\begin{aligned}\Delta(v) &= \left[\alpha\frac{\int_{\underline{v}}^{v} tdF(t)^{N-1}}{F(v)^{N-1}} + \beta(N-1)\frac{\int_{\underline{v}}^{v} tdF(t)}{F(v)}\right] - \\ &\quad - \left[\frac{N-1}{N}(\beta + \alpha(N-1))\frac{\int_{\underline{v}}^{v} tdF(t)^{N}}{F(v)^{N}} - \alpha(N-2)\frac{\int_{\underline{v}}^{v} tdF(t)^{N-1}}{F(v)^{N-1}}\right]\end{aligned}$$

Define a conditional distribution $\breve{F}(t) = \frac{F(t)}{F(v)}$, $t \in [\underline{v}, v]$, with mean $\breve{\mu} = \frac{\int_{\underline{v}}^{v} tdF(t)}{F(v)}$, the expected first-order statistic out of M samples $E\breve{v}_{(1,Z)} = \frac{\int_{\underline{v}}^{v} tdF(t)^{M}}{F(v)^{M}}$, and so on. After manipulations:

$$\Delta(v) = \alpha(N-1)E\breve{v}_{(1,N-1)} + \beta(N-1)\breve{\mu} - \frac{N-1}{N}(\beta + \alpha(N-1))E\breve{v}_{(1,N)}$$

Using (15) and (22), get

$$\Delta(v) = \frac{(\alpha + \beta)(N-1)}{N}E\breve{v}_{(2,N)} + \frac{\beta(N-1)}{N}\sum_{i=3}^{N} E\breve{v}_{(i,N)} > 0 \quad \square$$

VI.8 Proof of Proposition 8

(i) Consider a property reassignment mechanism where the auctioneer gathers players' reports $r_1, \ldots, r_n$, collects all N assets from all players, and reassigns these N assets to one of the players with probabilities $p_i(r_1, \ldots, r_n), i = 1, \ldots, N$, and acquires payments $c_i(r_1, \ldots, r_n), i = 1, \ldots, N$ from each player.[4] Let player i's type v_i be defined as the privately known "face" value v_i of the asset that she holds. Given the additive form (1) of i's gross total asset value $V_i(v_i, v_{-i})$, it is a linear transformation of the variables in equation (2.7) of Myerson (1981), so one can extend his arguments to expropriation auctions. Assuming all other players $-i$ report their values v_{-i} truthfully, the expected utility of player i with type v_i from reporting r_i is

$$\begin{aligned} EU_i(r_i|v_i) &= \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} \left(V_i(v_i, v_{-i})p_i(r_i, v_{-i}) - c_i(r_i, v_{-i})\right) dF_{-i}(v_{-i}) = \\ &= \alpha \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} v_i p_i(r_i, v_{-i}) dF_{-i}(v_{-i}) + \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} \left(\beta \sum_{-i} v_{-i} p_i(r_i, v_{-i}) - c_i(r_i, v_{-i})\right) dF_{-i}(v_{-i}) \end{aligned}$$

If instead player i's type were r_i which she reported truthfully, her expected utility would be

$$\begin{aligned} EU_i(r_i|r_i) &= \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} \left(V_i(r_i, v_{-i})p_i(r_i, v_{-i}) - c_i(r_i, v_{-i})\right) dF_{-i}(v_{-i}) = \\ &= \alpha \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} r_i p_i(r_i, v_{-i}) dF_{-i}(v_{-i}) + \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} \left(\beta \sum_{-i} v_{-i} p_i(r_i, v_{-i}) - c_i(r_i, v_{-i})\right) dF_{-i}(v_{-i}) \end{aligned}$$

Thus,

$$EU_i(r_i|v_i) = EU_i(r_i|r_i) + \alpha \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} (v_i - r_i) p_i(r_i, v_{-i}) dF_{-i}(v_{-i}) = EU_i(r_i|r_i) + \alpha (v_i - r_i) P_i(r_i)$$

where $P(r_i) = \int_{\underline{v}}^{\bar{v}} ... \int_{\underline{v}}^{\bar{v}} p_i(r_i, v_{-i}) dF_{-i}(v_{-i})$. Incentive compatibility requires that

$$EU_i(v_i|v_i) \geq EU_i(r_i|v_i) = EU_i(r_i|r_i) + \alpha(v_i - r_i)P_i(r_i)$$

Following the classical mechanism design arguments, one obtains that the expected payoff $EU_i(v_i) \equiv EU_i(v_i|v_i)$ of player i reporting truthfully her type v_i is given by

$$EU_i(v_i) = EU_i(\underline{v}) + \alpha \int_{\underline{v}}^{v_i} P_i(t) dt$$

