# Job security, asymmetric information, and wage rigidity*

Jonathan P. Thomas[†] Andy Snell[‡] Heiko Stüber[§]

June 6, 2019

**Abstract**

We consider, under both symmetric and asymmetric information, a labor market with directed search in which firms can commit to wage contracts but are constrained *not* to pay new hires less than ongoing hires. This constraint can be microfounded as a means of enhancing job security following the approach of Menzio and Moen (2010). Workers are risk averse, so there exists an incentive for firms to smooth wages over time and in the face of shocks to labor productivity. This leads to some downward rigidity in the wages of new hires and magnifies the response of unemployment and vacancies to negative shocks. We further show that the interplay with asymmetric information can substantially enhance wage rigidity and increase the responsiveness of unemployment and vacancies to productivity shocks. In an empirical exercise, we argue that downward — but not upward — real wage rigidity for new hires is apparent in Germany, and we find tentative evidence in favor of the model with asymmetric information.

*JEL Codes*: E32, J41.

*Keywords*: Labor contracts, business cycle, unemployment, equal treatment, downward rigidity, cross-contract restrictions.

*We thank Franck Malherbet, Arpad Abraham, Piero Gottardi, Mike Elsby, and Sevi Rodriguez Mora for their helpful comments; the participants at the IZA Workshop on "Wage Rigidities and the Business Cycle: Causes and Consequences"; and participants at seminars at the University of Edinburgh, the Bank of Spain, the Center for Macroeconomic Research, the University of Cologne, and the European University Institute, Florence. This work was supported by the Economic and Social Research Council [grant number ES/L009633/1] and the German Research Foundation under the priority programme 1764.

[†]Corresponding author: Jonathan P. Thomas, University of Edinburgh, 31 Buccleuch Place, Edinburgh, EH8 9JT, UK, Tel: +44 (0)131 6504515, Email: Jonathan.Thomas@ed.ac.uk.

[‡]Andy Snell, University of Edinburgh, Email: Andy.Snell@ed.ac.uk.

[§]Heiko Stüber, Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Institute for Employment Research (IAB), IZA, and Labor and Socio-Economic Research Center (LASER), Email: Heiko.Stueber@fau.de.

# 1 Introduction

The behaviour of real wages over the business cycle is critical to understanding the mechanisms that drive employment and output fluctuations. The procyclicality or otherwise of real wages was the subject of considerable debate following the publication of Keynes's *General Theory*, and it remains controversial.[1] In this paper, we develop a model that has implications for the cyclicality of real wages and for output volatility, but we emphasize *asymmetric* wage responses to different phases of the business cycle. Our model has partial "equal treatment" — the wages of new hires are constrained *not* to be below those of existing employees. This *no-undercutting* constraint binds in recessions. The implication is that if there is a reason for ongoing wages to be rigid — here, risk aversion — this will be transmitted to new hires' wages. The latter is important for employment fluctuations, as wages are allocative in our base model.

In adverse future states, because of the no-undercutting constraint, the firm will trade-off a desire to smooth the wages of workers in ongoing employment with the benefits from cutting the wage for new entrants. Treated on their own merit, the latter will receive a lower wage, but to avoid violating the constraint, the low wage will also be paid to incumbents, for whom a constant wage would be optimal from the insurance point of view. Then, the upshot is that there is some compromise and a degree of downward wage rigidity. The opposite is not true, however. In good states, there is no problem in paying a higher wage to new entrants than to incumbents, so the rigidity operates only in a downward direction.[2] Because the wage for new entrants is allocational, the downwardly rigid wage affects hiring and increases the variability of both unemployment and vacancies in response to productivity shocks.[3]

We extend the base model to incorporate asymmetric information about the state of nature (productivity) by assuming that firms are better informed, or at least that contracts cannot be conditioned on aggregate variables. In this case, we show that wages may be fully rigid downwards (to be precise: wages may fall but the rate of fall will be independent of the severity of negative shocks), thus further amplifying the variability of unemployment and vacancies. We show that it is the *interplay* between equal treatment and asymmetric information that leads to this result; without equal treatment, introducing asymmetric information has no impact on allocations. A rough intuition for the result is as follows:

[1] See Galí (2013) for a comparison of the cyclicality of real wages in the General Theory and in New Keynesian Models and, e.g., Pissarides (2009) for a discussion of more recent empirical evidence in the context of the "unemployment volatility puzzle" (Costain and Reiter (2008), Shimer (2005)).

[2] In our asymmetric information extension to the model, wages of new hires and incumbents remain linked also in upswings due to incentive compatibility constraints.

[3] Equal treatment can also lead to amplified unemployment fluctuations in competitive models, e.g., Thomas (2005) and Snell and Thomas (2010). See Gertler and Trigari (2009a) for a somewhat related mechanism within a search-matching model with staggered Nash bargaining rather than optimal contracting as employed here.

consider a bad state, say $x$, in which firms would like to cut new hire wages but cannot because they do not want to undercut incumbents — that is, we are in a region where the no undercutting constraint binds. Suppose there is also a worse state $x'$ in which wages for both incumbents and new hires are again equal but lower than in $x$, as arises in the symmetric information equilibrium. Ex post, the firm may prefer the contract associated with state $x'$ because it would lead to both lower new-hire wages *and* lower incumbent wages. Although ex ante, higher incumbent wages are better (as they offer better wage smoothing and hence reduce labor costs), ex post, with lower wages, the firm would save on wages for workers already in post. But the symmetric information equilibrium also has new hire wages that are higher than is optimal becuase of the no-undercutting constraint. Thus by switching to the $x'$ wages, the firm benefits on both counts: lower incumbent wages and a new-hire wage closer to the optimal one. This logic also applies even if wage differences are not those corresponding to the symmetric information solution. We show that the only incentive-compatible contract may involve a (large) range of shocks for which wages of incumbents and new hires — the latter are allocational — are not only equal but do not vary with the severity of the shock.

The no undercutting constraint can be microfounded as a means of enhancing job security following the logic of Menzio and Moen (2010), henceforth MM. In their paper, overlapping generations of two-period lived firms interact with infinitely lived workers in the context of a frictional labour market, as here, but where employment dynamics are driven by firm entry (each firm employs a fixed number of workers). Firms can commit to current and future wages but not to employment. In particular, they cannot commit *not* to lay off a worker. If the wage of new hires is below that of incumbents, the firm will have an incentive to replace its incumbents if it can find suitable applicants, and depending on matching probabilities, there will be a possibility that an incumbent is replaced. Workers will have a preference for a contract with no job insecurity, that is, one in which wages of future hires are never below their own wages; if this situation holds, then the firm will have no incentive to attempt to replace them. It may then be optimal for firms to offer such contracts because the ex ante costs of hiring are lower by a sufficient amount to offset having to forgo the potential benefit of a lower wage for new hires in some future states. That is, it may be optimal for a firm to use its ability to commit to wages that satisfy the no-undercutting constraint as a second-best policy, because of its inability to commit to not replacing incumbents.[4]

While this is not the main emphasis of the paper, we also consider a microfoundation for the no-undercutting constraint along the above lines, where the only cost to the worker of replacement is the immediate loss of earnings. We generalize the model to allow for firms to violate the constraint, so that worker replacement can occur, but when vacancy

[4]This type of argument was also made in Snell and Thomas (2010) in the context of a perfectly competitive labour market. MM's model, however, concerns a frictional labour market, and we follow their approach.

posting costs are not very low, the replacement risk is sufficiently high that firms optimally choose to avoid it. If there are additional costs (e.g., psychic, location costs) to the worker of replacement, then the argument is strengthened.[5]

In an empirical section, we argue that downward, but not upward, real rigidity for new hires is apparent in the German administrative IAB Employment History (BeH) panel dataset and that our model is broadly consistent with these empirical findings. We test a prediction of the model with asymmetric information that the degree of downward — but not upward — rigidity should not depend on the current shock realization, but on the predicted distribution. We find tentative evidence supporting this prediction.

In Snell et al. (2018), we also examined evidence of downward real rigidity in German data in the context of testing Snell and Thomas (2010). The latter model exhibits downward rigidity, but for a somewhat different reason than the frictional model of this paper. In Snell and Thomas (2010), equal treatment holds in *both* down- and upswings (whereas here, it holds only in downswings). Despite this symmetry, wage responses to productivity shocks are asymmetric; in upswings, the labour market clears and wages respond fully to shocks, whereas in downswings, the response to shocks is smaller — following a similar logic to that of the symmetric information version of our model presented here. In the current paper, we present new evidence of downward rigidity based on sectoral variances, and moreover, we test a prediction of the asymmetric information version of our model, mentioned above, that in downturns, wages should be responsive to the distribution of shocks but not their realization.

### 1.1 Related Literature

As mentioned above, the symmetric information version of our model is closely related to that of MM.[6] Menzio (2005) considers an asymmetric information model in which firms are informed about the current state of productivity and workers are not, and it exhibits equal treatment. It uses a bargaining model to show that firms satisfy equal treatment because workers receive, during bargaining, information about deals struck by the firm with other workers; an attempt to offer a higher wage when productivity is high, say, to

[5]Acharya and Wee (2018) argue that a significant amount of replacement hiring occurs by comparing total numbers of hires in excess of job gains at firms over time with data on quits. They also argue that evidence in Michaels et al. (2016), who focus on firms with zero net employment changes, is supportive of this point of view. In their theoretical model, firms may replace an incumbent if a higher productivity match for the position happens. Because they take a Nash bargaining approach to wage determination, replacement will occur as the firm will benefit from some of the extra match surplus, so, unlike here, the firm cannot commit to a wage policy that prevents replacement.

[6]We expand on the main differences in Section 2, but rather than firm entry being the driver of employment fluctuations, we assume a fixed number of firms operating subject to decreasing returns to scale. This allows us to develop a simple two-period model in a familiar demand/supply setting. In MM's numerical examples shocks are effectively zero probability events. In our two-period world it is straightforward to solve for equilibria with positive probability shocks.

a new hire, reveals that the firm has a positive shock and is willing to pay more and will cause other workers to re-open negotiations. In equilibrium, the firm offers a wage to all workers that is the lowest wage consistent with the observed history of the firm (there is a permanent idiosyncratic component). There are transitory shocks, but if these are not very persistent, the firm does not respond to them; the cost of responding to a positive shock involves paying all workers extra because of equal treatment, while the benefit in terms of additional hiring and retention is smaller when the shock is not expected to persist for long. By contrast, we develop and test a model of asymmetric wage rigidity using an explicit contracting framework. Nevertheless, a related logic applies across downswing states in that as the economy improves, the cost of rewarding incumbents along with new hires is greater than any benefit and, therefore, (by incentive compatibility) wages are constant across such states.

Other related work in which asymmetric information amplifies fluctuations includes Kennan (2010), who develops a model of procyclical information rents to firms: if a privately observed (to firms) component of match surplus has more dispersion when the aggregate state of the economy is better, and bargaining leads to an outcome in which firms capture the informational rent, wages are again relatively rigid, and procyclical rents to employer mean that employment fluctuations are magnified. Moen and Rosen (2011) analyze a model of moral hazard (unobservable worker effort) and competitive search and show that it introduces a counter-cyclical element to rents accruing to workers relative to a standard search-matching model, enhancing fluctuations in employment over the cycle. However, see also Guerrieri (2007) for a model in which workers have private information about match characteristics but which exhibits little amplification. Bruegemann and Moscarini (2010) derive a bound on extra employment amplification that can arise in frictional labor markets when there is acyclicality in worker *rents* (surplus relative to outside options) rather than wages per se, which is weaker than wage acyclicality when, as usual, outside options are procyclical. They argue that standard asymmetric bargaining models (where there is asymmetric information about match characteristics rather than an aggregate state) may achieve rent acyclicality but will not exceed their bound.

For the empirical results, we attempt to identify asymmetric responses of real wages to business cycle up- and downswings, in contrast to the empirical literature on wage stickiness, which typically has looked for evidence of downward real (and also nominal) rigidity by comparing empirical wage-change distributions with notional distributions, i.e., an attempt to capture how wage changes will be distributed in the absence of downward rigidities. An example is Dickens et al. (2007), who summarize results from the *International Wage Flexibility Project.* They use data from 16 OECD countries and find evidence of wage changes clustered around the expected inflation rate and fewer than the expected number of changes below that rate. This and similar evidence points to the existence of some real downward rigidity in individual wage changes in ongoing employment relationships. Our approach differs in that we concentrate on the wages of new hires (which are omitted by construction in the usual approach) and look at how wages respond to different phases of the cycle. See Basu and House (2016) for a recent survey of the literature relevant to downward nominal rigidity, which also considers how real labour costs are impacted by rigidities.

Recent evidence from a study of 15 European Union countries by Galuscak et al. (2012) suggests that new hire wages are intimately related to wage structures that already exist in the firm; moreover, this relationship is stronger in periods of labour market slack, which is a feature of the equilibrium we derive here. Galuscak et al. argue that fairness and incentive issues are important in leading to this linkage, which is consistent with evidence collected by Bewley (1999), who argue that internal equity considerations make it difficult for firms to employ new hires at a wage lower than that paid to incumbents. Gertler and Trigari (2009a) estimate the cyclicality of hiring wages in the U.S. by using Survey of Income and Program Participation data and argue that wages of new hires do not appear to be more procyclical than those of ongoing employees. Likewise, using the same data, Gertler et al. (2015) find that the composition of match quality explains the greater wage flexibility for new hires from unemployment.

An outline of the paper is as follows. In Section 2, we lay out the basic model that underlies the analysis. In Section 3, we introduce asymmetric information and show that it increases downwards rigidity. In Section 4, we test certain predictions of the model using German administrative data. Section 5 contains concluding comments.

## 2 The Model

There are two periods $t = 1, 2$, and a large number of identical firms and workers.[7] Each firm and worker lives for both periods, and the ratio of workers to firms equals $S$. We identify each firm with the entrepreneur who owns it; entrepreneurs do not supply labour. In each period, each firm operates a decreasing returns technology that produces a perishable good, with production function $f(n; x)$, where $n$ is the current number of workers employed at the firm, which we treat as a continuous variable, $x \in X$ is a productivity shock observable at the start of the period, and derivatives with respect to the first argument are $f' > 0$, $f'' < 0$, with $f(0; x) = 0$. (Hours per worker are not variable.) We assume that $x = x_0$ is fixed at $t = 1$, but at $t = 2$, $x$ is a random variable, common across firms, with finite support. Henceforth, $x$ without a 0 subscript will refer to the second period productivity shock. Each worker has a per-period utility of consumption function $v(c)$, with $v' > 0$ and $v'' < 0$. Workers cannot borrow or save, so they consume all their current income; we assume for simplicity that there is no discounting of the future by

[7]Formally, we will treat these as measures.

workers. Entrepreneurs, on the other hand, are risk-neutral, but they also have a zero discount rate (nothing depends on this, provided that discounting is symmetric). A worker who is unemployed in any period receives an income of $b$.

A firm has a wage policy $\sigma = \left(w_1, (w_{2,i})_{i=1,2}\right)$ to which it commits, where $i$ is the length of the worker's tenure and $w_{2,i}$ may be random (state contingent); so at $t = 1$, workers are offered a wage contract $(w_1, w_{2,2})$ and period-2 hires are offered $w_{2,1}$. We *impose* the no-undercutting constraint $w_{2,1}(x) \geq w_{2,2}(x)$, but consider how this can be microfounded in a later section. A worker who accepts a contract at $t = 1$ suffers exogenous separation from the firm at the end of the first period, with probability $\delta$. In this case, they will be in the same position as a worker who failed to gain employment in the first period; in the second period, such unattached workers seek work.

At the start of each period (in period 2, after $x$ is observed), search and matching occur (see Figure 1). We assume directed search (see Moen (1997), Acemoglu and Shimer (1999), and Rudanko (2009)). Briefly, an unemployed worker can apply for one job at a single firm in each period.[8] We rule out on-the-job search so that at $t = 2$, a worker cannot apply for a job if he or she is already employed. We identify the 'type' of a job with the utility $V$ a successful applicant obtains from it. The application succeeds with probability $p(\theta(V))$, where $\theta(V)$, "the expected queue length for the job," is the ratio of applicants to jobs of type $V$, that is, the inverse of labor market tightness.[9] (The determination of $\theta(V)$ is discussed below.) The function $p(\cdot)$ is assumed to be strictly decreasing, differentiable and such that $p(0) = 1$, $p(\infty) = 0$. Correspondingly, the firm fills a job of type $V$ with probability $q(\theta(V))$ where $q(\cdot)$ is strictly increasing, and satisfies $q(\theta) = p(\theta)\theta$, $q(0) = 0$ and $q(\infty) = 1$. Moreover, denoting the elasticity of $q$ with respect to $\theta$ by $\epsilon_q(\theta)$, $q(\theta)\epsilon_q(\theta)/(1-\epsilon_q(\theta))$ is assumed to be a decreasing function of $\theta$.[10]

Simultaneously with committing to a wage policy at the start of $t = 1$, firms choose how many new jobs $\overline{n}_i$ to create in period $i = 1, 2$, at a cost of $k > 0$ per job; $\overline{n}_2$ depends on the shock $x$. Unfilled jobs from the first period 'die' at the end of the period, along with filled jobs in which exogenous separation occurred (little depends on this assumption). The implication is that employment at the firm in period $i$ will increase by $q(\theta(V))\overline{n}_i$.[11]

[8] We do not consider search intensity on the worker side to be a choice variable. See e.g. Choi and Fernndez-Blanco, who consider optimal policy in a two-period directed search model with contract posting, as here, where search intensity depends on unemployment risk amongst other things.

