Local status and prospect theory

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Abstract

People are sometimes risk-averse in gains but risk-loving in losses. Such behavior and other anomalies underlying prospect theory arise from a model of local status maximization in which consumers compare their wealth with other consumers of similar wealth. This social explanation shares key features with the psychological explanation offered by Kahneman and Tversky.

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1 Introduction

Rather than being consistently risk-averse, people are sometimes risk-averse in gains but risk-loving in losses (Markowitz, 1952; Kahneman and Tversky, 1979). In their formulation of prospect theory, Kahneman and Tversky explain this anomaly by arguing that individuals tend to perceive changes rather than absolute values and have diminishing marginal sensitivity to changes. The utility function is therefore steepest in the region closest to current wealth where marginal sensitivity to change is greatest and flattens in either direction as the change in wealth becomes larger. Since small changes in either direction have a disproportionate impact on utility, consumers are more attracted to a certain small gain than the chance of a larger gain and are also more repelled by a certain small loss than the chance of a larger loss.

We show that a social explanation based on local status maximization shares key similarities with Kahneman and Tversky's psychological explanation. Global status models identify utility with the person's rank in the total distribution of wealth or consumption (Frank, 1985; Robson, 1992).[1] If the distribution of wealth is single-peaked the cumulative density function shifts from convexity to concavity at the mode, implying the modal person is risk averse in gains and risk loving in losses. We show that a much stronger result holds in a local status model. For any distribution of wealth everyone is risk averse in gains and risk loving in losses if they are sufficiently concerned with their status among others with similar wealth levels. Since utility for each person is determined by their rank within a reference group distributed most densely around their

[1] Concern for status can arise from signaling games (Veblen, 1899), competition for limited resources such as mates (Cole, Mailath, and Postlewaite, 1992), and other factors. We take a concern for status as given.

own wealth, people are most concerned with the large changes in status that occur in the immediate vicinity of current wealth. Just as in Kahneman and Tversky's explanation, small changes have a disproportionate impact on utility so people are risk-averse in gains but risk-loving in losses.

A second behavioral regularity addressed by prospect theory is the tendency to turn down any fair gamble with an equal chance of winning or losing (Markowitz, 1952; Kahneman and Tversky, 1979). To reconcile such "loss aversion" with risk-loving behavior in losses, prospect theory assumes a kink in the utility function at current wealth. Local status maximization does not produce this same kink but still offers insight into the phenomenon. If the wealth distribution is unimodal then the utility function's inflection point is between modal and current wealth, implying consumers with above-modal wealth have a locally concave utility function and will display some loss aversion. More generally, if the reference group is sufficiently concentrated around current wealth then for any wealth distribution the gains to anyone from a symmetric gamble are either negative or arbitrarily small.

Local status may also offer some insight into the popularity of insurance and lottery tickets. The tendency for consumers to simultaneously purchase both led Friedman and Savage (1947) to suggest a utility function that was first concave and then convex, the opposite of prospect theory. Kahneman and Tversky argue that such behavior arises not from the shape of the utility function but because people overweight both the small probability of winning a lottery and the small probability of events covered by insurance.[2] In a status model the shape of the utility function may still be relevant in explaining why some consumers purchase insurance while others purchase lottery tickets. If consumers

[2] This approach distinguishes prospect theory from Markowitz's (1952) theory which explains all three anomalies by a more complicated utility function centered around current wealth. Cumulative prospect theory (Tversky and Kahneman, 1992; Prelec, 1998) relies even more on the weighting function rather than the shape of the utility function to explain observed anomalies.

are interested in global status and the wealth distribution is single-peaked, consumers with below-modal wealth are in the convex region of the utility function and will be interested in some unfair lotteries, while consumers with above-modal wealth are in the concave region and will be interested in some unfair insurance policies. Local status implies similar though stronger results if the assumption that each person's reference group is centered around their own wealth is dropped. If the reference group is sufficiently concentrated around a higher wealth level the consumer will purchase a lottery ticket with a payoff exceeding that wealth level. Likewise, if the reference group is sufficiently concentrated around a lower wealth level the consumer will purchase insurance which prevents wealth from falling below that level.[3]

Regarding the general connection between status and nonstandard risk behavior, introducing status into the utility function clearly allows for complicated utility functions, especially if people care about both absolute wealth and status. For instance, Robson (1992) shows that the simultaneous purchase of insurance and lottery tickets is possible if utility is concave in wealth and convex in status. And Coelho and McClure (1998) show that the more complicated Markowitz (1952) utility function is possible if people are interested in absolute wealth, wealth relative to peers, and wealth of one's peer group relative to non-peers.[4] Even if status alone matters, any non-decreasing utility function is possible depending on the exact distribution of wealth (Kornienko, 2000). By concentrating on limiting behavior as status becomes more localized, we are able to make much more specific predictions. We show that concern for local status produces a particular set of behavior that, to varying degrees, is consistent with each of the principle anomalies underlying prospect theory.