As assets' "face" values are independent and privately known by their original holders, and the payoff of the player with the lowest possible value is always zero (i.e. $EU(\underline{v}) = 0$), and

[4]For example, $c_{i,WG}(r_i, r_{-i}) = B_w(r_i, r_{-i})$ in wallet games and $c_{i,PC}(r_i, r_{-i}) = B_w(r_i, r_{-i}) + \sum_{j \neq i} b(r_j)$ in predation contests, where $B_w(r_i, r_{-i})$ is the winner's payment as determined by the auction format, and $b(r_j)$ is the symmetric equilibrium bidding function in that format.

the expropriation auction allocates all net assets only to the player who is endowed with the highest privately known "face" value (so that $P_i(v) = F(v)^{N-1}$), and dropping subscripts, one obtains equation (18).

(*ii*) Integrating (18) by parts, and using a well-known formula

$$(N-1)Ev_{(1,N)} + Ev_{(2,N)} = NEx_{(1,N-1)} \tag{22}$$

one gets:

$$EW = \alpha N \int_{\underline{v}}^{\bar{v}} dF(v) \int_{\underline{v}}^{v} F(t)^{N-1} dt = \alpha \left(Ev_{(1,N)} - Ev_{(2,N)} \right) \tag{23}$$

Given (17) and (23), one gets (19). □

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A Online Appendix: Direct Derivations of the Expected Revenue

The expected revenue in each of the seven expropriation auctions is directly derived below, verifying the expected revenue equivalence result (19) in Proposition 8, as well as the player's expected payoff (18) is verified for the easiest cases of first-price sealed-bid auction formats.

Using integration by parts, one can simplify the following frequently appearing integral:

$$\begin{aligned}\int_{\underline{v}}^{v} F(t)^K dF(t)^L \int_{\underline{v}}^{t} \omega dF(\omega)^M &= \int_{\underline{v}}^{v} tF(t)^{K+M} dF(t)^L - \int_{\underline{v}}^{v} F(t)^K dF(t)^L \int_{\underline{v}}^{t} F(\omega)^M d\omega = \\ &= \frac{L}{K+L+M} \int_{\underline{v}}^{v} tdF(t)^{K+M+L} - \\ &\quad - \frac{L}{K+L}\left[\int_{\underline{v}}^{v} tdF(t)^{K+L+M} - F(v)^{K+L} \int_{\underline{v}}^{v} tdF(t)^M dt\right] = \\ &= \frac{L}{K+L}\left[F(v)^{K+L} \int_{\underline{v}}^{v} tdF(t)^M - \frac{M}{K+L+M} \int_{\underline{v}}^{v} tdF(t)^{K+M+L}\right] \end{aligned} \tag{24}$$

A.1 Ascending-Price (English) Predation Contest

This is an all-pay auction, where the winner pays the runner-up's bid. Given (2), (3), (4), and using (15), the expected revenue is:

$$\begin{aligned} ER &= 2Eb_{(2)} + \sum_{i=3}^{N-1} Eb_{(i)} + Eb_{(N)} = 2\left[\frac{\alpha+\beta}{2} Ev_{(2)} - (\alpha-\beta)\sum_{i=3}^{N} \frac{Ev_{(i)}}{i(i-1)}\right] + \\ &\quad + \beta \sum_{i=3}^{N-1} Ev_{(i)} + (\alpha-\beta)\sum_{i=3}^{N-1}\left[\frac{Ev_{(i)}}{i} - \sum_{j=i+1}^{N} \frac{Ev_{(j)}}{j(j-1)}\right] + \beta Ev_{(N)} + (\alpha-\beta)\frac{Ev_{(N)}}{N} = \\ &= (\alpha+\beta)Ev_{(2)} + \beta\sum_{i=3}^{N} Ev_{(i)} + (\alpha-\beta)\sum_{i=3}^{N}\frac{Ev_{(i)}}{i} - 2(\alpha-\beta)\sum_{i=3}^{N}\frac{Ev_{(i)}}{i(i-1)} - (\alpha-\beta)\sum_{i=3}^{N-1}\sum_{j=i+1}^{N}\frac{Ev_{(j)}}{j(j-1)} = \\ &= (\alpha+\beta)Ev_{(2)} + \beta\left(N\mu - Ev_{(1)} - Ev_{(2)}\right) + (\alpha-\beta)\sum_{i=3}^{N} Ev_{(i)}\frac{i-3}{i(i-1)} - \\ &\quad -(\alpha-\beta)\left(\frac{Ev_{(N)}}{N(N-1)} \times (N-3) + \frac{Ev_{(N-1)}}{(N-1)(N-2)} \times (N-4) + \ldots + \frac{Ev_{(4)}}{4(4-1)} \times 1\right) = \\ &= \alpha Ev_{(2)} + \beta N\mu - \beta Ev_{(1)} + (\alpha-\beta)\left[\sum_{i=4}^{N} Ev_{(i)}\frac{i-3}{i(i-1)} + Ev_{(3)}\frac{3-3}{3(3-1)}\right] - (\alpha-\beta)\sum_{i=4}^{N} Ev_{(i)}\frac{i-3}{i(i-1)} = \\ &= \beta N\mu - \beta Ev_{(1)} + \alpha Ev_{(2)} \end{aligned}$$