[9] For the moment, we suppress other arguments of $\theta(\cdot)$ corresponding to the economic environment.

[10] MM, who also assume this, point out that many standard matching processes satisfy these assumptions.

[11] Our base model differs from MM in the following principal respects. First, our workers are two-period lived rather than infinitely lived (firms in MM are two-period lived), and we have a two-period horizon (we extend this to multiple periods below). Second, rather than having firms of a fixed size (number of jobs) with constant productivity per filled job and free entry of firms, we suppose that there are a fixed number of firms, each with a decreasing returns to scale technology. The supply of jobs then varies not with variations in the number of firms entering the market but with firms' choices about how many jobs (or "vacancies") to create in each period. The fixed cost per job created replaces MM's assumption of a fixed cost incurred per firm that enters.

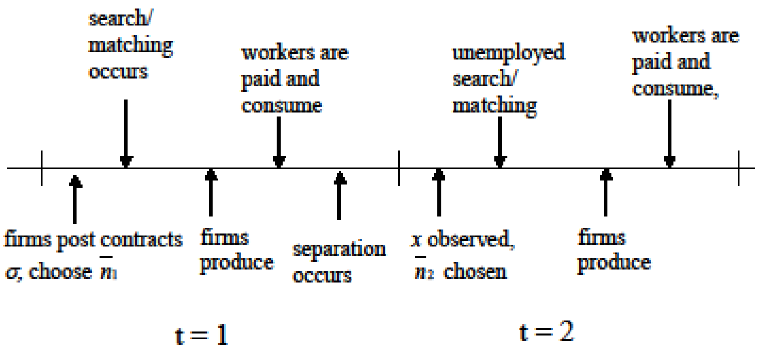


Figure 1: Timeline

Let $Z_1$ be the lifetime utility of a worker at the search stage in period 1 and $Z_2(x)$ be that of a worker in period 2 searching for work in state $x$. ($Z_1$ and $Z_2$ are the endogenous variables determining the economic environment the firm faces.) Define $Z = \left(Z_1, (Z_2\,(x))_{x\in X}\right)$. The value to a worker at $t = 1$ from being employed by a firm with wage policy $\sigma$ is then

$$V_1(\sigma; Z) := v\,(w_1) + E[\delta Z_2\,(x) + (1-\delta)v\,(w_{2,2}\,(x))] \tag{1}$$

where $E$ denotes the expectation.[12,13]

Let $U_1$ be the lifetime utility of a worker at $t = 1$ who fails to get a job:

$$U_1\,(Z) = v(b) + E\,[Z_2(x))]\,,$$

as currently, the worker receives $b$ and is able to search next period. Given $U_1$ and $Z_1$, the expected queue length for a job offering $V_1$ is assumed to satisfy:

$$\theta_1(V_1, Z_1, U_1) = \begin{cases} \theta : p(\theta)V_1 + (1-p(\theta))U_1 = Z_1, \text{ if } V_1 > Z_1 \\ 0, \text{ if } V_1 \leq Z_1 \end{cases} \tag{2}$$

The idea is that if the value of the job to a successful applicant, $V_1$, is greater than the value of search, $Z_1$, the expected queue length is driven up to the point where workers

[12]To avoid complicating the exposition, we will ignore the possibility that at the optimal period-2 wage, the firm will prefer to dismiss some of its incumbents. This situation will arise if $w_{22} > f'\left((1-\delta)\,n_1; x\right)$. In our simulations, parameters are chosen so that this scenario does not arise: We will assume throughout that positive hiring occurs in equilibrium. Given average annual turnover rates of around 30% in the U.S., for example, this assumption is not restrictive for any reasonable parametrization.

[13]Without loss of generality, we assume that $w_{22} \geq b$; otherwise, it would be in the worker's interest to quit.

are indifferent between applying for the job and searching somewhere else, and vice versa. The expected queue length for the job will be zero if the value of the job is less than (or equal to) the value of search.

For a worker at $t = 2$, the value from being employed at the wage $w_{2,1}$ is $v(w_{2,1})$, so the expected queue length for period-2 firms and workers for a job with wage $w_{2,1}$ is

$$\theta_2(w_{2,1}, Z_2) = \begin{cases} \theta : p(\theta)v\left(w_{2,1}\right) + (1 - p(\theta))v\left(b\right) = Z_2, & \text{if } v\left(w_{2,1}\right) > Z_2 \\ 0, & \text{if } v\left(w_{2,1}\right) \leq Z_2 \end{cases} \tag{3}$$

A firm's profit is

$$\begin{aligned} F\left(\sigma; \overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x\in X}; Z\right) = \left(f\left(n_1; x_0\right) - w_1 n_1 - k\overline{n}_1\right) + \\ E\left[\left(f\left((1-\delta)n_1 + n_2; x\right) - w_{2,2}(1-\delta)n_1 - w_{2,1}n_2 - k\overline{n}_2\right)\right] \end{aligned}$$

where $n_i$ is the number of new hires in period $i$ and is given by $n_i = q\left(\theta_i\right)\overline{n}_i$, $i = 1, 2$, where $\theta_i$ depends on $\sigma$, as given by $\theta_1(V_1\left(\sigma, Z\right), Z_1, U_1\left(Z\right))$ in (2) and $\theta_2(w_{2,1}, Z_2\left(x\right))$ in (3) above.

**Competitive Search Equilibrium**

We define an equilibrium as follows:

**Definition 1** *A symmetric no-undercutting equilibrium with positive hiring consists of search values* $Z = \left(Z_1, \left(Z_2\left(x\right)\right)_{x\in X}\right)$*, and a wage policy* $\sigma$ *satisfying* $w_{2,1}(x) \geq w_{2,2}\left(x\right)$, $x \in X$*, and job creation plan* $\left(\overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x\in X}\right)$ *with the following properties:*

(i) *Profit maximization*: For all $(\sigma'; \overline{n}_1', (\overline{n}_2'\left(x\right))_{x\in X})$ satisfying $w_{2,1}'(x) \geq w_{2,2}'\left(x\right)$, $x \in X$,

$$F((\sigma; \overline{n}_1, \left(\overline{n}_2\left(x\right)\right)_{x\in X}); Z) \geq F\left(\sigma'; \overline{n}_1', (\overline{n}_2'\left(x\right))_{x\in X}; Z\right);$$

and

(ii) *Consistency*: $\theta_1\left(V_1\left(\sigma, Z\right), Z_1, U_1\right) = S/\overline{n}_1$, and, for all $x$, $\theta_2(w_{2,1}, Z_2\left(x\right)) = S_2/\overline{n}_2\left(x\right)$ where $S_2 := \left(\left(1 - p\left(S/\overline{n}_1\right)\right) + \delta p\left(S/\overline{n}_1\right)\right)S$ is the number of workers (per firm) seeking work in period 2.

## 2.1 Characterization of Equilibrium Contracts

We proceed heuristically.[14] In period 2 in any state $x$, given $n_1$ and $w_1$, it can be shown that the firm must locally maximize profits plus weighted incumbent utility. In particular,

[14] The necessary conditions that follow in the text are derived formally in the Appendix.

it must maximize

$$\begin{aligned}
&f\left((1-\delta)n_1+n_2;x\right)-w_{2,2}(1-\delta)n_1-w_{2,1}n_2-k\overline{n}_2+\\
&\big(1/v'\left(w_1\right)\big)\,n_1\left((1-\delta)\,v\left(w_{2,2}\right)+\delta Z_2\left(x\right)\right), \qquad (4)
\end{aligned}$$

with respect to $\overline{n}_2, w_{2,1}, w_{2,2}, w_{2,1}\geq w_{2,2}$, where $n_2=q\left(\theta_2\left(w_{2,1},Z_2\left(x\right)\right)\right)\overline{n}_2=:\tilde{q}\left(w_{2,1},x\right)\overline{n}_2$. We write $\tilde{q}'\equiv\partial\tilde{q}/\partial w_{2,1}$. Note that the last term in (4) includes the continuation utility of an incumbent, taking into account the separation possibility and multiplied by the number of incumbents. The intuition here is that any change that affects the utility of the firm's old workers can be offset by a change in the first period wage, leaving $V_1$ unchanged (and, hence, $n_1$). Multiplying the utility change by the inverse of first-period marginal utility then converts it (for a small change) to the first-period wage savings per worker.

There are two cases to consider:

(A) If the *no-undercutting constraint* $w_{2,1}\geq w_{2,2}$ is not binding, then differentiating (4) with respect to $w_{2,2}$,

$$(1-\delta)n_1=n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\,v'\left(w_{2,2}\right)\right), \qquad (5)$$

so that $w_1=w_{2,2}$. Intuitively, the firm should stabilize the wages of the first period hires if there is no cost of doing this. In this case, also differentiating with respect to $w_{2,1}$, we obtain

$$f'\left((1-\delta)n_1+n_2;x\right)q'\overline{n}_2-w_{2,1}q'\overline{n}_2-q\overline{n}_2=0, \qquad (6)$$

and simplifying:

$$f'\left(n\right)\tilde{q}'-w_{2,1}\tilde{q}'-q=0,$$

where we write $n\equiv(1-\delta)n_1+n_2$ for total period-2 employment. Finally, differentiating with respect to $\overline{n}_2$,

$$f'\left(n\right)=w_{2,1}+k/q. \qquad (7)$$

We can combine these latter two to obtain

$$q^2\left(\tilde{q}'\right)^{-1}=k. \qquad (8)$$

Intuitively, in order to increase employment by one unit, the firm could open $1/q$ jobs at a cost of $k/q$. Alternatively a wage increase of $1/\left(\overline{n}_2\tilde{q}'\right)$, holding the number of jobs constant, accomplishes the same result by increasing the queue length and, hence, the probability that each existing job is filled, at a cost of $q\overline{n}_2\times1/\left(\overline{n}_2\tilde{q}'\right)=q/\tilde{q}'$. The two must be equal in equilibrium so that (8) follows.

In the proof of Proposition 1 it is shown that (8) can be solved to give a positively sloped locus of values for $n_2$ and $w_{2,1}$ compatible with equilibrium. This locus defines an upward-sloping "quasi-supply" curve of labor: when equilibrium $n_2$ is higher, it is more difficult to fill each job because the labor market is tighter ($\theta_2$ is lower, so $k/q(\theta_2)$ is higher); this makes wage increases, as a way to fill jobs, more attractive than creating jobs, and $w_{2,1}$ rises until the two methods cost the same. This locus is independent of the profitability of filling a job. We refer to it as the *unconstrained quasi-supply curve.* It corresponds to the solution to the first-order conditions in the case where the no-undercutting constraint $w_{2,1} \geq w_{2,2}$ is not imposed. (The two coincide in the current case because the constraint is not binding by assumption.) Combining this situation with the downward sloping (7), which is a standard labor demand equation, where the unit cost of increasing employment $k/q(\theta_2)$ (itself increasing as $n_2$ increases)[15] is added to the wage and yields a unique equilibrium for each productivity shock whenever the no-undercutting constraint does not bind.[16] As $x$ varies, only the labor demand curve shifts. Denote the solution of (7) and (8) by $\left(w_{2,1}^U(x, w_1, n_1), n_2^U(x, w_1, n_1)\right)$, where the $U-$superscript indicates that this is the solution to the FOCs in the unconstrained case.

Since in this case, $w_{2,1} \geq w_{2,2} = w_1$, we conclude that the intersection of (7) and (8) occurs at or above $w_1$.

(B) If, on the other hand, $w_{2,1} \geq w_{2,2}$ is binding at the optimum (when productivity is sufficiently low), the intersection of (7) and (8) occurs at a wage below $w_1$, but the wage can be shown to be above $w_{2,1}^U(x, w_1, n_1)$, while employment is below $n_2^U(x, w_1, n_1)$. In the proof, it is shown that $k < q^2/\tilde{q}'$. The unit cost of increasing employment through creating extra jobs, $k/q$, is lower than that through increasing wages, $q/\tilde{q}'$, so it would be cheaper to cut wages and increase jobs; however, this is not done because the wage cut has a negative externality on incumbents' wage smoothing. More intuitively, if productivity is low enough that the equilibrium hiring wage in the absence of the constraint $w_{2,1}^U$ is below $w_1$, then the no-undercutting constraint will be violated (recall that $w_{2,2}^U = w_1$). To satisfy the constraint, $w_{2,2}$ must be cut, which is costly because it reduces wage smoothing, so firms are less willing to let wages fall. Thus, below $w_1$, the equilibrium lies above the unconstrained quasi-labor-supply curve.

Consequently, taking $w_1$ as given, we can plot a *constrained quasi-supply* curve in $w_{2,1} - n_2$ space, which coincides with the unconstrained one above $w_1$, but below $w_1$, the curve lies above the unconstrained curve (it is the locus of points satisfying (30) in the Appendix). Equilibrium occurs at the intersection with the labor demand curve. As $x$ varies, the latter curve shifts. In Figure 2, a situation where the crossing point occurs below $w_1$ is illustrated.[17] The equilibrium values are at point A, rather than at the unconstrained

[15] As $n_2$ increases, we must have $p(\theta)$ increasing from $n_2 = p(\theta) S_2$, and hence, $\theta$ has fallen as $p' < 0$; thus $q(\theta)$ falls, given that $q' > 0$.

[16] The positions of these two curves depend only on $x$ and $n_1$, which implies the value of $S_2$.

[17] In simulations of the constrained quasi-supply curve, as $n_2$ falls, we find that wages eventually start to increase. The intuition is that the number of new hires falls sufficiently low such that the desire to insure incumbents dominates and the wage approaches $w_1$ as $n_2$ goes to zero.

solution. If $x$ is sufficiently high such that the intersection occurs above $w_1$, then the equilibrium will be at the unconstrained solution, $\left(w_{2,i}^U(x, w_1, n_1), n_2^U(x, w_1, n_1)\right)$. The proposition summarizes the discussion.

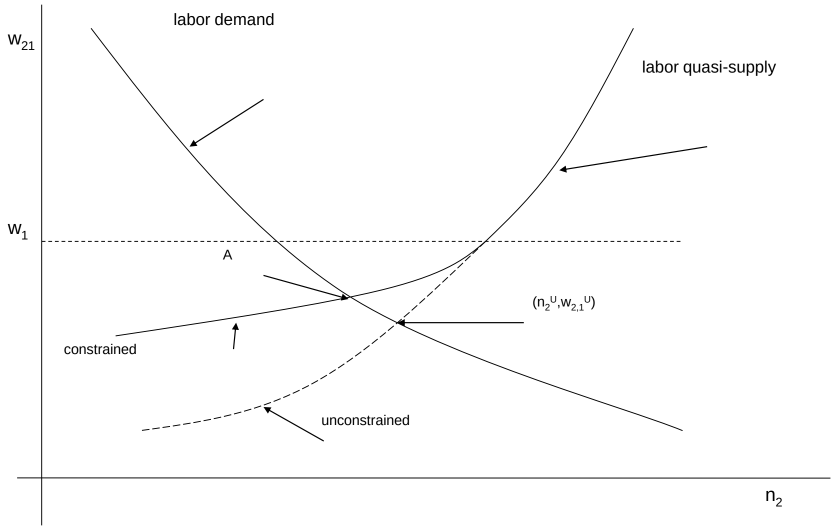


Figure 2: Constrained quasi-supply

**Proposition 1** *(a) If equilibrium hiring wages in period* 2 *are below period-*1 *wages,* $w_{2,1} < w_1$*, we have* $w_{2,1} > w_{2,1}^U(x; w_1, n_1)$ *and* $n_2 < n_2^U(x; w_1, n_1)$*: the wage for new hires is higher and employment is lower than they would be if firms were unconstrained;*[18] *moreover,* $w_{2,2} = w_{2,1} < w_1$*. Otherwise, (b) wages and employment are at the unconstrained levels:* $w_{2,1} = w_{2,1}^U(x; w_1, n_1)$ *and* $n_2 = n_2^U(x; w_1, n_1)$*, with* $w_{2,2} = w_1$*. Case (a) occurs when the labor demand curve intersects the unconstrained quasi-supply curve below* $w_1$*; otherwise, case (b) occurs.*

Wages are allocational[19] in period 2 so that the flatter quasi-supply in the region where there is downward pressure on wages will also imply more variable employment.[20] The result is unchanged if there is symmetric discounting. If discounting is asymmetric,

[18] If firms were not constrained in such a state, unless the state had a negligible probability, then the equilibrium two-period contract may be different, that is, $w_1$ and $n_1$ may differ. The proposition concerns the implied values of $w_{21}^U$ and $n_2^U$ in a hypothetical equilibrium that has the same period-1 values.