[3] In this context "framing" (Kahneman and Tversky, 1979) can be interpreted as affecting what reference group a person uses.

[4] They follow Duesenberry (1948) in measuring status by wealth relative to a mean rather than position in the wealth distribution so their results are not directly comparable with ours.

2 The Model

We consider a simple one period model in which individuals choose whether or not to take a gamble. Since there is only one period, consumption and wealth are synonymous. Wealth y is distributed according to the distribution function $F(\cdot)$ with density $f(\cdot)$ and support on the real numbers.[5] We assume that the reference group for each individual is different and in particular that individuals are more likely to compare their position with other individuals of similar wealth levels. The distribution of individuals in the reference group for individual with wealth level y_o is a function of both the overall distribution of wealth and of y_o. Let $g(y)$ represent the probability density that a given individual with wealth y will be in the reference group for an individual with initial wealth y_o. Let $G(\cdot)$ be the corresponding cumulative density function. We assume $g(\cdot)$ is symmetric around mean and mode μ_g, has standard deviation σ_g, and has support on the real numbers. For simplicity we assume both $f(\cdot)$ and $g(\cdot)$ are continuous. Combining the wealth and reference group distribution functions, the utility of wealth $\widehat{y}$ for an individual with wealth y_o is defined as

$$U(\widehat{y}) = K \int_{-\infty}^{\widehat{y}} f(y)g(y)dy \qquad (1)$$

where $K = (\int f(y)g(y)dy)^{-1}$ is a normalizing constant. Utility is simply the fraction of people in one's reference group who have lower wealth.[6]

This utility function clearly allows for a wide range of possible shapes and could change between convexity and concavity an unlimited number of times depending on the shapes of the wealth and reference group distributions. To make clearer predictions

[5] Negative values are included to reflect the possibility of indebtedness. Restricting wealth to be non-negative does not change the analysis.

[6] In his global status model Robson (1992) assumes utility is a convex rather than linear function of status. As long as the function is continuous, allowing for convexity or concavity does not affect any of the results except Proposition 2(i) which is overturned by sufficient convexity.

we investigate behavior as local status becomes increasingly important as measured by the standard deviation of the reference group distribution function. To avoid any strategic interactions, we assume that only one individual is offered a gamble.[7]

The following considers two decisions facing an individual. The first is to take a certain gain or a gamble offering a chance at a larger gain. The second is to take a certain loss or a gamble with a chance of a larger loss. If concern for status is sufficiently localized around current wealth then any individual chooses the certain smaller gain in the first case but the uncertain larger loss in the second case.

Proposition 1 *For any given y', y'', y_o, y^*, and y^{**} where $y' < y'' < y_o < y^* < y^{**}$ and any given $\alpha \in (0,1)$, if $\mu_g = y_o$ and σ_g is sufficiently small then (i) $\alpha U(y_o) + (1 - \alpha)U(y^{**}) < U(y^*)$ and (ii) $\alpha U(y') + (1-\alpha)U(y_o) > U(y'')$.*

Proof: (i) Suppose instead that the utility of the gamble is as high or higher than of the sure thing y^*. Then

$$\alpha \int_{-\infty}^{y_o} f(y)g(y)dy + (1-\alpha)\int_{-\infty}^{y^{**}} f(y)g(y)dy \geq \int_{-\infty}^{y^*} f(y)g(y)dy$$

which simplifies to

$$\frac{\alpha}{1-\alpha} \leq \frac{\int_{y^*}^{y^{**}} f(y)g(y)dy}{\int_{y_o}^{y^*} f(y)g(y)dy}.$$