A.2 Second-Price Sealed-Bid (Vickrey) Predation Contest

This is a second-price all-pay auction, so all players pay their bids as given by (6), except the winner (who has the highest value) pays the second-highest bid, and gets their bid refunded. Thus, the expected revenue is

$$ER = N\int_{\underline{v}}^{\bar{v}} b(v)dF(v) - Eb_{(1)} + Eb_{(2)}$$

Step 1: calculate the expected equilibrium bid, using (24):

$$\begin{aligned}
\int_{\underline{v}}^{\bar{v}} b(v)dF(v) &= \frac{\alpha+\beta}{2}\int_{\underline{v}}^{\bar{v}} vdF(v) - \frac{\alpha-\beta}{N}\frac{N-2}{2}\int_{\underline{v}}^{\bar{v}} F(v)^{-\frac{N}{2}}dF(v)\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}} = \\
&= \frac{\alpha+\beta}{2}\mu - \frac{\alpha-\beta}{N}\frac{N-2}{2}\left[\frac{N}{N-2}\int_{\underline{v}}^{\bar{v}} vdF(v) - \frac{2}{N-2}\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}}\right] = \\
&= \frac{\alpha+\beta}{2}\mu - \frac{\alpha-\beta}{2}\mu + \frac{\alpha-\beta}{N}\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}} = \beta\mu + \frac{\alpha-\beta}{N}\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}}
\end{aligned}$$

Step 2: using (24), calculate the expected highest bid:

$$\begin{aligned}
Eb_{(1)} &= \int_{\underline{v}}^{\bar{v}} b(v)dF(v)^N = \frac{\alpha+\beta}{2}\int_{\underline{v}}^{\bar{v}} vdF(v)^N - \frac{\alpha-\beta}{N}\frac{N-2}{2}\int_{\underline{v}}^{\bar{v}} F(v)^{-\frac{N}{2}}dF(v)^N\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}} = \\
&= \frac{\alpha+\beta}{2}Ev_{(1)} - \frac{\alpha-\beta}{N}\frac{N-2}{2}\left[-\int_{\underline{v}}^{\bar{v}} tdF(t)^N + 2\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}}\right] = \\
&= \frac{\alpha+\beta}{2}Ev_{(1)} + \frac{\alpha-\beta}{N}\frac{N-2}{2}Ev_{(1)} - (N-2)\frac{\alpha-\beta}{N}\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}} = \\
&= \left(\alpha - \frac{\alpha-\beta}{N}\right)Ev_{(1)} - (N-2)\frac{\alpha-\beta}{N}\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}}
\end{aligned}$$

Step 3: using (22) and (24), calculate the expected second-highest bid:

$$
\begin{aligned}
Eb_{(2)} &= \int_{\underline{v}}^{\bar{v}} b(v)N(N-1)(1-F(v))F(v)^{N-2}dF(v) = \\
&= \frac{\alpha+\beta}{2}\int_{\underline{v}}^{\bar{v}} vN(N-1)(1-F(v))F(v)^{N-2}dF(v) - \\
&- \frac{\alpha-\beta}{N}\frac{N-2}{2}\int_{\underline{v}}^{\bar{v}} F(v)^{-\frac{N}{2}}N(N-1)(1-F(v))F(v)^{N-2}dF(v)\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}} = \\
&= \frac{\alpha+\beta}{2}Ev_{(2)} - \tfrac{(\alpha-\beta)(N-1)(N-2)}{2}\left[\int_{\underline{v}}^{\bar{v}} F(v)^{\frac{N}{2}-2}dF(v)\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}} - \int_{\underline{v}}^{\bar{v}} F(v)^{\frac{N}{2}-1}dF(v)\int_{\underline{v}}^{v} tdF(t)^{\frac{N}{2}}\right] = \\
&= \frac{\alpha+\beta}{2}Ev_{(2)} - \frac{\alpha-\beta}{2}(N-1)(N-2)\left[\frac{2}{N-2}\int_{\underline{v}}^{\bar{v}} vdF(v)^{\frac{N}{2}} - \frac{N}{(N-1)(N-2)}\int_{\underline{v}}^{\bar{v}} vdF(v)^{N-1} + \right. \\
&\quad \left. + \frac{1}{N}\int_{\underline{v}}^{\bar{v}} vdF(v)^{N} - \frac{2}{N}\int_{\underline{v}}^{\bar{v}} vdF(v)^{\frac{N}{2}}\right] = \\
&= \frac{\alpha+\beta}{2}Ev_{(2)} - \frac{\alpha-\beta}{2}(N-1)(N-2)\left[-\frac{NEv_{1,N-1}}{(N-1)(N-2)} + \frac{Ev_{(1)}}{N} + \frac{4}{N(N-2)}\int_{\underline{v}}^{\bar{v}} vdF(v)^{\frac{N}{2}}\right] = \\
&= \frac{\alpha+\beta}{2}Ev_{(2)} - \frac{\alpha-\beta}{2}(N-1)(N-2)\left[-\frac{(N-1)Ev_{(1)}+Ev_{(2)}}{(N-1)(N-2)} + \frac{Ev_{(1)}}{N} + \frac{4}{N(N-2)}\int_{\underline{v}}^{\bar{v}} vdF(v)^{\frac{N}{2}}\right] = \\
&= \beta Ev_{(2)} + (\alpha-\beta)\frac{N-1}{N}Ev_{(1)} - (\alpha-\beta)\frac{2(N-1)}{N}\int_{\underline{v}}^{\bar{v}} vdF(v)^{\frac{N}{2}}
\end{aligned}
$$

Step 4: combine all components:

$$
\begin{aligned}
ER &= N\int_{\underline{v}}^{\bar{v}} b(v)dF(v) - Eb_{(1)} + Eb_{(2)} = \\
&= \left[\beta N\mu + (\alpha-\beta)\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}}\right] - \left[\left(\alpha - \frac{\alpha-\beta}{N}\right)Ev_{(1)} - (\alpha-\beta)\frac{N-2}{N}\int_{\underline{v}}^{\bar{v}} tdF(t)^{\frac{N}{2}}\right] + \\
&+ \left[\beta Ev_{(2)} + (\alpha-\beta)\frac{N-1}{N}Ev_{(1)} - (\alpha-\beta)\frac{2(N-1)}{N}\int_{\underline{v}}^{\bar{v}} vdF(v)^{\frac{N}{2}}\right] = \\
&= \beta N\mu - \beta Ev_{(1)} + \alpha Ev_{(2)}
\end{aligned}
$$

A.3 First-Price Sealed-Bid Predation Contest

This is an all-pay auction, where everyone pays own bid as given by (7). Using (22) and (24), derive the expected revenue:

$$
\begin{aligned}
ER &= N\int_{\underline{v}}^{\bar{v}} b(v)dF(v) = \\
&= (N-1)(\alpha(N-1)+\beta)\int_{\underline{v}}^{\bar{v}} F(v)^{-N}dF(v)\int_{\underline{v}}^{v} tdF(t)^N - \\
&\quad -\alpha N(N-2)\int_{\underline{v}}^{\bar{v}} F(v)^{-(N-1)}dF(v)\int_{\underline{v}}^{v} tdF(t)^{N-1} = \\
&= (N-1)(\alpha(N-1)+\beta)\left[-\frac{N}{1-N}\int_{\underline{v}}^{\bar{v}} tdF(v) + \frac{1}{1-N}\int_{\underline{v}}^{\bar{v}} tdF(t)^N\right] - \\
&\quad -\alpha N(N-2)\left[-\frac{N-1}{2-N}\int_{\underline{v}}^{\bar{v}} vdF(v) + \frac{1}{2-N}\int_{\underline{v}}^{\bar{v}} vdF(v)^{N-1}\right] = \\
&= (N-1)(\alpha(N-1)+\beta)\left[\frac{N}{N-1}\mu - \frac{1}{N-1}Ev_{(1)}\right] - \alpha N(N-2)\left[\frac{N-1}{N-2}\mu - \frac{1}{N-2}Ev_{(1,N-1)}\right] = \\
&= \beta N\mu - (\alpha(N-1)+\beta)Ev_{(1)} + \alpha\left((N-1)Ev_{(1)} + Ev_{(2)}\right) = \\
&= \beta N\mu - \beta Ev_{(1)} + \alpha Ev_{(2)}
\end{aligned}
$$

