Search and Matching - Ph.D. Training Course
Lecture 4: Search and Sorting

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Search and Sorting

- Big focus in labor: unemployment
- Less focus: "unsuitable" employment
- Examples:
  - Dentist working at a fast-food restaurant
  - Ph.D. economist working as taxi driver
- Why is this hard: observational problems (output hard to observe)
- Need more theory to understand this
- Frictions: induce mismatch (but other things do as well).

Sorting and Search Frictions: The Basics

We keep the basic elements of the framework before, but

- Each worker has a type x; distr. H_w
- Each job has a type y; distr. H_m
- The output is $f(x, y)$ [same as $V(m, w)$ with men and women]
- Matching through matching function (directed or random).
- succesful: firm gets $f(x, y) - w$ and worker gets w (risk-neutrality).
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- Some prob $s \geq 0$ that job survives to next period.
- unsuccessful: workers unemployment payoff $b \geq 0$, firms get 0.
- Potentially try next period again (discount $\delta \in [0, 1)$).

Sorting

How does sorting work now? Who get's matched with whom? Why?
Recall from frictionless matching: PAM if $f_{xy} > 0$.
Things change with frictions:

- It is not only important which partner one gets,
- But it is also important whether one gets a partner at all.
- The second part tends to favor NAM, because the highest types have most to loose and are most likely to accept lower matches if that helps them getting matched.
- Most easily explained in directed search.

Sorting in Directed Search

Sorting in Directed Search.(based on Eeckhout-Kircher ECTR. See also Shi 01, Shimer 05)

Assume bilateral meetings. (otherwise auctions, see Eeckhout-Kircher JET)

Firm y posts (w, x) combination to maximize:

$$\max_{x,w} m(\lambda(x,w))[f(x,y) - w] \text{ s.t. } n(\lambda(x,w))w = U(x).$$

$$\Leftrightarrow \quad \max_{x,\lambda} m(\lambda)f(x,y) - \lambda U(x)$$

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FOC at optimal $\lambda = \Lambda(y)$ and $x = \mu(y)$:

$$\begin{aligned} m'(\Lambda)f(\mu,y) &= U(\mu) \\ m(\Lambda)f_x(x,y) &= \Lambda U'(\mu). \end{aligned}$$

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SOC according to Hessian:

$$\begin{pmatrix} m''(\Lambda)f(\mu,y) & m'(\Lambda)f_x(\mu,y) - U'(\mu) \\ m'(\Lambda)f_x(\mu,y) - U'(\mu) & m(\Lambda)f_{xx}(\mu,y) - \Lambda U''(\mu) \end{pmatrix}$$

Can be done. Real complication: deal with possible non-differentiabilities.

Sorting and Directed Search

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The wages and matching probabilities are also easily described (diff equ)
Discuss: what happens as short side of the market gets matched for sure...

Sorting: Random Search

Sorting with Random Search:

- Downside for theory: much harder (illustrate matching bands)
- Applied upside: breaks perfect matching (feature of data)
- Canonical Model: Shimer-Smith ECTR
- Considitions for increasing matching bands (PAM):
  - f sm, f_x log-sm, f_{xy} log-sm,... (implies f log $-sm$)
- More interesting for applied work:
  - Can we identify the production function from observed data?
  - Can we say whether sorting is positive, negative, etc?
  - Can we say how much value is lost from mismatch?
  - How much could the market improve (increase b, not done yet)?

Identificaiton of Sorting under Random Search

Identification of Sorting with Random Search:

Fixed search costs $c > 0$ (Atakan ECTR, Eeckhout-Kircher REStud, Gautier-Teulings)

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- Worker's type:

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- Firm's type:

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- With $s\delta = 1$: firm type cannot be identified
- With $s\delta < 1$: firm type is identified by excess payments (what workers get beyond their reservation wage Hagedorn-Law-Manovskii)

Other Identification Strategies

Other ways of identification:

- Hight and width of wage function (use picture)
  (Gautier-Teulings: mismatch costs $\approx$ unemployment costs)
- Similar types of co-workers (de Melo)
- Speed of sorting with search intensity (Lentz...)

Problematic:

- Correlation of worker and firm fixed effects
  (reason: non-monotonicity of wage function)

Different reason for mismatch: shocks or learning

Open questions about sorting:

- How to handle on-the-job search (important for wage dispersion, recently introduced by Lise-Robin, Hagedorn-Law-Manovskii, Gautier-Teulings...)
- How to handle ideosyncratic and aggregate shocks (Lise-Robin)
- To use it for sensible policy questions:
  - What is the effect of higher unemployment insurance
  - What is the effect of job protection....

Different way to think about mismatch:

- Shocks to types or learning
- Long literature going back to Waldmann...
- Short exposition based on my own work
- Message:
  - Combining search and shocks might be important
  - Small improvements on any of these can be a great dissertation
  - Keep relevance in mind
  - Keep data in mind