[19] I.e., firms hire until the marginal product net of the hiring cost $(k/q)$ is equal to the new hire wage.

[20] E.g., take the matching function $m(u, \nu) = u\nu / \left(u^l + \nu^l\right)^{1/l}$, where $u$ is the number of workers searching and $\nu$ is the number of vacancies, where we set $l = 0.5$ (Hagedorn and Manovskii (2008) calibrate $l = 0.407$), with a log production function subject to uniformly distributed multiplicative productivity shocks, CRRA utility and a coefficient of risk aversion of 2, $\delta = 0.1$ (approximate annual separation rate in our German data), $\beta = 0.9$, a replacement rate of 43%, and we calibrate $k$ to yield an average period-2 unemployment rate of 7.5%. The standard deviation of unemployment in the region where the no-undercutting constraint is binding is approximately twice that in the unconstrained model. The effect is smaller, however, under

then we show in Section A.1 of the Appendix that the reference wage in period 2, which determines the regime (and $w_{2,2}$ when no undercutting does not bind), differs from $w_1$.

## 2.2 Multi-period Extension

The model extends readily in the obvious way to multiple periods (with long-lived firms and workers). In the Appendix[21] we show that Proposition 1 extends to this case, where we define undercutting in terms of discounted wages costs rather just than the current wage. If no-undercutting in this sense is imposed, incumbents' wages are always no higher than new hire wages and fall only to maintain this relationship, otherwise remaining constant. Moreover, in downturns, wages do not fall as far as firms would like them to in the following sense: if new hire wages fall between periods $t$ and $t+1$, they are above the relevant unconstrained quasi-supply curve at $t+1$; when new hire wages rise between the two periods, however, they *will* lie on the relevant unconstrained quasi-supply curve at $t+1$.

## 2.3 Endogenizing the no-undercutting constraint

As discussed in the introduction, and following MM, the no-undercutting constraint can be endogenized under certain circumstances, by assuming that employment is "at will", and firms can costlessly replace incumbent workers by cheaper new hires if they are able to match with the latter. Since this is an ex ante risk for period-1 hires, it may be better for the firm to avoid this by satisfying $w_{2,1} \geq w_{2,2}$, even though this is ex post costly for the firm which would like to bring in cheaper new hires. The idea is that firms can commit to future wages but cannot commit *not* to replace. The undercutting constraint thus acts as second-best way of committing to not replacing incumbents. In the Appendix, we generalize the model to allow for $w_{2,1} < w_{2,2}$ and hence replacement to occur. In our simulations we can compute profits of a firm that deviates, from an equilibrium with no undercutting, by offering a contract with $w_{2,1} < w_{2,2}$ in some states. If these are smaller than the initial equilibrium profits then the equilibrium remains an equilibrium without the constraint provided contracts are at will. In our simulations this holds provided $k$ is not too small, *ceteris paribus.* Intuitively, not too many vacancies are posted so that $\theta$

alternative parameterizations. With a Cobb-Douglas matching technology with $p(\theta) = M\theta^{\eta-1}$, $q(\theta) = M\theta^{\eta}$, where $M = 1/10$ and $\eta = 1/2$ (this is the same specification used in MM's example) and $v(c) = c^{0.5}$, and setting $\delta = 0.3$ (appropriate for US annual data), we obtain a much smaller increase of approximately 25%. This result is partly attributable to a lower risk-sharing motive, but the higher separation rate means that in bad states, the incentive to bring in new hires at a lower wage is stronger.

[21] As the main insights of the multi-period model are already apparent in the more easily interpretable two-period model, we do not go into detail in the text. The principal qualitative difference is that there may be multiple incumbent wages at each date and that the new hire wage is no longer fully allocative, as future cohorts may be paid more than a newly hired cohort will and may have different associated hiring costs (see Kudlyak (2014)). The appendix treatment also generalizes the model to asymmetric discount factors between firms and workers.

is high and the replacement risk is substantial. This increases the ex ante benefit from satisfying the constraint.

# 3 Asymmetric Information

So far, we have seen that equal treatment leads to a degree of downward real rigidity. We now consider adding asymmetric information about the period-2 state $x$, and we argue that for a wide range of adverse shocks, this state may lead to a period-2 wage that is completely rigid for incumbents and, more importantly, for new hires. Moreover, under the assumptions of Proposition 2 below, period-2 wages remain allocational, which leads to enhanced employment variability. We assume that it is always optimal to satisfy the no-undercutting constraint.

We will assume that in period 2, ongoing hires in a firm can observe only wages $w_{2,1}$ and $w_{2,2}$ but cannot observe $x$ (nor $Z_2$ so they cannot infer $x$). Additionally, they cannot observe the total employment or vacancies at the firm (we relax this later). Equivalently, we assume that such variables are not contractible. The resultant incentive compatibility constraints on the contract imply that the equilibrium contract exhibits a much higher degree of wage rigidity and employment and vacancy fluctuations than induced by equal treatment alone.

In the symmetric information model, when the no-undercutting constraint is binding so that $w_{2,1} = w_{2,2}$, as $x$ varies, we pick off points on the quasi-supply curve as in Figure 2. Moving to the asymmetric information model, if wages vary with the state as in the symmetric information solution, then the firm has an incentive to claim that the state corresponding to a lower wage has occurred, as not only is the incumbent wage reduced, which is an unambiguous benefit to the firm, but the new hire wage is also reduced, which is a benefit locally (as $w_{2,1}$ is higher than the committed new hire wage). To satisfy incentive compatibility, then, the wage must be constant across a wide range of states. Because the wage is allocational, this translates into large employment movements.

## 3.1 Incentive Constraints

As before, assuming that a firm's profit is

$$F\left(\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X};Z\right)=\left(f\left(n_1\right)-w_1n_1-k\overline{n}_1\right)+E[F^{(x)}]$$

where $F^{(x)}$ is period-2 profits in state $x$ and is given by

$$F^{(x)}\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right):= \\ \left(f\left((1-\delta)n_1+n_2\left(x\right);x\right)-w_{2,2}\left(x\right)(1-\delta)n_1-w_{2,1}\left(x\right)n_2\left(x\right)-k\overline{n}_2\left(x\right)\right)$$

(recall $n_i$ is the number of *new hires* in period $i$, and is given by $n_i = q\left(\theta_i\right)\overline{n}_i$, $i = 1, 2$, where $\theta_i$ depends on $\sigma$, as given by $\theta_1(V_1\left(\sigma, Z\right), Z_1, U_1\left(Z\right))$ in (2) and $\theta_2(w_{2,1}, Z_2\left(x\right))$ in (3) above). We now have the firm's maximization problem as

$(\sigma;\overline{n}_1,(\overline{n}_2\left(x\right))_{x\in X})$ maximizes $F((\sigma;\overline{n}_1,(\overline{n}_2\left(x\right))_{x\in X});Z)$ subject to the incentive compatibility constraints for all $x$,

$$F^{(x)}\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right)= \\ \max_{x',\overline{n}_2'}\left\{\left(f\left((1-\delta)n_1+n_2';x\right)-w_{2,2}(x')(1-\delta)n_1-w_{2,1}(x')n_2'-k\overline{n}_2'\right)\right\}$$

where $n_2' = q\left(\theta_2\right)\overline{n}_2'$ and $\theta_2 = \theta_2(w_{2,1}(x'), Z_2\left(x\right))$, and the no-undercutting condition is $w_{2,1} \geq w_{2,2}$ for all $x$. That is, the firm has a menu of wage profiles $\left(w_{21}\left(x\right), w_{22}\left(x\right)\right)$ to choose from and will optimize vacancies, given its choice;[22] incentive compatibility requires that the firm prefers the wage profile associated with the current state to any other.

We now assume that $X \subset R_+$, and that $f$ is differentiable and increasing in $x$. We can establish the following:

**Proposition 2** ***(Asymmetric information)****(i) If firms are unconstrained, then introducing asymmetric information does not affect the equilibrium. (ii)* (wage floor) *Suppose in the constrained asymmetric information model with a single period-*2 *productivity state*[23] $\hat{x}$*, that there is an equilibrium with no undercutting and the no-undercutting constraint binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states,* $\hat{x}-\varepsilon$ *and* $\hat{x}+\varepsilon$ *(i.e., with expected value* $\hat{x}$*), and assuming that there exists* $\overline{\varepsilon}$ *such that for* $\varepsilon \in [0, \overline{\varepsilon})$*, the equilibrium is unique and continuous in* $\varepsilon$*, period-*2 *wages are* constant *across these states, provided that the perturbation* $\varepsilon$ *is sufficiently small.*[24] *Period-2 wages are allocational. (iii)* (upward flexibility) *In the constrained asymmetric information model, at the highest* $w_{22}$*, i.e., for* $x \in \arg\max_{x'} w_{22}\left(x'\right)$,

[22] These are ex post (after the period-2 state is observed) constraints; for simplicity, we assume that $n_1$ is contractible. Otherwise, the incentive compatibility constraints should be expressed in terms of an ex ante constraint that requires that should the firm deviate at date 1 (i.e., possibly changing $n_1$) and in any period-2 state, it cannot increase its discounted expected profit. Since in the latter case, the ex post constraints will also hold, the results will be very similar.

[23] Obviously asymmetric information does not bite until we perturb the equilibrium to have multiple states.

[24] For ease of presentation, the proposition considers the case where there is a single period-2 state $\hat{x}$ in the initial situation. If there are other states in which the no-undercutting constraint is not binding, the argument can be extended straightforwardly. The argument also extends readily to non-equi-probability perturbations.

*$w_{21}(x) \geq w^U_{2,1}(x, w_1, n_1)$ with equality if the no-undercutting constraint is not binding; $w_{22}(x) \leq w_1$, all $x$.*

Part (i) of Proposition 2 considers the nature of the contract with asymmetric information but in the absence of the no-undercutting constraint. The firm will offer a non-contingent period-2 contract wage to period-1 hires (equal to $w_1$) but will be unrestricted in offering the optimal hiring wage to period-2 workers. Since a stable wage for incumbents is optimal and incentive-compatible, the solution will be identical to the unconstrained solution considered earlier.

Part (ii) considers what happens in the constrained case, where asymmetric information now matters: if there are two states close to each other and the no-undercutting constraint is binding, then wages are non-contingent (which has direct implications for hires).

While the formal proposition requires the variance of the shocks to be small, simulations suggest that the optimal contract has a fixed period-2 wage for a very wide range of shocks, and where there are multiple shocks. To see the intuition for the proposition, consider the constrained solution under symmetric information: suppose there are two states $x_1$ and $x_2$ at $t = 2$ and that we are in the region where the no-undercutting constraint is binding in both states, $w_{1,2}(x) = w_{2,2}(x)$, $x = x_1, x_2$. If the wage varies with the state, say if $w_{1,2}(x_1) = w_{2,2}(x_1) < w_{1,2}(x_2) = w_{2,2}(x_2)$, in state $x_2$, the firm will prefer to "announce" state $x_1$: it benefits from paying a lower wage to its existing employees. In addition, because the no-undercutting constraint is binding, the optimal wage for new hires (i.e., ignoring the no-undercutting constraint) would be lower than at the constrained solution, and the firm will benefit from a lower wage considering new hires. Therefore, for both reasons, period-2 profits increase. Consequently, the constrained solution will violate incentive compatibility, but a similar logic applies more generally when wages vary at all across the two states, since announcing the lower wage state always maximizes ex post profits. Thus the only incentive compatible contract has a constant wage.

This argument works for a small difference in the two shocks; however, for a very wide variation in shocks, the lower $w_{1,2}$ in the symmetric information equilibrium might be so low — below the optimal level in the other state — that switching to it reduces profits from new hires. This fall in profits is unlikely to outweigh the gains from cutting $w_{2,2}$ though, as the latter are first-order and large, while around the optimal hiring wage the change in profits on cutting $w_{2,1}$ will be second-order.[25] An incentive compatible contract

[25]For very high rates of turnover (such that incumbents become a very small fraction of the workforce) and for large negative shocks such that wages are not very close together in the constrained solution, the latter solution *will* satisfy incentive compatibility. However, in our simulations with parameterizations as in Footnote 20, constant wage contracts remain optimal across negative shocks, where the worst shock is up to 50% below the best shock, even when the turnover rate is as high as 80%. For lower turnover rates, the range of shocks where constant wages are optimal is still higher.

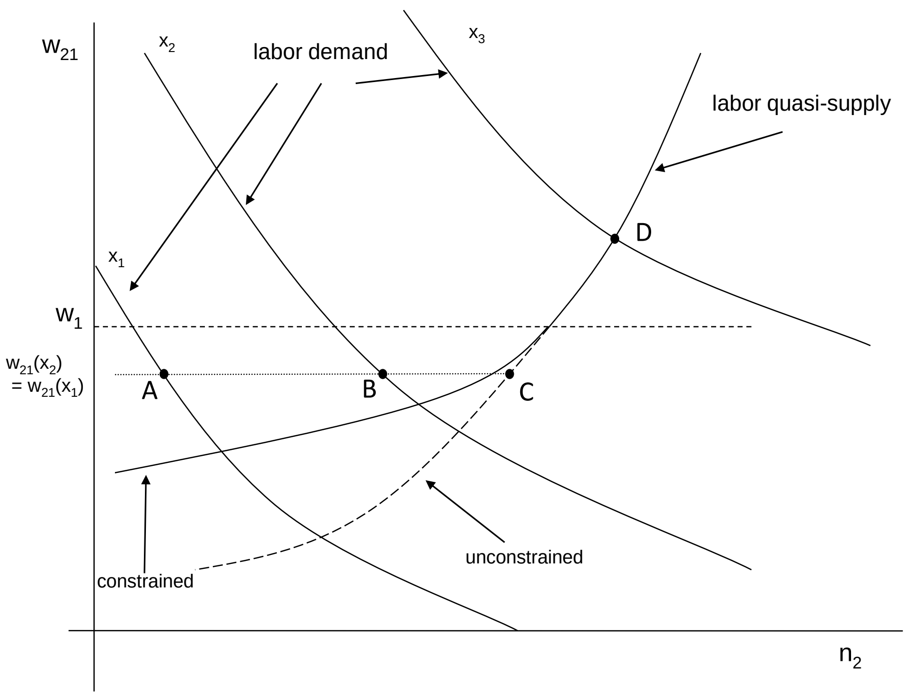


Figure 3: A rigid wage under asymmetric information

is illustrated by points A and B in Figure 3, assuming there are only two states, $x_1$ and $x_2$.[26]

Part (iii) says that in the state with the highest $w_{22}$, if the no-undercutting constraint is not binding, new hire wages are at the unconstrained solution (where labour demand and unconstrained quasi-supply curves intersect), and if it is binding, wages are at least at this level. Intuitively, continuing the previous discussion, suppose that there is a third state $x_3 > x_1$, $x_2$, but such that $w_{2,1}(x_3) = w_{2,2}(x_3)$ is equal to the common wage in $x_1$ and $x_2$, and suppose that this state of nature improves (i.e., consider perturbing the model by increasing $x_3$ holding all else constant). As $x_3$ increases, the new-hire wage that is optimal in the absence of the constraint $w^U_{2,1}(x_3, w_1, n_1)$, that is, ignoring the no-undercutting and incentive compatibility constraints in that state, rises above the constant wage (say $\overline{w}$) for the lower two states. It is clearly incentive compatible to have $w_{2,1}(x_3)$ at the optimal level (see point D in Figure 3) but $w_{2,2}(x_3) = \overline{w}$: announcing a lower state from state $x_3$ will reduce profits ($w_{2,1}$ will be at a suboptimal level, while $w_{22}$ will be the same). In fact, the

[26] The level of the wage floor will depend on the severity of the distribution where the constrained regime applies, as roughly speaking, the wage floor averages across the wages on the constrained supply curve in this region. Below, we proxy for this distribution with a linear function of forecast productivity that is conditional on being in the constrained region.

firm can do even better: $w_{2,2}$ will be slightly higher than $\overline{w}$.[27] For sufficiently favorable $x_3$, $w_{2,2}$ can increase all the way to $w_1$ without violating incentive compatibility, but as shown in general in the proposition, it is never optimal to exceed $w_1$. Nevertheless, incumbent wages are procyclical — though within the restricted interval of wages $[\overline{w}, w_1]$ — over a wider range of "positive" shocks than in the symmetric information case, something that may accord better with empirical evidence.[28]

When there is just one state in which wages exceed a wage floor, the latter logic also implies that the constrained quasi-supply curve now coincides with the unconstrained one for a range of wages below $w_1$, down to the "wage floor" $\overline{w}$ (in contrast to the symmetric information case).[29] Therefore, the region of "flexibility" for new hire wages extends further (i.e., wages are initially more flexible downwards, but then fully rigid) than in the symmetric information case. Consider point C in Figure 3: if there is a state with demand curve passing through this point, the fact that incentive compatibility lowers the incumbent wage even in such a state implies that the no-undercutting constraint first binds only at lower levels of the new hire wage so that $w_{2,1}$ will be set at this level.