Substitute the equilibrium bidding function (7) into player's expected payoff function:

$$
\begin{aligned}
EU(v) &= \alpha vF(v)^{N-1} + \beta(N-1)F(v)^{N-2}\int_{\underline{v}}^{v} tdF(t) - \\
&\quad -b(v)F(v)^{N-1} - (N-1)F(v)^{N-2}\int_{\underline{v}}^{v} b(t)dF(t)
\end{aligned}
$$

Using (24), after some manipulations, get

$$
\begin{aligned}
\int_{\underline{v}}^{v} b(t)dF(t) &= \frac{N-1}{N}(\alpha(N-1)+\beta)\int_{\underline{v}}^{v} F(t)^{-N}dF(t)\int_{\underline{v}}^{t}\omega dF(\omega)^N - \\
&\quad -\alpha(N-2)\int_{\underline{v}}^{v} F(t)^{-(N-1)}dF(t)\int_{\underline{v}}^{t}\omega dF(\omega)^{N-1} = \\
&= (\alpha(N-1)+\beta)\int_{\underline{v}}^{v} tdF(t) - \frac{\alpha(N-1)+\beta}{N}\frac{\int_{\underline{v}}^{v} tdF(t)^N}{F(t)^{N-1}} + \\
&\quad +\alpha\frac{\int_{\underline{v}}^{v} tdF(t)^{N-1}}{F(t)^{N-2}} - \alpha(N-1)\int_{\underline{v}}^{v} tdF(t) = \\
&= \beta\int_{\underline{v}}^{v} tdF(t) - \frac{\alpha(N-1)+\beta}{N}\frac{\int_{\underline{v}}^{v} tdF(t)^N}{F(t)^{N-1}} + \alpha\frac{\int_{\underline{v}}^{v} tdF(t)^{N-1}}{F(t)^{N-2}}
\end{aligned}
$$

Thus, the equilibrium bidder's payoff is:

$$
\begin{aligned}
EU(v) &= \alpha v F(v)^{N-1} + \beta(N-1)F(v)^{N-2}\int_{\underline{v}}^{v} t dF(t) - \\
&- \left[\frac{N-1}{N}(\alpha(N-1)+\beta)\frac{\int_{\underline{v}}^{v} t dF(t)^N}{F(v)} - \alpha(N-2)\int_{\underline{v}}^{v} t dF(t)^{N-1}\right] - \\
&-(N-1)F(v)^{N-2}\left[\beta\int_{\underline{v}}^{v} t dF(t) - \frac{\alpha(N-1)+\beta}{N}\frac{\int_{\underline{v}}^{v} t dF(t)^N}{F(t)^{N-1}} + \alpha\frac{\int_{\underline{v}}^{v} t dF(t)^{N-1}}{F(t)^{N-2}}\right] = \\
&= \alpha v F(v)^{N-1} - \alpha\int_{\underline{v}}^{v} t dF(t)^{N-1} = \alpha\int_{\underline{v}}^{v} F(t)^{N-1} dt
\end{aligned}
$$

A.4 Ascending-Price (English) Wallet Game

This a second-price winner-pay auction, so the expected revenue is the expected price paid by the winner, i.e. $Eb_{(2)}$. Using (15), rewrite the price paid by the winner in (11) as

$$
\begin{aligned}
ER &= \beta\sum_{j=3}^{N} Ev_{(j)} + (\alpha+\beta)Ev_{(2)} = \beta(N\mu - Ev_{(1)} - Ev_{(2)} + (\alpha+\beta)Ev_{(2)} = \\
&= \beta N\mu - \beta Ev_{(1)} + \alpha Ev_{(2)}
\end{aligned}
$$