Let $r = (y^* - y_o)/\sigma_g$. By Chebyshev's inequality and the symmetry of $g(y)$ around y_o, $1 - G(y_o + r\sigma_g) \leq 1/(2r^2)$ or $1 - G(y^*) \leq 1/(2r^2)$. Let $\overline{f}_{[a,b]}$ and $\underline{f}_{[a,b]}$ represent the respective maximum and minimum values of $f(y)$ over the range $[a,b]$. Then

$$\frac{\int_{y^*}^{y^{**}} f(y)g(y)dy}{\int_{y_o}^{y^*} f(y)g(y)dy} \leq \frac{\overline{f}_{[y^*,y^{**}]}\int_{y^*}^{y^{**}} g(y)dy}{\underline{f}_{[y_o,y^*]}\int_{y_o}^{y^*} g(y)dy}$$

[7] If more than one individual makes a choice then in equilibrium each individual must react rationally to the choices of others, potentially altering behavior (Harbaugh, 1996).

$$\leq \frac{\overline{f}_{[y^*,y^{**}]}(1-G(y^*))}{\underline{f}_{[y_o,y^*]}(G(y^*)-1/2)}$$
$$\leq \frac{\overline{f}_{[y^*,y^{**}]}}{\underline{f}_{[y_o,y^*]}(r^2-1)}$$

so

$$\frac{\alpha}{1-\alpha} \leq \frac{\overline{f}_{[y^*,y^{**}]}}{\underline{f}_{[y_o,y^*]}(r^2-1)}.$$

As σ_g approaches zero r^2 becomes arbitrarily large so this does not hold.

(ii) Suppose instead that the utility of the sure thing y'' is higher than the gamble so that

$$\int_{-\infty}^{y''} f(y)g(y)dy \geq \alpha \int_{-\infty}^{y'} f(y)g(y)dy + (1-\alpha)\int_{-\infty}^{y_o} f(y)g(y)dy$$

which simplifies to

$$\frac{\alpha}{1-\alpha} \geq \frac{\int_{y''}^{y_o} f(y)g(y)dy}{\int_{y'}^{y''} f(y)g(y)dy}.$$

By Chebyshev's inequality and the symmetry of $g(y)$ around y_o, $G(y_o - r\sigma_g) \leq 1/(2r^2)$ or $G(y'') \leq 1/(2r^2)$. Analogous to the steps in (i) above, this implies

$$\frac{\alpha}{1-\alpha} \geq \frac{\overline{f}_{[y',y'']}}{\underline{f}_{[y'',y_o]}(r^2-1)}$$

which does not hold as σ_g approaches zero. ■

The example of Figure 1 shows the impact of local status. Wealth follows a normal distribution with mean 10 and standard deviation 1 while the reference group density function is a normal distribution with mean y_o and standard deviation 1. Combining these two factors, the utility function for an individual with wealth y_o is $U(\widehat{y}) = \int^{\widehat{y}} \phi(10,1)\phi(y_o,1)dy / \int \phi(10,1)\phi(y_o,1)dy$ where $\phi(\mu,\sigma)$ is the normal distribution with mean μ and standard deviation σ. The three curves capture utility functions for individuals with wealth levels $y_o = 9$, $y_o = 10$, and $y_o = 11$ as shown from left to right. Note that under global status maximization everyone shares the same utility function $U(y) = \int^{\widehat{y}} \phi(0,1)dy$ so individuals with wealth above the inflection point tend to be risk averse and individuals with wealth below the inflection point tend to be risk loving. Under local status maximization people are more likely to compare themselves with others of similar wealth so position in the overall wealth distribution is less important. As status concerns become more localized the inflection point for each individual's utility function becomes closer and closer to y_o and individuals become more generally risk averse in gains and risk loving in losses.

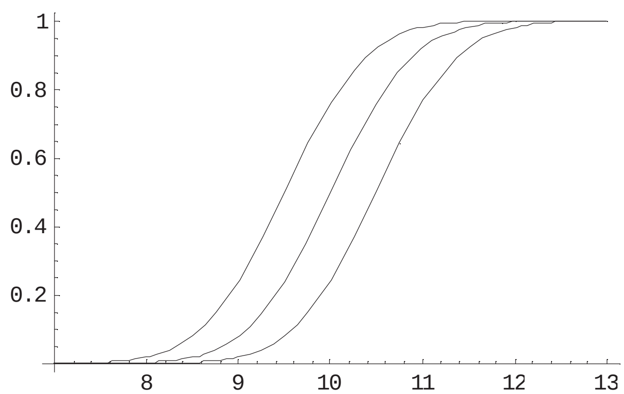

Figure 1: Utility functions with local status concern

The fact that the inflection point is not exactly at y_o is relevant for loss aversion. Prospect theory argues that marginal utility is steeper in losses than in gains in the area of y_o, implying symmetric gambles are rejected. For individuals with above-modal wealth the inflection point can be arbitrarily close to y_o but is below y_o, implying the utility function is concave at current wealth. Looking at Figure 1, this is apparent for $y_o = 11$.[8] This concavity implies local risk aversion and rejection of small symmetric gambles. The following proposition shows that individuals will avoid symmetric gambles