### Discussion

It is useful to contrast our result with earlier models in the asymmetric information implicit contracting literature, such as Grossman and Hart (1981). In the latter, a firm employs risk-averse workers with a decreasing returns to scale production function, as here, and likewise with asymmetric information where the firm knows the state. If the firm is risk neutral, then the first-best contract can be implemented, but if the firm is risk averse, it would prefer to lay-off some workers in some productivity states where it would be efficient to employ them (in that their marginal products exceed their reservation wage).

[27] There will now be a cost of deviating by announcing a lower state, given that the new hire wage will fall below the optimal level, so $w_{2,2}(x_3)$ can increase towards $w_1$, increasing incumbent wage costs by a corresponding amount (recall that $w_{2,2} = w_1$ will improve ex ante profits). Hence, $w_{2,2}(x_3)$ will be set to exactly satisfy the incentive compatibility constraint subject to not exceeding $w_1$. Initially, this scenario is a comparison between a second-order cost and a first-order gain, so the increase in $w_{2,2}$ is itself second-order to avoid violating the incentive constraints.

[28] Using the same calibrations as in Footnote 20, we find that the standard deviation of unemployment in the rigid wage region is increased by approximately 60% relative to the constrained model. Once the spot wage rises above the wage floor, incumbent wages are also increasing in the shock, but by a smaller amount, as explained in the text; in the simulations, incumbent wages increase up to the point where the new hire wage is approximately 10% higher than $w_1$.

[29] If there are multiple states with wages above the wage floor, we can establish the following result (details available on request). For any equilibrium satisfying monotonicity in the sense that whenever $w_{2,2}(x) > w_{2,2}(x')$, $Z_2(x) \geq Z_2(x')$ and $w_{2,1}(x) \geq w_{2,1}(x')$, and also no undercutting binds in $x$ if and only if $w_{2,2}(x)$ is below some critical $w_{2,2}$ (which can be the empty set), then only downward incentive compatibility constraints can bind, and for all states $x$ where no undercutting is not binding, $w_{2,1}(x) \leq w^{**}(x)$. That is, new hire wages are no higher than the unconstrained level. Moreover, if only local downward constraints bind (as is true in our simulations) and $w_{2,2} < w_1$ for higher states (higher by $w_{2,2}$ ranking), it is a strict inequality: $w_{2,1}(x) < w^{**}(x)$. The intuition here is that cutting $w_{2,1}(x)$ a small amount below $w^{**}(x)$ imposes only a second-order cost in state $x$, but announcing $x$ in a higher state will suffer a first-order cost by this change; this cut would relax the incentive compatibility constraint and permit a higher $w_{2,2}(x')$.

The idea is that if a firm fully insures workers (i.e., across states and whether workers are employed or not) then to implement efficiency the difference between the wage for an employed worker and what an unemployed worker receives, must equal the reservation wage. This will induce the firm to employ up to the point where the reservation wage equals the marginal product, as in the first best. However the firm would bear all the risk; a risk averse firm would optimally set the contract to shift some risk to workers, and to implement this under asymmetric information the above difference must exceed the reservation wage, shifting some risk from the firm to workers when productivity is low. But this implies the firm will employ fewer workers than is efficient in some states.

This model differs from the current one, aside from having a risk averse firm, in that it is effectively a one-period setting in which a firm has a pool of workers associated with it with which it contracts (the firm and workers enter into a contract before the state is known, but workers may be immobile once contracted). Not all workers need be employed in all states, although the firm can insure those who do not have a job. In our case, by contrast, the employment decision is a vacancy/hiring one rather than an employment/layoff one; moreover the insurance of workers concerns the period 2 wage of incumbents hired in period 1.[30]

Our base assumption that firm employment is unobservable to workers or not contractible, as in e.g., Grossman and Hart (1983),[31] contrasts with work such as Chari (1983), Green and Kahn (1983), and, in a single worker model, as is often considered in this literature, observing employment (hours of work) is inevitable of course. In practice, however, the level of employment in a firm can be difficult to define precisely. For example, if the relevant employment level is at the plant, the firm may be able to move production to other companies or plants within the same company, making it difficult to condition on employment (as argued by Stiglitz (1986)). However, we also consider an extension below in which we allow contracts to depend on employment levels, and we show that (when shocks are not too far apart) a similar logic applies locally and that wages are essentially constant.

Of course if the aggregate state is contractible in some way, then the asymmetric information problem would be resolved. Outside conditions, such as labor market tightness or the value of $Z_2$, may be difficult to contract over; in the equilibrium the equilibrium wage itself is not informative as it is constant across the bad states. An approach adopted in the literature which implies that asymmetric information may nonetheless affect output when the aggregate state is observable, is to assume that the distribution of idiosyncratic shocks

[30] If our model were one-period, or equivalently, if turnover was 100%, then the constrained quasi-supply curve coincides with the unconstrained one, and there is no incentive to deviate from the optimal symmetric information contract to benefit from savings on incumbent wages, so the optimal contract is implementable.

[31] Grossman and Hart (1983) consider a single worker model in which a worker is either employed or unemployed, or equivalently as they argue, a firm with many workers but where the level of employment is, as here, not contractible.

*depends* on the aggregate shock. Extending Grossman and Hart (1981), Grossman et al. (1983) use the idea is that there may be aggregate states where there is no asymmetric information, and employment is efficient, and other states where firms are subject to asymmetric shocks which are not known to workers at those firms. Because, in the latter aggregate state, employment is inefficiently low on average due to the logic explained above, unemployment varies with the aggregate state.

In our model, consider multiple aggregate period 2 states, but suppose there are multiple sectors which are independent but only aggregate unemployment is observable. Sectoral productivity will be the aggregate one plus an iid sectoral shock say. In high aggregate productivity states most sectors receive positive shocks relative to period 1, and so will be paying new-hire wages above the incumbent wage. Here there is no issue with asymmetric information. In the low productivity states, sectors will mostly have low productivities relative to period 1. Workers will be able to deduce from aggregate unemployment what the aggregate state is, and so if unemployment contingent contracts are feasible, worse aggregate states will be associated with lower wages. However across negative shock sectors in a particular aggregate state wages will be the same. We use a test based on a related idea — where predictions, rather than observations, of the aggregate state are used — in Section 4.3.

#### 3.1.1 Employment-Contingent Contracts

The above concerns the case in which no variables that are observable to both parties can be contracted upon. While in a model such as this, which features a frictional labor market, it is plausible to suppose that it may be difficult to condition contracts on aggregate labor market variables such as wages offered by other firms, employment at the firm in which the worker is employed may be a variable that could be conditioned upon. Intuitively, in a low-productivity state, employment could be specified to be inefficiently low to discourage the firm from underreporting productivity in a better state to avail itself of lower wages, given that such inefficiency harms profits more in the better state. We next consider how matters change if employment contingent contracts are possible; for small variations in productivity, in fact, it does not affect the constant wage result.

**Proposition 3** ***(Contractible employment levels)*** *In the constrained asymmetric information model* where period-2 employment is contractible *and with a single period-2 productivity state $\hat{x}$, suppose that for given parameter values, there is a unique equilibrium and that the no-undercutting constraint binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states, $x' = \hat{x} - \varepsilon$ and $x'' = \hat{x} + \varepsilon$ (i.e., with expected value $\hat{x}$), and assuming the differentiability of equilibrium val-*

*ues,*[32] *equilibrium period-*2 *wages are approximately* constant *across these states, provided that the perturbation* $\varepsilon$ *is sufficiently small; formally,* $\lim_{\varepsilon\to 0_+}\left(w_{2,1}\left(x''\right)-w_{2,1}\left(x'\right)\right)/2\varepsilon = 0$.

A rough intuition for this result is as follows: Given that for a small perturbation in both states $x'$ and $x''$, the no-undercutting constraint continues to bind, and wages for incumbents and new hires are equal. If in the lower-productivity state, wages are lower by more than a second-order amount, there will be, as earlier, a first-order incentive for the firm in $x''$ to announce $x'$, as there is a benefit both in terms of lower wages for period-1 hires and in terms of reducing the hiring cost for new hires. To prevent this, hiring can be reduced in $x'$, which would be costly in the state $x''$, but it must be reduced by a large amount, given that hiring is initially (in the unperturbed equilibrium) optimal; this cut in hiring will also impose *first-order* costs in $x'$, swamping any benefit from the lower wages (which are second-order).

# 4 Testing the Model's Predictions

In this section, we present a variety of tests of the salient features of our model. Our focus is on the new hire wage because it is allocational in the second period of the two-period model.

## 4.1 The Data

For our empirical exercises, we use the IAB Beschäftigten-Historik (BeH, version 10.01), the Employee History File of the Institute for Employment Research (IAB) of the German Federal Employment Agency. The BeH covers all workers who were at least once employed subject to social security in Germany since 1975.[33] Not covered are self-employed, civil servants (Beamte), family workers assisting in the operation of a family business, and regular students. The BeH includes roughly 80% of the German workforce. To protect data privacy, we are not allowed to work with the universality of the BeH. Therefore, we use a 20% random sample of all workers that worked full-time during at least one year since 1975.[34]

The BeH is organized by employment spells. A *spell* is a continuous period of em-

[32] That is, assuming that $Z_2$ is a differentiable function of $\varepsilon$ in a neighbourhood of 0.

[33] The BeH also covers marginal part-time workers employed since 1999.

[34] More precisely, we focus on "regular workers" according to the definition used in the Administrative Wage and Labor Market Flow Panel (AWFP) dataset (see Seth and Stüber, 2017): a regular worker is employed full time and belongs to person group 101 (employee s.t. social security without special features), 140 (seamen) or 143 (maritime pilots). Therefore, all (marginal) part-time employees, employees in partial retirement, interns, etc., are not considered regular workers.

ployment within an establishment in a particular calendar year. Hence, the maximum spell length is 366 days. For each identified full-time worker, the BeH has a record of all existing employment spells — including part-time employment, apprenticeships, etc. For our analyses, we restrict our attention to employment spells of full-time workers[35] aged 16 to 65 years from West Germany for the period from 1978 to 2014. We keep employment spells only if the workers are employed on December $31^{st}$ of the respective year.[36]

We define a newly hired spell as a worker's first spell at the establishment.[37] Hence, a worker's tenure in an establishment that spans more than one calendar year will consist of multiple spells, with the first being classified as a new hire spell.

Our dependent variable is the real average daily wage of a worker over any spell. As the earnings data are right-censored at the contribution assessment ceiling ("Beitragsbemessungsgrenze"), only non-censored wage spells are considered in the analyses.[38] To calculate the average daily real wage and real output per capita in 2010 prices, we use the German Consumer Price Index (CPI, see Table A2).

As a proxy for aggregate productivity, we use West German GDP per capita. GDP data were obtained from the German Federal Statistical Office and the Federal Statistical Offices of the Federal States. In an initial test of downward rigidity, we also make use of the aggregate unemployment rate, which we obtained from the Federal Unemployment Agency (see Table A2).

The final dataset used in our analyses contains over 97.8 million employment spells for nearly 9 million workers working for more than 2.8 million establishments. The BeH contains an establishment identifier, but henceforth, we refer to establishments as "firms" in keeping with the phrasing used in the discussion of the theory.[39]

[35]The BeH documents only total spell earnings, not hours worked in that spell. We therefore consider only full-time workers, as these workers' hours are likely to be acyclical. In earlier work that is available upon request, we analyse the time series properties of an extraneous estimate of the average hours worked in a year by full-time employees in Germany. We find cyclicality — in the sense of having a significant correlation with output — to be relatively weak.

[36]This specification implies that we only ever have a maximum of one spell per worker per year, so when we compute yearly averages over spells, we do not more heavily weight those workers with multiple within-year spells. It also excludes most short-lived spells in the data, particularly temporary summer work.

[37]Re-hires are therefore not identified as new hires. Our decision to treat returning workers as incumbents is because of the relatively short time of absence; 70% of returners returned after an absence of less than one year, and returners' average length of time away is approximately 20 months. This suggests that these spells are for workers who have long-term relationships with the establishment and whose absences were temporary (for reasons such as paternity/maternity leave).

[38]We drop spells with wages $\geq$ 0.98 * the contribution assessment ceiling. Dropping top-coded spells leads to an under-representation of highly qualified workers, making the results somewhat less generalizable. Because the wages of highly qualified workers are less likely to be covered by a collective bargaining agreement (see, e.g., Düll, 2013) and because uncovered wages are more flexible than covered wages (see, e.g., Devereux and Hart, 2006), we likely slightly underestimate the wage cyclicality. For a quantitative evaluation of the effect of dropping censored spells, see, for example, Appendix A of Stüber and Beissinger (2012).

[39]The main results of this paper hinge on estimates that control for match fixed effects, with the un-

## 4.2 Extracting Composition-Bias-Free Estimates of New Hire Wages

We wish to test the model's predictions concerning the cyclical behavior of new hire wages. To do this, one must extract estimates of these wages from the panel data, controlling for composition bias. Following Solon et al. (1994), this can be achieved with a two-step method. In the first stage, year effects are extracted from the panel using year dummies while controlling for worker-firm characteristics. In the second stage, the year effects are treated as composition-controlled estimates of the average new hire wage in each year. In the two-period asymmetric information model, new hires come from unemployment, not from other firms. Hence, the wage year effects that we would like to identify are those for new hires arriving directly from unemployment. We define these hires as workers who were without a job for over four weeks before arriving at the firm.

As noted above, it is important to control for as much worker-firm heterogeneity as possible, and a natural way to do so is to use worker-firm (match) fixed effects (MFE) as well as proxies for returns to tenure and experience. It is widely believed that match quality is procyclical (see the discussion in Gertler et al. (2016)), and failing to control for it may lead to misleading inferences in this respect (Gertler and Trigari, 2009b).

In the first stage, the primary specification to be estimated is the panel regression

$$w_{ijt} = m_{ijt} + \sum_{\tau=1}^{T} \beta_\tau^I I_t^\tau + \sum_{\tau=1}^{T} \beta_\tau^E E_t^\tau + \sum_{\tau=1}^{T} \beta_\tau^U U_t^\tau + \sum_{k=1}^{2} \lambda_k age_{it}^k + \sum_{k=1}^{4} \phi_k ten_{ijt}^k + v_{ijt}, \quad (9)$$

where $w_{ijt}$ is the log of the real average daily wages of worker $i$ in firm $j$ during year $t$, and $v_{ijt}$ is an error term assumed to be orthogonal to the regressors.

The equation allows for three distinct sets of year effects written in the first three summation terms. The first consists of the dummies $I_t^\tau$ ($\tau = 1, \ldots, 37$) with coefficients $\beta_\tau^I$ where $I_t^\tau$ equals one if $t = \tau$ and the worker is an incumbent, but is zero otherwise. The $\beta^I$ coefficients are the incumbents' year effects. The second (third) set of dummies $E_t^\tau$ ($U_t^\tau$) take the value of one if the wage is from an ee (ue) new hire and $t = \tau$, but is equal to zero otherwise. The $\beta_t^E$ ($\beta_t^U$) are the corresponding year effects. The variable $age_{it}$ is the worker's age in years, and $ten_{ijt}$ is the worker's firm tenure measured in days at the end of the spell. Finally, $m_{ijt}$ is an MFE. Note that this effect controls for (estimates) the sum of a firm $j$'s effect plus a worker $i$'s effect plus a match quality effect. While the use of MFEs is a general way to absorb heterogeneity in the panel, a drawback is that if new hire wages are excessively sensitive to the state of the cycle at entry and if (part of) this

derlying assumption being that matches are with establishments, not firms. However, even if matches are formed at the firm level, then using worker-establishment fixed effects will absorb them in any event; their use in this case may be inefficient but will not bias the estimated year effects.

effect remains constant throughout the entire relationship with the firm, then it will be absorbed into the MFE and will not appear as "excess" new hire cyclicality; for example, if new hire effects are procyclical and permanent, they will be observationally equivalent to procyclical match quality effects. This is one of a number of problems that makes a rigorous test of equal treatment difficult and is one reason why we do not execute such tests in this paper.