A.5 Second-Price Sealed-Bid (Vickrey) Wallet Game

This is a second-price winner-pay auction, so the expected revenue is the expected value of the second-order statistics of the bidding function (13). Using (22) and (24), get:

$$
\begin{aligned}
ER &= \int_{\underline{v}}^{\bar{v}} b(v)N(N-1)(1-F(v))F(v)^{N-2}dF(v) = \\
&= (\alpha+\beta)Ev_{(2,N)} + \beta N(N-1)(N-2)\int_{\underline{v}}^{\bar{v}}(1-F(v))F(v)^{N-3}dF(v)\int_{\underline{v}}^{v} tdF(t) = \\
&= (\alpha+\beta)Ev_{(2,N)} + \beta N(N-1)\int_{\underline{v}}^{\bar{v}} dF(v)^{N-2}\int_{\underline{v}}^{v} tdF(t) - \beta N(N-2)\int_{\underline{v}}^{\bar{v}} dF(v)^{N-1}\int_{\underline{v}}^{v} tdF(t) = \\
&= (\alpha+\beta)Ev_{(2,N)} + \beta N(N-1)\left[\int_{\underline{v}}^{\bar{v}} vdF(v) - \frac{1}{N-1}\int_{\underline{v}}^{\bar{v}} vdF(v)^{N-1}\right] - \\
&\quad -\beta N(N-2)\left[\int_{\underline{v}}^{\bar{v}} vdF(v) - \frac{1}{N}\int_{\underline{v}}^{\bar{v}} dF(v)^{N}\right] = \\
&= (\alpha+\beta)Ev_{(2,N)} + \beta N(N-1)\mu - \beta N Ev_{(1,N-1)} - \beta N(N-2)\mu + \beta(N-2)Ev_{(1,N)} = \\
&= \beta N\mu + (\alpha+\beta)Ev_{(2,N)} - \beta\big((N-1)Ev_{(1,N)} + Ev_{(2,N)}\big) + \beta(N-2)Ev_{(1,N)} = \\
&= \beta N\mu - \beta Ev_{(1)} + \alpha Ev_{(2)}
\end{aligned}
$$

A.6 First-Price Sealed-Bid and Descending-Price (Dutch) Wallet Games

This is a first-price winner-pay auction, so the expected revenue is the expected value of the first-order statistics of the bidding function (14). Using (22) and (24), get:

$$
\begin{aligned}
ER &= \int_{\underline{v}}^{\bar{v}} b(v)dF(v)^N = \alpha\int_{\underline{v}}^{\bar{v}} F(v)^{-(N-1)}dF(v)^N\int_{\underline{v}}^{v} tdF(t)^{N-1} + \beta(N-1)\int_{\underline{v}}^{\bar{v}} F(v)^{-1}dF(v)^N\int_{\underline{v}}^{v} tdF(t) = \\
&= \alpha\left[N\int_{\underline{v}}^{\bar{v}} vdF(v)^{N-1} - (N-1)\int_{\underline{v}}^{\bar{v}} vdF(v)^N\right] + \beta(N-1)\left[\frac{N}{N-1}\int_{\underline{v}}^{\bar{v}} vdF(v) - \frac{1}{N-1}\int_{\underline{v}}^{\bar{v}} vdF(v)^N\right] \\
&= \alpha N Ev_{(1,N-1)} - \alpha(N-1)Ev_{(1,N)} + \beta N\mu - \beta Ev_{(1,N)} = \\
&= \beta N\mu - (\beta + \alpha(N-1))Ev_{(1,N)} + \alpha((N-1)Ev_{(1,N)} + Ev_{(2,N)}) = \\
&= \beta N\mu - \beta Ev_{(1)} + \alpha Ev_{(2)}
\end{aligned}
$$

Substitute the equilibrium bidding function (14) into the player's payoff function:

$$
\begin{aligned}
EU(v) &= \alpha vF(v)^{N-1} + \beta(N-1)F(v)^{N-2}\int_{\underline{v}}^{v} tdF(t) - F(\tilde{v})^{N-1}\left[\alpha\frac{\int_{\underline{v}}^{v} tdF(t)^{N-1}}{F(v)^{N-1}} + \beta(N-1)\frac{\int_{\underline{v}}^{v} tdF(t)}{F(v)}\right] = \\
&= \alpha vF(v)^{N-1} - \alpha\int_{\underline{v}}^{v} tdF(t)^{N-1} = \alpha\int_{\underline{v}}^{v} F(t)^{N-1}dt
\end{aligned}
$$