[8] If, unlike this example, the distribution of wealth were skewed so that the mode was below the median, then the utility function would be concave at current wealth for most consumers.

in the range where the wealth distribution is decreasing, as occurs for individuals with above-modal when the wealth distribution is unimodal. It also shows more generally that any symmetric gamble offers no better than arbitrarily small gains if the reference group density function is sufficiently concentrated around current wealth. This weaker statement holds regardless of the distribution of wealth and regardless of the individual's wealth level.[9]

Proposition 2 *If* $\mu_g = y_o$ *then (i)* $\frac{1}{2}U(y_o - x) + \frac{1}{2}U(y_o + x) < U(y_o)$ *if* $f(y)$ *is strictly decreasing on* $[y_0 - x, y_0 + x]$ *and (ii)* $\frac{1}{2}U(y_o - x) + \frac{1}{2}U(y_o + x) - U(y_o) < \varepsilon$ *for any given* $x > 0$ *and any given* $\varepsilon > 0$ *if* σ_g *is sufficiently small.*

Proof: (i) The net gain from the gamble is

$$\begin{aligned} & \frac{1}{2}U(y_0 - x) + \frac{1}{2}U(y_0 + x) - U(y_0) \\ = & \frac{K}{2}\left(\int_{-\infty}^{y_0 - x} f(y)g(y)dy + \int_{-\infty}^{y_0 + x} f(y)g(y)dy - 2\int_{-\infty}^{y_0} f(y)g(y)dy\right) \\ = & \frac{K}{2}\left(\int_{y_0}^{y_0 + x} f(y)g(y)dy - \int_{y_0 - x}^{y_0} f(y)g(y)dy\right) \\ < & \frac{K}{2}\left(\overline{f}_{[y_0, y_0 + x]}\int_{y_0}^{y_0 + x} g(y)dy - \underline{f}_{[y_0 - x, y_0]}\int_{y_0 - x}^{y_0} g(y)dy\right). \end{aligned}$$

Since $f(y)$ is continuous and decreasing on $[y_0 - x, y_0 + x]$, $\underline{f}_{[y_0 - x, y_0]} = \overline{f}_{[y_0, y_0 + x]} = f(y_0)$, and since $g(y)$ is symmetric around y_0, $\int_{y_0}^{y_0 + x} g(y)dy = \int_{y_0 - x}^{y_0} g(y)dy$. The inequality therefore reduces to $\frac{1}{2}U(y_0 - x) + \frac{1}{2}U(y_0 + x) - U(y_0) < 0$.

(ii) First note that

$$\lim_{\sigma_g \to 0} U(y_o) = \lim_{\sigma_g \to 0} \frac{\int_{-\infty}^{y_o} f(y)g(y)dy}{\int_{-\infty}^{y_o} f(y)g(y)dy + \int_{y_o}^{\infty} f(y)g(y)dy}.$$

[9] The simplest way to explain any residual aversion to symmetric gambles is a healthy skepticism by consumers that the gamble is absolutely fair. Alternatively, the utility function may include a non-status component that incorporates global risk aversion. For instance, the utility function could be $U(\widehat{y}) = K\int_{-\infty}^{\widehat{y}} f(y)c(y)dy + v(\widehat{y})$ where $v(\cdot)$ is concave. Inclusion of such a non-status component guarantees loss aversion while not affecting the other propositions if the non-status component is sufficiently small.