### 4.3 Testing the Model

A key prediction common to all versions of our model is that new hire wage growth is relatively rigid in downturns but moves closely with productivity in upturns. To get an initial grasp on whether or not our data support this feature, we look at cross-sector new hire wage growth variance over time.

Suppose the German economy consists of several "sectors" — that is, smaller economies, each with its own distinct and separate labour market. Additionally, suppose that sectoral productivity reacts to aggregate productivity in a heterogenous fashion. In this world, we would expect the cross-sectional (across-sector) variance of new hire wage growth to be higher in upturns than in downturns.[40] We test this simple idea by disaggregating the single new hire (from unemployment) year dummies $U_t^{\tau}$ in (9) into 29 sectoral dummies, according to the classification defined in Appendix A1. Hence, we initially extract 29 time series of new hire wage year effects — one for each sector. We first-difference each sector year effect to obtain a new hire wage growth rate and compute a cross-sector standard deviation for each of the 37 available years, which we denote by $\sigma_{wt}$.[41] To proxy — somewhat crudely — cyclical movements in productivity, we use i) the de-meaned aggregate West German unemployment rate ($\widetilde{u}_t$) and ii) the growth rate of West German GDP per capita ($\widetilde{\Delta y_t}$). We refer to years when demeaned unemployment (GDP growth) is below (above) zero as "upswings" and, vice versa, as "downswings". Line 1 in Table 1 shows the results of regressing a) $\sigma_{wt}$ on $\widetilde{u}_t$, b) $\sigma_{wt}$ on $\widetilde{u}_t$ when $\widetilde{u}_t < 0$, c) $\sigma_{wt}$ on $\widetilde{u}_t$ when $\widetilde{u}_t > 0$, and d) $\sigma_{wt}$ on a dummy that is one in upswings and zero otherwise. Line 2 gives analogous results using GDP growth as a cyclical indicator in place of unemployment (with, of course, upswings/downswings defined here as periods of positive/negative growth).[42]

Regarding unemployment, the table shows that there is a clear-cut negative relationship between aggregate unemployment and cross-sectoral wage growth volatility, although

[40] Consider the case where, in the asymmetric information model, all sectors start in the same position. If aggregate productivity falls, then most sectors will be in a downturn and are likely to have wage growth (negative) at the wage floor, whereas if aggregate productivity rises, wage growth in a sector will depend on the realization of the sectoral productivity growth when it is positive.

[41] In the following, we assume the (small sample) measurement error arising from using estimated rather than actual variance is uncorrelated with the regressors. Estimates of annual new hire wage growth are of course derived from very large samples and raise no such issues.

[42] T-ratios are computed using robust standard errors.

Table 1: The Relationship Between New Hire Wage Growth Volatility and Unemployment and GDP Growth

| Regression of $\sigma_{wt}$ on | *a)* | *b)* | *c)* | *d)* |
|---|---|---|---|---|
| $\widetilde{u}_t$ | $-1.27$ | $-2.05$ | $-0.15$ | 0.04 |
| | (4.02) | (2.24) | (0.50) | (2.59) |
| $\widetilde{\Delta y_t}$ | 0.52 | 0.95 | 0.03 | 0.02 |
| | (1.83) | (1.76) | (0.09) | (1.22) |
| *Regression of* $\Delta w_t^n$ *on* | | | | |
| $\Delta\widetilde{u}_t$ | $-0.87$ | $-1.30$ | $-0.61$ | .003 |
| | (3.44) | (4.57) | (1.94) | (0.56) |
| $\widetilde{\Delta y_t}$ | 0.42 | 0.55 | 0.11 | 0.015 |
| | (4.45) | (2.92) | (.81) | (3.68) |

Note: T-ratios in brackets.

as columns *b)* and *c)* show, this holds in upswings only. The final column shows that volatility is significantly higher in upswings — during these years, it almost doubles (the estimated intercept is around 0.04). Line 2 shows there is less co-movement with GDP growth. There is a positively (borderline) significant relationship with wage growth volatility — a result confirmed by separating upswing and downswing years, as we do in columns *b)* and *c)*. However, column *d)* shows that although the volatility of wage growth is higher in upswings, this increase is not statistically significant. Notwithstanding the latter result, these findings offer some indicative support for downward real wage rigidity.

We may also use the aggregate series for new hire wages extracted from (9), denoted by $w_t^n$, to test certain aspects of the model. We begin by looking again at the broad question of whether or not new hire wages are relatively rigid in recessions and relatively flexible in booms. We repeat the regressions *a)* to *d)* in Table 1 above, but this time, with $\Delta w_t^n$ as the regressand and with $\Delta u_t$ replacing $u_t$ as a regressor.[43] The results — in the lower part of the Table — are broadly indicative of downward real wage rigidity although the findings using GDP growth as a cyclical indicator are more definitive than those using unemployment.

Above, we have used observables to definitively indicate up and downswing years, but it would be interesting to see if the results hold up for a latent variable approach instead. We allow the mean of $\sigma_{wt}$ (alternatively, $\Delta w_t^n$) to take two values according to whether a latent indicator variable is positive or negative, where this indicator variable is, in turn, linear in $\widetilde{u}_t$ (alternatively, $\widetilde{\Delta y_t}$). (Full details of this model are given below for a case that nests the current one). The results for these two cyclical indicators and for the two wage

[43]It is more common in this context to regress wages on unemployment in levels rather than first differences. This step does not change the results qualitatively. However, we prefer the current specification over levels because the latter has a highly persistent error term that causes problems for inference.

measures are in Table 2.

Table 2: A Switching Regime Model for the Means of $\sigma_{wt}$ and $\Delta w_t^n$.

| Volatility Measure (Cyclical Indicator) | $\sigma_{wt}(\widetilde{u_t})$ | $\sigma_{wt}(\widetilde{\Delta y_t})$ | $\Delta w_t^n(\widetilde{u_t})$ | $\Delta w_t^n(\widetilde{\Delta y_t})$ |
|---|---|---|---|---|
| Upswing Mean | 0.12 | 0.17 | 0.020 | 0.017 |
| | (6.49) | (22.15) | (8.47) | (6.86) |
| Downswing Mean | 0.03 | 0.04 | −0.00 | −0.01 |
| | (8.68) | (8.15) | (1.23) | (2.42) |

Note: T-ratios in brackets.

The table shows that the volatility and mean of wage growth are both higher in the upswing regime, regardless of which of the two cyclical variates is used to determine the regime. Furthermore, in all cases, a bounds test[44] that these (four) differences are zero is roundly rejected. Finally, the two downswing estimates of wage growth are insignificant and negative, respectively, whilst their upswing counterparts are both significantly positive.

We now turn to a test of the more specific predictions of our theory. To this end, we focus on the predictions of the two-period asymmetric information model; this model has the most interesting and distinct implications for new hire wages. As before, we concentrate on new hire wages because they are allocational in our model. The model has two possible wage "regimes" under no undercutting in its second period: a "spot rate" regime and a "constrained" regime. In the spot regime, the new hire wage is determined at the intersection of the labour demand curve and the unconstrained quasi-supply curve, as depicted, for example, by point D in Figure 3. However, if productivity falls in the second period to the extent that the no-undercutting constraint binds, wages fall at a rate that is independent of the current state; under asymmetric information, forecasted productivity, not actual productivity, matters in this regime.[45] Our challenge here is to examine the extent to which these features are present in the time series observations on new hire wages that we obtained from the panel.

We begin with a simple model for the "spot" wage, which, as noted above, lies on the unconstrained quasi-supply curve in Figure 3. The first-order driving force in our model for any time $t$ endogenous variable — including the spot wage — is the current state of productivity. Lagged productivities obviously also matter, but we take these as being of second-order importance. The theory implies that the spot wage is increasing in productivity. The simplest possible model for the log of the new hire (spot) wage ($w_t^s$) is

[44] The variance of the difference between two random variables $a-b$ (say) cannot exceed $var(a)+var(b)+2var(a)var(b)$. We use this upper bound to compute the smallest possible t-test of the difference between up and downswing means.

[45] The constrained regime is also one of equal treatment — as noted already in the text, this prediction is difficult to test rigorously, and we do not address it in this paper.

therefore the constant elasticity specification

$$w_t^s \approx \alpha + \gamma x_t$$

where $x_t$ is the log of productivity. Clearly, $w_t^s$ is a latent variable; in the spot regime, it is equal to the (log of the) new hire wage $w_t^n$, but in the constrained regime, it lies below it.

It is not straightforward to test the two-period model with time series data. However, given the previous discussion, a reasonable interpretation of the model's period-two wage might be

$$w_t^n = \alpha + \gamma x_t \quad if \quad \alpha + \gamma x_t > w_{t-1}^n \tag{10}$$

$$\Delta w_t^n = -\theta(w_{t-1}^n - \alpha - \gamma \widehat{x_t}) \qquad if \qquad \alpha + \gamma x_t < w_{t-1}^n \tag{11}$$

where $\widehat{x_t}$ is the forecast of productivity conditional on being in the constrained regime.[46]

In period 1, the model is — by assumption — in the spot regime. If this regime also holds in period 2, then wage growth will be proportional to productivity growth, $\Delta x_t$.[47] If, by contrast, wages are constrained in period 2, then wage growth is negative and proportional to $w_{t-1}^n - \alpha - \gamma \widehat{x_t}$.

In general, there is no way of knowing a priori which regime exists in any particular year.[48] Therefore, we use the following latent variable switching model. Let $I_t$ be an indicator defined on the real line that takes positive (negative) values when we are in the spot (constrained) regime. The model is

[46] See Footnote 26.

[47] In a multi-period setting, we may also move from a constrained regime to a spot regime in consecutive observations. If such occurrences are frequent, there will be downward bias in the estimated spot regime productivity elasticity.

[48] Given that when new hire wages rise, we know we must be in the unconstrained regime, it would be tempting to use this fact to split the sample to run separate regressions. However, this approach would be a mistake for two reasons. First, years in which new hire wages fall are not necessarily constrained years (if the fall is relatively modest). Second, using information on a regressand to preselect a regression subsample is well known to lead to biased estimates (upwards in this case). We can, of course, preselect the sample for regression purposes using the value of (presumed exogenous) regressors — an exercise we describe in the text below — but this approach does not achieve an exact split into constrained and unconstrained years.

$$I_t = const + \delta(\gamma x_t - w_{t-1}^n) + u_t \tag{12}$$

$$\Delta w_t^n = const + \beta^u \Delta x_t + \theta^u(\gamma \widehat{x_t} - w_{t-1}^n) + v_t \quad \text{if} \quad I_t > 0 \tag{13}$$

$$\Delta w_t^n = const + \beta^d \Delta x_t + \theta^d(\gamma \widehat{x_t} - w_{t-1}^n) + v_t \quad \text{if} \quad I_t < 0 \tag{14}$$

$$u_t \sim IIDN(0,1), v_t \sim IIDN(0, \sigma_v^2) \tag{15}$$

If the two-period model with asymmetric information is true, then we have $i)$ $\beta^d = 0$, $ii)$ $\beta^u = \gamma, iii)$ $1 > \theta^d > 0$, $iv)$ $\theta^u = 0$.[49]

We do not have a measure of West German productivity (TFP), so in the following, we proxy for it using West German GDP per capita and henceforth refer to $x_t$ as simply "output".

There are problems implementing and interpreting (12) to (15). The assumptions behind the model (particularly the i.i.d. normality of errors) are quite strong. The model requires a proxy for forecasted output $\widehat{x_t}$. Our output and wage series are $I(1)$ and are not cointegrated, so we cannot use their levels on the RHS of (12) or in our estimate of $\widehat{x_t}$. Finally, there are eight parameters (including two intercepts that allow for differential long-run wage growth in upswings and downswings), which "stretches" the information in our 38 data points somewhat.

To help ameliorate the scarcity of data points, we calibrate $\gamma$ using values from simulations to save having to estimate it. The elasticity of the spot wage with respect to model productivity from these simulations generally lays in the region of 0.6 to 1.0 for a wide array of parameter values and productivity processes. We therefore set $\gamma$ to 0.8 and subsequently check how sensitive the results are to changing that number by $+/-$ 0.2 (i.e., to $\gamma = 0.6$ and $\gamma = 1$). We should also note at this point that the scarcity of observations is a pervasive problem in business cycle analyses and is not limited to this study.

The lack of cointegration between wages and output is hardly surprising; it is unlikely that there is a single common stochastic trend driving both the wage series we have extracted from a sample of workers and economy-wide output. Nonetheless, it does mean that even if we know the value of $\gamma$, we cannot use the levels of wages and output in the model without first rendering them stationary. To obtain stationary measures of the cyclical component of these variables, we follow standard macroeconomic procedures and use HP filtered (log) output and (log) wages.

Whilst we cannot test the i.i.d. assumptions behind the model, we can test for the normality it requires. Applying Shapiro-Wilk tests for the null of normality to wage

[49] $1 > \theta^d$, as the slope of the quasi-supply curve is less than that of the unconstrained curve in this region.

growth, output growth and actual and forecasted wage pressure gives p-values of 0.84, 0.32, 0.33 and 0.58, respectively.

To obtain an estimate of forecasted output, we use the fitted value from an AR(2) model for (HP filtered) log output.[50] The regression coefficients on the two lags are almost exactly of equal and opposite signs (p-value for this test is 0.85), and imposing this equality as a constraint gives

$$x_t^h = const + 0.43\Delta x_{t-1}^h + \varepsilon_t \tag{16}$$

$$(4.55) \tag{17}$$

where here and henceforth, superscript $h$ denotes an HP filtered quantity.

In addition to estimating the full model, we also estimate a restricted version in which we ignore the impact of forecasted output in downturns by setting the $\theta's$ to zero. This exercise is similar to that conducted in Table 1 in the sense we regress wage growth on a cyclical indicator (this time output growth), with the crucial difference being that the two regimes are endogenously determined via our latent variable model. The models are estimated by ML, and the results for the restricted version are presented in Table 3.

Table 3: ML Estimates of the Model

| | $\beta^u$ | $\beta^d$ | $\theta^u$ | $\theta^d$ | $t_\delta$ |
|---|---|---|---|---|---|
| Coefficient | .50 | .17 | – | – | 6.26 |
| | (8.22) | (2.49) | | | |
| Coefficient | .47 | .12 | .17 | .42 | 4.62 |
| | (6.39) | 1.68 | (1.21) | (2.66) | |

Note: T-ratios in brackets.

The values of $t_\delta$ in the table indicate that delta is significant in both restricted and unrestricted models. This, in turn, implies that our cyclical indicator $\gamma\widehat{x}_t - w_{t-1}^n$ is a significant determinant of the current state (i.e., an upswing or a downswing). The restricted model results in the first line of the table, reinforcing the findings above, namely, that in a downswing, wage growth is poorly related to output growth, whilst in an upswing, it is closely related to it.

The lower part of Table 3 presents the results for the model including forecasted

[50]Strictly speaking, we should use only data points from the constrained regime, and obviously, in the current setup, we cannot do this. However, the AR coefficients are stable over sub-periods. Furthermore, in an additional exercise, we used the latent variable model's fitted probabilities to determine the most likely regime in each year and split the sample accordingly. The estimated AR process using only the data points classified as being constrained was practically identical to that from the full sample.

output $\widehat{x_t^h}$, offering some support for the model's predictions. In particular, $\theta^u$ is insignificant, $\theta^d$ is significant, $\beta^u$ is positive and highly significant (albeit smaller than $\gamma$) and $\beta^d$ is small in magnitude and of borderline significance. However, these inferences must be treated with caution not just because of the restrictive assumptions of the model and the small sample but also because $\widehat{x_t^h}$ is a generated regressor. This issue does not occur when testing the null of $\theta^u = 0$ or $\theta^d = 0$ (see Pagan (1984)) but may appear for inferences about other (non-generated) regressors. Generally, however, the finding in the generated regressor literature is that the latter problem is insubstantial. Nonetheless, we carried out an alternative exercise to check the impact of generated regressors. We re-estimated the model replacing $\widehat{x_t^h}$ with $\Delta x_{t-1}^h$ and allowed this variable and $w_{t-1}^n$ to enter (13) and (14) with free coefficients. The estimates of these two terms i) were jointly significant (insignificant) in down (upswings), ii) were "correctly" signed, and iii) passed an LR test of the theory's restriction that their ratio should equal $\gamma\hat{\rho}$, where $\hat{\rho}$ is the estimate of the coefficient in (16). Modulo the degrees of freedom issue, this exercise provides some reassurance that our inferences are not likely to be affected by the generated regressor problem.

A further concern is the low degrees of freedom caused by the endogenous switching process. To partially address this issue, we carried out a further sensitivity analysis; in the same vein as in the early part of this section, we split the sample a priori into constrained/unconstrained data points according to whether or not HP filtered output was below/above zero. The reasoning here is that whilst this split is "noisy", it at least achieves parsimony with respect to degrees of freedom. The estimates (available on request) were very close to their latent variable counterparts, and despite higher standard errors, the test results were the same.

Finally, we note that the results for both restricted and unrestricted models (again available on request) are qualitatively unchanged when we decrease/increase $\beta$ to 0.6/1.0.[51]

# 5 Concluding Comments

We have considered a simple frictional model of the labor market, in which a constraint on not undercutting existing workers leads to a degree of downward wage rigidity for new hires. The rigidity arises from worker risk aversion and a desire to limit temporal wage variation for incumbent workers, which also transmits to new hires in downturns. Because period-two new hire wages are allocational, the response of unemployment and vacancies to negative shocks is amplified. We further show that the interplay with asymmetric information can substantially enhance downward wage rigidity and increase the respon-

[51]Of course, the model also requires that $\beta^u = \gamma$, so this lack of sensitivity is not all good news for its predictions. However, the maximized likelihood was relatively flat over the relevant ranges, suggesting that $\beta$ was poorly identified.

siveness of unemployment and vacancies to productivity shocks. We argue that downward, but not upward, real wage rigidity for new hires is apparent in the German BeH panel dataset, in line with the model's predictions, and moreover we find tentative support for the asymmetric information version of the model.

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# A Extensions

## A.1 Multi-period Extension to Base Model

We extend the model in a straightforward way to $T > 2$ periods. At the start of each period $t$ a productivity shock $x_t \in X$ is drawn, according to a Markov process with transition probabilities $\Pi = [\pi_{xx'}]_{x,x' \in X}$, where again $x = x_0$ is fixed at $t = 1$. Per-period production and utility functions are as before, but we allow for respective discount factors for firms and workers $\beta_f$ and $\beta_w$, $0 < \beta_f, \beta_w < 1$.

A firm's wage policy, to which it commits, is $\sigma = \left(w_1, (w_{2,i})_{i=1,2}, ..., (w_{T,i})_{i \leq T}\right)$, where $w_{t,i}$ now denotes the wage paid at $t$ to a worker with $i$ periods tenure, so that $w_{t,1}$ is the wage paid to a new hire at $t$, etc. Firms also choose how many new jobs $\overline{n}_t$ to create in period $t$ at a cost of $k > 0$ per job. Each $w_{t,i}$ and $\overline{n}_t$ is a function of the history of shocks $x^t := (x_1, \ldots, x_t)$. As before, an employed worker suffers exogenous separation from the firm at the end of a period with probability $\delta$. Such workers join the existing unemployed in searching for work from the start of the next period. A worker who is unemployed in any period receives an income of $b$.

Search and matching occurs in a similar manner to the earlier model. In the spirit of the no-replacement motivation for the no undercutting constraint, we generalize the latter. A firm satisfies no undercutting, that is, will *not* have an incentive to replace a particular cohort having tenure $i > 1$ at date $t$, if the discounted continuation costs, taking into account separation, do not exceed those for a new hire[52]:

$$E[\sum_{\tau=t}^{T} (\beta_f(1-\delta))^{\tau-t} w_{\tau,\tau-t+i} \mid x^t] \leq E[\sum_{\tau=t}^{T} (\beta_f(1-\delta))^{\tau-t} w_{\tau,\tau-t+1} \mid x^t]. \tag{18}$$

We require that (18) holds for all $t$, $1 < t \leq T$, all $i$, $1 < i \leq t$, and all $x^t$.

Let $Z_t(x^t)$ be the lifetime utility of a worker searching in period $t$. Define $Z := (Z_1, Z_2, \ldots, Z_T)$ (suppressing dependence on $x^t$ where no ambiguity arises). The value to a worker at $t$ with tenure $i$ from being employed by a firm with wage policy $\sigma$ is defined recursively by

$$V_{t,i}(\sigma; Z, x^t) := v\left(w_{t,i}\left(x^t\right)\right) + \beta_w E[\delta Z_{t+1} + (1-\delta) V_{t+1,i+1}(\sigma; Z) \mid x^t], \tag{19}$$

for $t = 1, \ldots, T$, $i \leq t$, with $Z_{T+1} = V_{T+1,i}(\sigma; Z) = 0$, $i \leq T+1$. Likewise let $U_t$ be the lifetime utility of an unemployed worker at $t$ who fails to find a job:

$$U_t\left(Z, x^t\right) = v(b) + \beta_w E\left[Z_{t+1} \mid x^t\right].$$

[52] This inequality is relevant provided that the firm will not try to replace either cohort in the future. Since we will look for contracts that satisfy no-undercutting constraints at all dates, this will hold.

Given $U_t$ and $Z_t$, the expected queue length for a job offering $V_{t,1}$ to a new hire at $t$ is assumed to satisfy:

$$\theta(V_{t,1}, Z_t, U_t) = \begin{cases} \theta : p(\theta)V_{t,1} + (1 - p(\theta))U_t = Z_t, \text{ if } V_{t,1} > Z_t \\ 0, \text{ if } V_{t,1} \leq Z_t \end{cases} \tag{20}$$

A firm's profit at $t$ is:

$$F_t(\sigma; (\overline{n}_1, \overline{n}_2, \ldots, \overline{n}_T); Z) = f\left(\sum_{i=1}^{t} n_{t,i}; x_t\right) - \sum_{i=1}^{t} w_{t,i} n_{t,i} - k\overline{n}_t$$

where $n_{t,i}$ is the number of workers in the firm in period $t$ with tenure $i$, and is given by $n_{t,i} = (1-\delta)^{i-1} q(\theta_{t-i+1}) \overline{n}_{t-i+1}$, $i = 1, \ldots, t$, where, from (20), $\theta_{t-i+1} = \theta(V_{t-i+1,1}(\sigma; Z), Z_{t-i+1}, U_{t-i+1}(Z))$.

We define an equilibrium analogously with the two-period case:

**Definition 2** *A symmetric stationary competitive search equilibrium with no undercutting and positive hiring consists of search values $Z = (Z_1, Z_2, \ldots, Z_T)$, and a wage policy $\sigma$ satisfying (18) for all $t, i < t$ and $x^t$, and job creation plan $(\overline{n}_1, \overline{n}_2, \ldots, \overline{n}_T)$, $\overline{n}_t > 0$ all $t = 1, \ldots, T$, with the following properties:*

(i) *Profit maximization*:[53] For all $(\sigma'; (\overline{n}'_1, \ldots, \overline{n}'_T))$ satisfying (18) for all $t, i < t$ and $x^t$,

$$E\sum_{t=1}^{T} \beta_f^{t-1} F_t(\sigma; (\overline{n}_1, \overline{n}_2, \ldots, \overline{n}_T); Z) \geq E\sum_{t=1}^{T} \beta_f^{t-1} F_t(\sigma'; (\overline{n}'_1, \ldots, \overline{n}'_T); Z); \tag{21}$$

(ii) *Consistency*: $\theta(V_{t,1}(\sigma, Z), Z_t, U_t) = S_t/\overline{n}_t$, where $S_t := (1 - p(S_{t-1}/\overline{n}_{t-1}))(1-\delta) S_{t-1} + \delta S$ is the number of workers (per firm) seeking work in period $t$.

Proceeding as in the two-period model, we define a firm's job filling probability as a function of the current wage holding its future wages constant: $\tilde{q}(w_{t,1}, x^t) := q(\theta(V_{t,1}(\sigma, Z), Z_t(x^t), U_t(x^t, Z)))$, where the dependence on $w_{t,1}$ is via $V_{t,1}$ from (19) holding $V_{t+1,i+1}$ constant. We write $\tilde{q}' \equiv \partial\tilde{q}/\partial w_{t,1}$.

As in the proof of Proposition 1 if the firm can set $w_{t,1}$ without constraint it must satisfy $q^2(\tilde{q}')^{-1} = k$; the equilibrium wage and employment must then be at an intersection of the locus of values for $n_t$ and $w_{t,1}$ (the *unconstrained quasi-supply curve*) which satisfy this equation, and the labor demand curve.[54]

[53] We have not explicitly defined profits for contracts which violate no undercutting, but they are defined analogously to the two-period case, and deviations of this type are understood to be included in (18).

[54] The "labor demand" curve now includes a forward looking element which takes into account the

**Proposition 4** *A symmetric equilibrium with positive hiring has the following characterization. (a) (Evolution of incumbent wages) For $i \geq 1$, define $\tilde{w}_{t+1,i+1}(x^t, x)$ to be the solution to*

$$\beta_w v' \left(\tilde{w}_{t+1,i+1}\right) = \beta_f v' \left(w_{t,i}\right)$$

*(i.e., when $\beta_w = \beta_f$, $\tilde{w}_{t+1,i+1} = w_{t,i}$). Then*

$$w_{t+1,i+1} = Min\{\tilde{w}_{t+1,i+1}, w_{t+1,1}\}.$$

*(b) (Evolution of new hire wages) If $\beta_w v' \left(\tilde{w}_{t+1,1}\right) > \beta_f v' \left(w_{t,1}\right)$ (i.e., when $\beta_w = \beta_f$, if new hire wages fall at $t+1$: $w_{t+1,1} < w_{t,1}$), then $w_{t+1,1}$ lies above the unconstrained quasi-supply curve*[55]*; if $\beta_w v' \left(\tilde{w}_{t+1,1}\right) \leq \beta_f v' \left(w_{t,1}\right)$ (when $\beta_w = \beta_f$, if new hire wages rise or are constant at $t+1$), $w_{t+1,1} \geq w_{t,1}$, then $w_{t+1,1}$ lies on the unconstrained quasi-supply curve.*

## A.2 Endogenizing the no-undercutting constraint $w_{2,1} \geq w_{2,2}$

We consider how the model changes if we allow firms to set $w_{2,1} < w_{2,2}$, but also assume that the firm cannot commit *not* to replace workers by cheaper new hires, as in MM and Snell and Thomas (2010). Our argument is to show that under certain circumstances, though not all, a firm will nevertheless want to satisfy the constraint $w_{2,1} \geq w_{2,2}$ to avoid uncertainty created by employment risk. Satisfying the constraint may be costly in some states in the sense that the firm, if it could commit by some other means not to replace incumbent workers, would prefer to set $w_{2,1} < w_{2,2}$.

In more detail, suppose that employment is "at will", so during the matching stage of the second period (after observing $x$), the firm can dismiss a worker without compensation; crucially, suppose that the firm can dismiss a worker after matching with a worker who can replace the original worker.[56] Specifically, at $t = 2$, suppose that unemployed workers can apply for jobs that are already filled; if there is a successful applicant, the firm can, by at-will contracting, choose whether to replace the incumbent or not. If $w_{2,1} \geq w_{2,2}$ firms

reduction in future hiring costs due to an extra worker taken on today, and any difference in future wage costs between a hire made today and one made next period (i.e., until the no undercutting constraint induces equality between the two):

$$w_{t,1} = f_t' - k/q_t + E\left(\beta_f \left(1-\delta\right) k/q_{t+1} + \sum_{\tau=t+1}^{T} \beta_f^{\tau-t} \left(1-\delta\right)^{\tau-t} \left(w_{\tau,\tau-t} - w_{\tau,\tau-t+1}\right)\right),$$

where $q_t \equiv q\left(\theta_t\right)$.

[55] In the sense that $q^2 \left(\tilde{q}' \left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} > k$, where $\tilde{q}\left(w_{t,1}, x^t\right)$ is the probability that a firm that unilaterally varies only the initial wage of a new hire equilibrium contract fills the job.

[56] Less relevant is the decision of the worker to quit if we assume a worker can quit without penalty, but will remain unemployed in the second period. This situation implies that the only participation constraint that matters for period-1 hires is the period-1 constraint. An alternative assumption that leads to this implication is that a worker who changes jobs incurs a high mobility cost. In either case we will ignore the worker quit decision.

will have no incentive to do this (and unemployed workers no incentive to apply for such positions), but for $w_{2,1} < w_{2,2}$ the incentive exists to replace. In the latter case, then, to the extent that the matching process succeeds in selecting a successful applicant, the incumbent is at risk of losing her position. If $w_{2,1} < w_{2,2}$, then a filled job is as attractive as an unfilled one from the point of view of an applicant, as the new hire wage $w_{2,1}$ is assumed to apply to any new hire, and assume there is no cost associated with receiving applicants for filled jobs.[57]

In the expression for the value to a worker at $t = 1$ from being employed by a firm with wage policy $\sigma$, if replacement occurs in some states, that is, if $w_{2,1} < w_{2,2}$, then in such states, the term inside the square brackets in (1) must be replaced by

$$\delta Z_2(x) + (1-\delta)q(\theta_2)\,v(b) + (1-\delta)(1-q(\theta_2))\,v(w_{2,2}(x)).$$

This expression reflects the additional risk $q(\theta_2)$ to a surviving worker of being replaced by a successful applicant.

Likewise, in any state where replacement occurs, the expression for second-period profit is replaced by

$$f((1-\delta)n_1 + n_2; x) - w_{2,2}(1-q(\theta_2))(1-\delta)n_1 - w_{2,1}(n_2 + q(\theta_2)(1-\delta)n_1) - k\overline{n}_2,$$

where $q(\theta_2)(1-\delta)n_1$ is the number of incumbents who are replaced by new hires, and $n_2 = q(\theta_2)\overline{n}_2$ is the number of new hires *into newly created jobs*.[58]

The *consistency condition for equilibrium* must be generalized, so that in any state for which replacement occurs,

$$\theta_2(w_{2,1}(x), Z_2(x)) = S_2/(\overline{n}_2(x) + (1-\delta)q(S/\overline{n}_1)\overline{n}_1).$$

### A.2.1 Replacement in State $x$

Next, we characterize outcomes when replacement does occur.[59] We now identify the unconstrained solution with what the firm can achieve when it can *commit* not to replace more expensive workers.

[57] To be clear, and following MM, in this case, a filled job will attract the same number of applicants as any newly created unfilled job and will have the same probability of a successful applicant being found and, hence, of the incumbent losing his/her position.

[58] MM introduce a sunspot into their model, which allows the firm to randomize between replacement and no replacement. They can then show that an equivalent of (4) must be maximized across replacement/no-replacement regimes and derive analytical sufficient conditions for no replacement to be optimal.

[59] The proof of Proposition 1 assumed that there is no replacement in period 2 in *any* state; even with replacment in some states, the statement still holds for non-replacement states $x$: if there is replacement in some state $x' \neq x$, it modifies the expectation term in (22) and (25), but they cancel one another out).

**Proposition 5** *Suppose that replacement occurs in state $x$. Then, for a given $w_1$ and $n_1$, the wage for new hires is lower (and employment is higher) than they would be if firms were able to commit, $w_{2,1} < w^U_{2,1}(x; w_1, n_1) < w_1$; moreover, $w_{2,2} = w_1$.*

Intuitively, cutting the new hire wage makes a job less attractive, and therefore, given that replacement occurs, the risk of replacement decreases; this positive externality on incumbents makes a wage that is lower than the unconstrained (commitment) wage optimal. The firm should stabilize the wages of the first-period hires because there is no cost of doing this — given that the replacement probability is independent of $w_{2,2}$ whenever $w_{2,1} < w_{2,2}$.

When does replacement improve profits? In our simulations reported above we were able to check this directly. In general, however, consider first the limiting case of a competitive labour market, as in Snell and Thomas (2010). In this case, if $w_{2,1} < w_{2,2}$ in some state, all incumbents will be replaced, provided that $w_{2,1}$ is not below the supply price of unemployed workers, as the firm can then hire as many new hires as it wants. Since the supply price of an unemployed worker in period 2 will be at least as great as what a replaced worker would expect to obtain (we are assuming that the latter will remain unemployed), changing the contract so that $w_{2,2} = w_{2,1}$ clearly does not leave the firm worse off, as it faces the same costs at period 2. Period-1 hires will weakly prefer this contract because they are not replaced. Thus, satisfying the no-undercutting constraint is weakly dominant (and strictly so if the supply price of the unemployed exceeds what a replaced worker obtains). However, this logic may not extend to the frictional labour market; suppose that the cost of vacancy creation is sufficiently low such that $\theta$ is low in equilibrium, that is, $q(\theta)$ is low. Then, the probability of replacement, $q(\theta)$, may be such that a firm is better off by setting $w_{2,1} < w_{2,2}$ and offering full insurance to an incumbent if he/she remains in the firm, but with a small risk of replacement, and offering a lower wage to new hires. In this case, the no-undercutting constraint is (optimally) violated.[60]

# B Proofs

## B.1 Proof of Proposition 1

**Proof.** We derive the necessary conditions by considering the following Lagrangian, assuming that there is an interior solution.