Let $r = \sigma_g^{-1/2}$ so by Chebyshev's inequality and the symmetry of $g(y)$, $G(y_o - r\sigma_g) \leq 1/(2r^2)$ or $G(y_o - \sigma_g^{1/2}) \leq \sigma_g/2$. Then

$$\begin{aligned}\int_{-\infty}^{y_o} f(y)g(y)dy &\leq \overline{f}_{[-\infty,y_o-r\sigma_g]}\sigma_g/2 + \overline{f}_{[y_o-r\sigma_g,y_o]}(1/2-\sigma_g/2)\\ \int_{-\infty}^{y_o} f(y)g(y)dy &\geq \underline{f}_{[-\infty,y_o-r\sigma_g]}\sigma_g/2 + \underline{f}_{[y_o-r\sigma_g,y_o]}(1/2-\sigma_g/2).\end{aligned}$$

Since $\lim_{\sigma_g\to 0} \overline{f}_{[y_o-r\sigma_g,y_o]} = f(y_o)$ therefore $\lim_{\sigma_g\to 0} \int_{-\infty}^{y_o} f(y)g(y)dy \leq f(y_o)/2$ and $\lim_{\sigma_g\to 0} \int_{-\infty}^{y_o} f(y)g(y)dy \geq f(y_o)/2$, implying $\lim_{\sigma_g\to 0} \int_{-\infty}^{y_o} f(y)g(y)dy = f(y_o)/2$. By the same logic $\lim_{\sigma_g\to 0} \int_{y_o}^{-\infty} f(y)g(y)dy = f(y_o)/2$, so $\lim_{\sigma_g\to 0} U(y_o) = (f(y_o)/2)/f(y_o)$ $= 1/2$. Since it is clear that $\lim_{\sigma_g\to 0} U(y_o - x) = 0$ and $\lim_{\sigma_g\to 0} U(y_o + x) = 1$, therefore $\lim_{\sigma_g\to 0} \left(\frac{1}{2}U(y_o - x) + \frac{1}{2}U(y_o + x) - U(y_o)\right) = 0$. ■

To explain the popularity of both insurance and lotteries, prospect theory argues that people overweight small probabilities. Status models offer a more limited explanation that may capture why some people are willing to gamble while others take insurance. As mentioned, if status concerns are global and the wealth distribution is single-peaked, people below the mode are in the convex region of the utility function and people above the mode are in the concave region, suggesting the former will be more disposed toward gambling and the latter toward insurance, though exact behavior will depend on the odds and payoffs. If the reference group function is sufficiently diffuse then local status and global status are equivalent so the same property holds in our model. As status becomes more concentrated around current wealth Proposition 1 implies neither gambling nor insurance has much appeal. Proposition 1 assumed that the reference group function was centered around current wealth which is consistent with Markowitz and with Kahneman and Tversky. The following proposition considers what happens when the reference group function is centered elsewhere. We find that when individuals compare themselves to a group with higher wealth they will gamble and when they compare themselves to a group with lower wealth they will purchase insurance.

Proposition 3 *For any given y', y'', and $y_o \in (y', y'')$ and any given $\alpha \in (0, 1)$, if σ_g is sufficiently small then (i) $\alpha U(y') + (1 - \alpha)U(y'') > U(y_o)$ if $\mu_g \in (y_o, y'')$ and (ii) $\alpha U(y') + (1 - \alpha)U(y'') < U(y_o)$ if $\mu_g \in (y', y_o)$.*

Proof: (i) Since $\mu_g \in (y_o, y'')$, $\lim_{\sigma_g \to 0} U(y') = 0$, $\lim_{\sigma_g \to 0} U(y'') = 1$, and $\lim_{\sigma_g \to 0} U(y_o)$ $= 0$, so the inequality holds. (ii) Since $\mu_g \in (y', y_o)$, $\lim_{\sigma_g \to 0} U(y') = 0$, $\lim_{\sigma_g \to 0} U(y'') =$ 1, and $\lim_{\sigma_g \to 0} U(y_o) = 1$ so the inequality holds. ■

3 Conclusion

By introducing local status into a standard status utility model this note revealed a close connection between status concerns and prospect theory. Prospect theory argues that consumers are disproportionately concerned with small gains and small losses in consumption since their sensitivity to change decreases as changes become larger. We showed that a similar effect arises in a model of local status maximization for social rather than psychological reasons. Since individuals are most likely to compare their status with others of comparable wealth levels, they are most concerned with small changes around their current wealth, and are therefore risk loving in losses and risk averse in gains. We also showed that other anomalies underlying prospect theory may reflect local status concerns. Regarding loss aversion, sufficient concern for local status implies that symmetric gambles either reduce utility or offer arbitrarily small gains. Regarding the coexistence of insurance and lotteries, consumers whose reference group is wealthier than they are will tend to buy lottery tickets, while consumers whose reference group is poorer than they are will tend to buy insurance.

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