[60] This situation may seem somewhat paradoxical, as a low $k$ implies that search becomes more competitive; but while this is true for new hires, the replacement probability for incumbents will fall to zero, as the ratio of new advertised jobs to existing ones goes to infinity, i.e., the model is not continuous at $k = 0$.

$$\mathcal{L} = \left(f\left(\tilde{q}_1\left(V_1\right)\overline{n}_1\right) - w_1\tilde{q}_1\left(V_1\right)\overline{n}_1 - k\overline{n}_1\right) + E_{x'}[(f\left((1-\delta)\tilde{q}_1\left(V_1\right)\overline{n}_1 + \tilde{q}\left(w_{2,1},x'\right)\overline{n}_2;x'\right) - w_{2,2}(1-\delta)\tilde{q}_1\left(V_1\right)\overline{n}_1$$

where $\tilde{q}_1\left(V_1\right)$ is defined analogously to $\tilde{q}\left(w_{2,1},x\right)$, $\lambda_{x'}$ is the multiplier on the $w_{2,1} \geq w_{2,2}$ constraint in state $x'$ and recall $V_1 = v\left(w_1\right) + E[\delta Z_2\left(x'\right) + (1-\delta)v\left(w_{2,2}\left(x'\right)\right)]$. This expression leads to the FOCs:

$$\tilde{q}_1'v'\left(w_1\right)\overline{n}_1(f'\left(n_1\right) - w_1 + E_{x'}[f'\left(n;x'\right)(1-\delta) - w_{2,2}\left(x'\right)(1-\delta)]) - \tilde{q}_1\left(V_1\right)\overline{n}_1 = 0 \tag{22}$$

$$f'\left(n;x\right)\tilde{q}\left(w_{2,1},x\right) - w_{2,1}\tilde{q}\left(w_{2,1},x\right) - k = 0 \tag{23}$$

$$f'\left(n;x\right)\tilde{q}'\overline{n}_2 - \tilde{q}\left(w_{2,1},x\right)\overline{n}_2 - w_{2,1}\tilde{q}'\overline{n}_2 + \lambda_x = 0 \tag{24}$$

$$\begin{aligned}&\tilde{q}_1'v'\left(w_{2,2}\left(x\right)\right)(1-\delta)\overline{n}_1(f'\left(n_1\right) - w_1 + \\ &E_{x'}[f'\left(n;x'\right)(1-\delta) - w_{2,2}\left(x'\right)(1-\delta)]) - \lambda_x - (1-\delta)\tilde{q}_1\left(V_1\right)\overline{n}_1 = 0\end{aligned} \tag{25}$$

together with the complementary slackness conditions. Note that (23) implies (7) in the text.

From (22) and (25),

$$\frac{v'\left(w_1\right)}{v'\left(w_{2,2}\right)}\left(q_1 + \frac{\lambda_x}{\overline{n}_1(1-\delta)}\right) = q_1. \tag{26}$$

Using this to eliminate $\lambda_x$ in (24):

$$f'\left(n;x\right)\tilde{q}'\overline{n}_2 - \tilde{q}\left(w_{2,1},x\right)\overline{n}_2 - w_{2,1}\tilde{q}'\overline{n}_2 + q_1\overline{n}_1(1-\delta)\left(\frac{v'\left(w_{2,2}\right)}{v'\left(w_1\right)} - 1\right) = 0. \tag{27}$$

There are two cases.

A. If $\lambda_x = 0$, then (26) $w_1 = w_{2,2}$, and (27) implies (6) in the text, and hence, we get (8). We characterize points that satisfy (8). For clarity, we let $\tilde{w}_{2,1}$ and $\tilde{\theta}_2$ denote the individual firm's values. Then

$$\tilde{q}' = \frac{dq}{d\theta_2}\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,1}}\,|_{Z_2\text{ constant}}\,.$$

From (3),

$$\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,1}}\,|_{Z_2\text{ constant}} = -\frac{pv'\left(w_{2,1}\right)}{\frac{dp}{d\theta_2}\left(v\left(w_{2,1}\right) - v\left(b\right)\right)},$$

and differentiating $q = p \cdot \theta_2$ to eliminate $\frac{dp}{d\theta_2}$, we obtain

$$\tilde{q}' = -\frac{dq}{d\theta_2} \frac{p\theta_2 v'(w_{2,1})}{\left(\frac{dq}{d\theta_2} - p\right)(v(w_{2,1}) - v(b))}. \tag{28}$$

After rearrangement,

$$\frac{q^2}{\tilde{q}'} = q^2 \frac{\left(1 - \frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)}{\theta_2 \frac{dq}{d\theta_2}} \frac{v(w_{2,1}) - v(b)}{v'(w_{2,1})}.$$

From our assumption on $q$, $q^2$ is increasing in $\theta_2$, and the second term in the product is also increasing in $\theta_2$ by assumption (it is the inverse of $q(\theta)\epsilon_q(\theta)/(1-\epsilon_q(\theta))$) while the final term is increasing in $w_{2,1}$. Thus, the locus of values of $\theta_2$ and $w_{2,1}$ such that (8) holds is negatively sloped. Recall that $n_2 = p(\theta_2) S_2$, and as $p' < 0$, there is a one-to-one negative relationship between $n_2$ and $\theta_2$. Therefore, (8) can be solved to give a positively sloped locus of values for $n_2$ and $w_{2,1}$ that is compatible with equilibrium.

Next, (23) is negatively sloped in $n_2$–$w_{2,1}$ space by $f'' < 0$ and $q(\theta_2) = q\left(p^{-1}(n_2/S_2)\right)$, $q' > 0$, $p' < 0$. Therefore, $(w_{2,1}, n_2)$ is at the unique intersection point, denoted by $\left(w_{2,1}^U(x; w_1, n_1), n_2^U(x; w_1, n_1)\right)$ in the text. Since $w_{2,1} \geq w_1$ implies $\lambda_x = 0$ (see next line), claim (b) is established.

B. If $\lambda_x > 0$, then $w_{2,2} = w_{2,1}$ and from (26) $w_1 > w_{2,2} = w_{2,1}$, and (27) implies

$$(1-\delta)n_1 - \left(f'(n; x)\tilde{q}'\overline{n}_2 - w_{2,1}\tilde{q}'\overline{n}_2 - q\overline{n}_2\right) = n_1\left(1/v'(w_1)\right)\left((1-\delta)v'(w_{2,1})\right). \tag{29}$$

(This equation also follows from differentiating (4) with respect to $w_{2,1}$ after setting $w_{2,1} = w_{2,2}$.) Thus, eliminating $f'$ using (23), and using $n_2 = q\overline{n}_2$,

$$1 + \frac{\left(1 - k\tilde{q}'/q^2\right)n_2}{n_1(1-\delta)} = \frac{v'(w_{2,1})}{v'(w_1)}, \tag{30}$$

so that as $w_{2,1} < w_1$, $k\tilde{q}'/q^2 < 1$, i.e., $k < q^2/\tilde{q}'$. Holding $n_2$ (and hence $\theta_2$) constant, $q^2/\tilde{q}'$ is increasing in $w_{2,1}$, so the locus of points $(n_2, w_{2,1})$ satisfying (30) must lie above — $w_{2,1}$ is higher — that defined by (8). At $w_{2,1} = w_1$ we have $k\tilde{q}'/q^2 = 1$, so the two loci coincide. Thus, the downward sloping (23) must intersect (30) at a higher wage and a lower value for $n_2$ than it would intersect (8). Thus, claim (a) is established.

Since $\lambda_x > 0$ if and only if $w_{2,1} < w_1$, the final claim of the proposition follows. ■

### B.2 Proof of Proposition 2

**Proof.** (i) In the unconstrained model, consider an equilibrium in the absence of asymmetric information. We have that $w_{2,2}$ is independent of the period-2 state $x$, and $w_{2,1}$

is chosen independently of $w_{2,2}$ to minimize the cost of hiring a new worker in state $x$. With asymmetric information, the firm has no incentive to misreport since the wage paid to non-separated period 1 hires is constant, while any different $w_{2,1}$ can only increase new hire costs. The result follows.

(ii) Let $x' := \hat{x} - \varepsilon$, $x'' := \hat{x} + \varepsilon$. Consider an arbitrary sequence $\{\varepsilon_s\}_{s=0,1,\ldots}$, $\varepsilon_s > 0$, $\varepsilon_s \to 0$; we show that there is some $\bar{s}$ such that for $s \geq \bar{s}$, wages are equal in both states: $w_{2,2}(x') = w_{2,2}(x'') = w_{2,1}(x') = w_{2,1}(x'')$.[61] By the assumptions of continuity and the binding no-undercutting constraint at $\hat{x}$,

$$\lim_{s\to\infty} w_{2,2}\left(x'\right) = \lim_{s\to\infty} w_{2,2}\left(x''\right) = \lim_{s\to\infty} w_{2,1}\left(x'\right) = \lim_{s\to\infty} w_{2,1}\left(x''\right) = \hat{w}_{2,2} = \hat{w}_{2,1}, \tag{31}$$

where the original equilibrium corresponding to $\hat{x}$ is denoted by $\hat{}$. In what follows, we will deal with the case where $w_{2,2}(x') \leq w_{2,2}(x'')$ infinitely often as $s = 0, 1, \ldots$, so we consider below the circumstances in which this is true; the arguments apply equally to the opposite case. To consider this case, we define

$$C\left(w_{2,1}, x''\right) := \left(k/q\left(\theta_2(w_{2,1}, Z_2\left(x''\right))\right) + w_{2,1}\right)$$

and

$$w^{**}\left(x''\right) \in \arg\min_{w_{2,1}} \left(k/q\left(\theta_2(w_{2,1}, Z_2\left(x''\right))\right) + w_{2,1}\right) \tag{32}$$

where $\theta_2(w_{2,1}, Z_2(x''))$ is as defined in (3); $C(w_{2,1}, x'')$ is the cost per period-2 hire in state $x''$ ($k/q + w$ is the total cost of a new hire), while $w^{**}(x'')$ is the wage that minimizes this cost. It is independent of the number of hires, and the cost is strictly convex in $w_{2,1}$ (hence, $w^{**}(x'')$ is unique).

To see this, as earlier, write $q(\theta_2(w_{2,1}, Z_2(x''))) \equiv \tilde{q}(w_{2,1}, x'')$, so

$$\begin{aligned} \frac{dC\left(w_{2,1}, x''\right)}{dw_{2,1}} &= -\frac{k\tilde{q}'}{\tilde{q}^2} + 1 \\ &= -\frac{k}{\tilde{q}^2} \frac{\theta_2 \frac{dq}{d\theta_2}}{\left(1 - \frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)} \frac{v'\left(w_{2,1}\right)}{v\left(w_{2,1}\right) - v\left(b\right)} + 1, \end{aligned} \tag{33}$$

using (28). Given that $\tilde{q}' > 0$ (a higher wage increases the job-filling rate), the second term in the product is $q(\theta_2)\,\epsilon_q(\theta_2) / (1 - \epsilon_q(\theta_2))$ and therefore is decreasing in $\theta_2$ (by assumption) and, hence, also decreasing in $w_{2,1}$, while the final term in the product is also decreasing in $w_{2,1}$, we have

$$\frac{d^2C\left(w_{2,1}, x''\right)}{dw_{2,1}^2} > 0. \tag{34}$$

[61] The dependence of values on $\varepsilon_s$ will mostly be left implicit to avoid the notation becoming more cluttered.

Additionally, given the assumption that the no-undercutting constraint is strictly binding initially, we have $\hat{w}_{2,1} > w^{**} := w^{**}(x)$ (the value for $w^{**}(x'')$ when $\varepsilon = 0$, being equal to the optimal hiring wage in the unperturbed model), and therefore, by (31) and the continuity of $w^{**}(x')$ and $w^{**}(x'')$ in $\varepsilon$ (by the Theorem of the Maximum, as they are both unique by the strict convexity of $C$ and $C$ is continuous in $Z$ and, hence, in $\varepsilon$),

$$\lim_{s\to\infty} w^{**}\left(x'\right) = \lim_{s\to\infty} w^{**}\left(x''\right) = w^{**} < \hat{w}_{2,1}. \tag{35}$$

Profits in period 2, in state $x''$, are

$$\max_{n_2}\left(f\left((1-\delta)n_1 + n_2; x''\right) - w_{2,2}(x'')(1-\delta)n_1 - C\left(w_{2,1}(x''), x''\right) n_2\right).$$

In state $x''$, the firm can claim that $x'$ occurred and make nonnegative savings in wages paid to incumbents because $w_{2,2}(x') \leq w_{2,2}(x'')$. It follows that we must have

$$C\left(w_{2,1}\left(x''\right), x''\right) \leq C\left(w_{2,1}\left(x'\right), x''\right) \tag{36}$$

since otherwise, by announcing $x'$, hiring costs are reduced as well.

There are three possibilities to consider, and at least one of which must occur infinitely often along the sequence $s = 0, 1, \ldots$. First, $w_{2,1}(x') < w_{2,1}(x'')$. From (36), $w_{2,1}(x') < w^{**}(x'')$ by (34). But as $s \to \infty$, a contradiction occurs in view of $\lim_{s\to\infty} w_{2,1}(x') = \hat{w}_{2,1}$ and (35).

On the other hand, if $w_{2,1}(x') > w_{2,1}(x'')$, then by (36) and (34), $w_{2,1}(x') > w^{**}(x'')$. However, we have

$$w_{2,1}\left(x'\right) > w_{2,1}\left(x''\right) \geq w_{2,2}\left(x''\right) \geq w_{2,2}\left(x'\right),$$

where the second inequality follows from no undercutting and the final inequality by hypothesis. However, consider a change where $w_{2,1}(x')$ is cut to $w_{2,1}(x'')$ and $w_{2,2}(x')$ is increased to $w_{2,2}(x'')$ if it is initially below this value. This changed contract satisfies no undercutting and (trivially) incentive compatibility. The decrease in $w_{2,1}(x')$ reduces hiring costs by (31) and (35), which imply $w_{2,1}(x') > w^{**}(x')$ for a large $s$. Additionally, for $s$ large enough, $w_{2,2}(x'') < w_1(\varepsilon_s)$ by the binding no-undercutting constraint in Problem A (from Proposition 1, this implies $\hat{w}_{22} < \hat{w}_1$), (31) and, by assumption, $\lim_{s\to\infty} w_1(\varepsilon_s) = \hat{w}_1$ using an obvious notation. Then, $v'' < 0$ implies that a small reduction in $w_1$ to leave $V_1$ constant will reduce expected wages while leaving hiring constant. Therefore, for a large enough $s$, the contract is not optimal, contrary to the assumption. The final possibility has $w_{2,1}(x') = w_{2,1}(x'')$. By no undercutting, then,

$$w_{2,1}\left(x'\right) = w_{2,1}\left(x''\right) > w_{2,2}\left(x''\right) = w_{2,2}\left(x'\right),$$

where the final equality follows by incentive compatibility (otherwise, $x'$ would be announced because incumbent wages would be lower), and the inequality is strict by the assumption that it not a constant wage contract. Similar to the previous case, both $w_{2,2}(x'')$ and $w_{2,2}(x')$ can be increased by the same small amount without violating incentive compatibility or no undercutting, which is compensated by a small reduction in $w_1(\varepsilon_s)$, reducing expected wages paid to period-1 hires. Thus, again, the equilibrium contract is not optimal, contrary to assumption.

(iii) Period-2 profits from the contract for state $x$ in state $x'$ can be written as

$$\pi\left(x,x'\right) := \max_{n_2}\{f\left((1-\delta)n_1+n_2;x'\right) - w_{2,2}(x)(1-\delta)n_1 - C\left(w_{2,1}(x),x'\right)n_2\}.$$

We proceed in a number of steps. (a) Suppose that there is a binding incentive compatibility constraint between states $x'$ and $x''$ such that $\pi(x',x') = \pi(x'',x')$ and $C(w_{2,1}(x'),x') > C(w_{2,1}(x''),x')$, so the firm benefits from announcing $x''$ in state $x'$ from the point of view of new hire costs. Incentive compatibility implies $w_{2,2}(x') < w_{2,2}(x'')$. Then, consider replacing the $x'$ contract by that at $x''$ (holding $n_1$ constant). This must trivially satisfy incentive compatibility and no undercutting and leave ex post profits unchanged. However, since $w_{2,2}$ is increased in state $x'$, ex ante utility $V_1$ rises, which reduces period-1 hiring costs; hence, profits increase, contrary to optimality. We conclude that $\pi(x',x') = \pi(x'',x')$ implies $C(w_{2,1}(x'),x') \le C(w_{2,1}(x''),x')$, and hence, by incentive compatibility, $w_{2,2}(x') \ge w_{2,2}(x'')$ (and if the first inequality is strict or an equality, so is the second, and vice versa).

(b) Let $X' \subseteq X$ be such that for $x \in X'$, $w_{2,2}(x) > w_1$. We show that $X' = \varnothing$. For $x' \in X'' := X\backslash X'$, $x \in X'$, we cannot have $\pi(x',x') = \pi(x,x')$, since $w_{2,2}(x') < w_{2,2}(x)$, contradicting (a). Hence, $\pi(x',x') > \pi(x,x')$ (incentive compatibility is slack). Hence, we can find (by $X$ finite) an $\eta > 0$ such that $\pi(x',x') \ge \pi(x,x') + \eta$ for all $x' \in X''$, $x \in X'$. Next, cut $w_{2,2}(x)$ by $\varepsilon < \eta\left((1-\delta)n_1\right)^{-1}$ for all $x \in X'$; this does not affect incentive compatibility between $x, x'' \in X'$ as profits change by the same amount in each state, and by construction of $\varepsilon$, $\pi(x',x') > \pi(x,x')$, $x' \in X''$, $x \in X'$. As $\pi(x,x)$ is increased for each $x \in X'$ by $\varepsilon(1-\delta)n_1$, $\pi(x,x) > \pi(x',x)$, $x' \in X''$, as the RHS is unchanged and a weak inequality held before the change. Thus, (global) IC is satisfied. No undercutting is satisfied because only $w_{22}$ is cut. If $X' \neq \emptyset$, for a small enough $\varepsilon$, this uniform cut in $w_{2,2}$ in all states where $w_{2,2} > w_1$ and a corresponding increase in $w_1$ to leave $V_1$ unchanged increases profits by standard consumption smoothing arguments (hold $n_1$ constant), i.e., a profitable deviation that is contrary to the assumption. We conclude that $X' = \emptyset$, i.e., $w_{2,2}(x') \le w_1$ all $x' \in X$.

(c) Let $\hat{X} := \arg\max_{\hat{x}} w_{22}(\hat{x})$. If this is a singleton, $\{x\}$, then by part (a), there is no other state $x'$ with $\pi(x',x') = \pi(x,x')$. It follows that provided that the no undercutting constraint is slack in state $x$, $w_{2,1}(x) = w^{**}(x)$ and, hence, $w_{21}(x) = w^{U}_{2,1}(x, w_1, n_1)$, as otherwise, if $w_{2,1}(x) \neq w^{**}(x)$ a small enough change in $w_{21}$ towards $w^{**}$ increases profits in state $x$ (by the strict convexity of $C(\cdot, x)$), satisfies no undercutting, violates no $\pi(x', x') \geq \pi(x, x')$ constraint for all $x' \neq x$, and relaxes $\pi(x, x) \geq \pi(x', x)$ for $x' \neq x$. If no undercutting binds in state $x$, this argument implies $w_{2,1}(x) \geq w^{**}(x)$, as $w_{2,1}$ can be increased if $w_{21} < w^{**}$ and, hence, $w_{21}(x) \geq w^{U}_{2,1}(x, w_1, n_1)$.

If $\hat{X}$ is not a singleton, by a similar argument, consider $x \in \hat{X}$ such that $w_{2,1}(x) \neq w^{**}(x)$. If no undercutting is not binding at state $x$, change $w_{2,1}(x)$ towards $w^{**}(x)$ by an amount $\varepsilon$ such that $C(w_{2,1}(x), x)$ falls. Again, by part (a) for all $x' \notin \hat{X}$, we have $\pi(x', x') > \pi(x, x')$, and provided that $\varepsilon$ is small enough, these incentive compatibility and no undercutting constraints are not violated. If any incentive compatibility constraint for $x'' \in \hat{X}$ is violated, replace $w_{21}(x'')$ by the new value of $w_{21}(x)$; this increases ex post profits in $x''$ and does not affect period 1, as $w_{22}$ is unchanged. Profits are increased by this change, contrary to the assumption. Hence, $w_{2,1}(x) = w^{**}(x)$ for all $x \in \hat{X}$. If no undercutting binds at the lowest $w_{2,1}(x)$, $x \in \hat{X}$, again, $w_{2,1}(x) \geq w^{**}(x)$. ■

## B.3 Proof of Proposition 3

**Proof.** Incentive compatibility in state $x''$ requires that

$$\begin{aligned}(f((1-\delta)n_1 + n_2(x''); x'') - w_{2,2}(x'')(1-\delta)n_1 - C(w_{2,1}(x''), x'')\, n_2(x'')) \geq \\ (f((1-\delta)n_1 + n_2(x'); x'') - w_{2,2}(x')(1-\delta)n_1 - C(w_{2,1}(x'), x'')\, n_2(x')),\end{aligned} \tag{37}$$

where $C(\cdot, \cdot)$ is the total cost of a new period 2 hire as defined as in the proof of Proposition 2, and hiring in state $x'$ is denoted $n_2(x')$, etc. We will write $w_{2,1}(x')$ as $w'_{2,1}$ etc. to simplify notation below.

We start by assuming that the optimal contract is differentiable (from the right) at $\varepsilon = 0$. Consider $\varepsilon$ small and take a first-order approximation for (37) around the initial equilibrium[62] at $\hat{x}$, where (37) trivially holds with equality (and where as in the proof of Proposition 2 we use a ˆ to denote the corresponding initial equilibrium contract) and defining deviations as $\Delta w'_{2,2} := w'_{2,2} - \hat{w}_{2,2}$ etc., and where $\Delta x''(= -\Delta x') := x'' - \hat{x} = \varepsilon$: $f'(\Delta n''_2 - \Delta n'_2) - (1-\delta)n_1 \left(\Delta w''_{2,2} - \Delta w'_{2,2}\right) - \frac{\partial C}{\partial w} n_2 \left(\Delta w''_{2,1} - \Delta w'_{2,1}\right) - C(\Delta n''_2 - \Delta n'_2) \geq 0$, with the reverse inequality implied by incentive compatibility in state $x'$, so given that $f' = C$ in the initial equilibrium ($\hat{n}_2$ is chosen efficiently given $\hat{w}_{2,1}$ in the absence of incentive compatibility constraints), we get

$$-(1-\delta)n_1 \left(\Delta w''_{2,2} - \Delta w'_{2,2}\right) - \frac{\partial C}{\partial w} n_2 \left(\Delta w''_{2,1} - \Delta w'_{2,1}\right) = 0. \tag{38}$$

[62]That is, we omit terms of order smaller than $\varepsilon$ in the expressions that follow. We assumed that the equilibrium of the model is differentiable in $\varepsilon$ on an interval $[0, \bar{\varepsilon})$ (from the right at 0), so that in particular $C$ is also differentiable in $x$. In the approximation $\partial C / \partial x$ cancels.

Suppose that $\Delta w''_{2,2} < \Delta w'_{2,2}$; we will establish a contradiction. Since $\frac{\partial C}{\partial w} > 0$ (at the initial equilibrium), (38) implies $sgn\left(\Delta w''_{2,2} - \Delta w'_{2,2}\right) = -sgn\left(\Delta w''_{2,1} - \Delta w'_{2,1}\right)$. Hence $\Delta w''_{2,1} > \Delta w'_{2,1}$; thus $w''_{2,2} < w'_{2,2}$ and $w''_{2,1} > w'_{2,1}$ and

$$w''_{2,2} < w'_{2,2} \leq w'_{2,1} < w''_{2,1},$$

where the weak inequality follows by no undercutting in state $x'$.

Consider the following change to the contract (use a $\tilde{}$ to denote this new contract): set wages in $x''$ to equal those in $x'$: increase $w''_{2,2}$ to $\tilde{w}''_{2,2} := w'_{2,2}$ and reduce $w''_{2,1}$ to $\tilde{w}''_{2,1} = w'_{2,1}$; hold $n_1$ constant, set $n_2$ in each state to maximize period 2 profits given $w'_{2,1}$ and $\tilde{w}''_{2,1}$, and change $w_1$ to $\tilde{w}_1$ to keep $V_1$ constant. The cut in $w''_{2,1}$ reduces hiring costs by, for $\varepsilon$ small enough, $w''_{2,1} > w^{**}(x'')$ (the latter being the new hire cost minimizing wage in state $x''$, using notation and the argument in the proof of Proposition 2 above) and as $\tilde{n}''_2$ is chosen optimally, profits on new hires in $x''$ must rise. Likewise as $\tilde{n}'_2$ is chosen optimally profits in $x'$ cannot fall. Incentive compatibility is satisfied trivially. From $V_1$ constant (which implies constant vacancy creation and hence constant period 1 vacancy costs)

$$v(\tilde{w}_1) - v(w_1) + 0.5\beta(1-\delta)\left(v\left(w'_{2,2}\right) - v\left(w''_{2,2}\right)\right) = 0. \tag{39}$$

By $w^*_{2,2} < w^*_1$, $w'_{2,2} < w_1$; also $w'_{2,2} < \tilde{w}_1$ for $\varepsilon$ small enough, so

$$w_1 > \tilde{w}_1 > w'_{2,2} > w''_{2,2}.$$

It follows from (39) and by $v'' < 0$ that

$$w_1 - \tilde{w}_1 > 0.5(1-\delta)\left(w'_{2,2} - w''_{2,2}\right);$$

thus the change in costs of period 1 hires is

$$n_1\left(\tilde{w}_1 - w_1 + 0.5(1-\delta)\left(w'_{2,2} - w''_{2,2}\right)\right) < 0.$$

Thus the new contract is more profitable than the putative equilibrium one, a contradiction. This establishes that $\Delta w''_{2,2} < \Delta w'_{2,2}$ is not possible. Similarly $\Delta w''_{2,2} > \Delta w'_{2,2}$ yields a contradiction. Thus $\Delta w''_{2,2} = \Delta w'_{2,2}$ and so by (38) $\Delta w''_{2,1} = \Delta w'_{2,1}$. It follows that $\left(\Delta w''_{2,1} - \Delta w'_{2,1}\right)/(2\varepsilon) = 0$, which establishes the claim.

Now we allow for the contract to be non-differentiable in $\varepsilon$ (from the right) at $\varepsilon = 0$. It must be (right) continuous at $\varepsilon = 0$, as otherwise profits would also be discontinuous, while a simple constant wage contract would be continuous so would do better.[63] Consider

[63] Profits are bounded above by a contract which ignores the incentive constraint, which would be continuous, so any discontinuity must imply profits jump down for $\varepsilon > 0$. Holding wages constant across states and setting period 2 employment efficiently at those wages as in the construction in the proof of Proposition 2 would satisfy incentive constraints and lead to profits varying continuously; hence this would

a sequence for $\varepsilon \equiv (x'' - x')/2$: $\{\varepsilon_\nu\}$, $\varepsilon_\nu \to 0$ as $\nu \to \infty$. Assume that the no undercutting constraint binds (so that $w_{22} = w_{21} =: w_2$ say) in both states along the sequence (cf. proof of Proposition 2) and that only the downward incentive constraint binds (i.e., (37)). Then by standard arguments $w_2'' \geq w_2'$ and $n_2''$ is at the optimal level given $w_2''$.[64] The other possibilities can be dealt with in an analogous manner. We again suppress the explicit dependence of the optimal contract on $\varepsilon_\nu$ for notational simplicity. We suppose, contrary to hypothesis, that

$$0 < \limsup_{\nu \to \infty} \left| w_2'' - w_2' \right| / \varepsilon_\nu. \tag{40}$$

Rearranging (37):

$$f\left((1-\delta)n_1 + n_2''; x''\right) - f\left((1-\delta)n_1 + n_2'; x''\right) - C\left(w_{2,1}\left(x''\right), x''\right) n_2\left(x''\right) + \\ C\left(w_{2,1}\left(x'\right), x''\right) n_2\left(x'\right) - (1-\delta)n_1\left(w_2'' - w_2'\right) \geq 0. \tag{41}$$

By (40) we can take a subsequence such that $\lim_{\nu\to\infty}\left(w_2'' - w_2'\right)/\varepsilon = a$ where $|a| > 0$, and where $n_1$ converges to say $\tilde{n}_1$, we get after dividing (41) by $\varepsilon_\nu$ and taking the limit:

$$\liminf_{\nu \to \infty} [(f\left((1-\delta)n_1 + n_2''; x''\right) - f\left((1-\delta)n_1 + n_2'; x''\right) - C\left(w_2'', x''\right) n_2'' + \\ C\left(w_2', x''\right) n_2')/\varepsilon_\nu] \geq (1-\delta)\tilde{n}_1 a. \tag{42}$$

By $w_2'' - w_2' \geq 0$, $a > 0$. In other words, assuming for small $\varepsilon$ we have lower wages in state $x'$ than in $x''$ by a first-order amount, implies that the R.H.S. of (42) is positive, that is, there is a (first-order) incentive in state $x''$ to underreport $x$ to benefit from lower wage costs; to offset this (i.e., to preserve incentive compatibility) the level of new hires in state $x'$ needs to be sufficiently different (below in this case) that in $x''$ to lead to a fall in profits from new hires that is also first-order. We show that such a difference in hires would also imply, contrary to optimality, that a deviation contract is profitable which avoids the costs of distorted employment, where wages are constant and employment in state $x'$ is set at an efficient level given wages.

Consider then the following possible deviation contract. In state $x'$ set $w_{2,1} = w_{2,2} = w_2''$, and set $n_2$ at the profit maximizing level in state $x'$ for $w_2''$, say $\tilde{n}_2'$. Change $w_1$ to leave $V_1$ unchanged (and leave hiring in period 1 the same). In period 2 this contract differs only in state $x'$, satisfies no undercutting, and is incentive compatible as wages are the same across states and $n_2$ is chosen optimally in each state. Considering only incumbents the wage increase from $w_2'$ to $w_2''$ must increase profits once the reduction in $w_1$ is taken into account ($w_2' < w_1$ implies that more smoothing reduces wage costs). As overall profits cannot be improved by any deviation, the change in profits in state $x'$ ignoring incumbents

be a profitable deviation.

[64] I.e., it maximizes $f\left((1-\delta)n_1 + n_2''; x''\right) - C\left(w_2'', x''\right) n_2''$.

must be nonpositive, i.e.,

$$
\begin{aligned}
&0 \geq \qquad (43)\\
&\left(f\left((1-\delta)n_1+\tilde{n}_2';x'\right)-C\left(w_2'',x'\right)\tilde{n}_2'\right)-\left(f\left((1-\delta)n_1+n_2';x'\right)-C\left(w_2',x'\right)n_2'\right)\geq\\
&\left(f\left((1-\delta)n_1+n_2'';x'\right)-C\left(w_2'',x'\right)n_2''\right)-\left(f\left((1-\delta)n_1+n_2';x'\right)-C\left(w_2',x'\right)n_2'\right),
\end{aligned}
$$

where the second inequality follows by definition of $\tilde{n}_2'$ yielding at least as much profit as $n_2''$ at $w_2''$. Dividing the R.H.S. of (43) by $\varepsilon_\nu$, note that this differs from the term in square brackets in (42) only by the argument in $x$, so that given differentiability of $f$ and $C$ in $x$ the two expressions differ by a term of order less than $\varepsilon_\nu$.[65] So taking the limit as $\nu \to \infty$, we get the same value, which is a contradiction as from (42) it is at least $(1-\delta)\tilde{n}_1 a > 0$, whereas from (43) it is nonpositive. ■

## B.4 Proof of Proposition 4.

**Proof.** (a) 1. We use a variational argument. Starting from the optimal contract, consider frontloading wages between $t$ and $t+1$ in some state $x_{t+1} = x \in X$ (we hold $x^t$ and $x$ fixed throughout the proof). Reduce the wage for some cohort with tenure $i+1$ at $t+1$ after state $x$ by a small amount $\Delta$, and increase the wage for this cohort at $t$ by $\eta$ so as to leave the worker indifferent; do not change the contract, or vacancy creation, otherwise. This implies that

$$-\pi_{x_t x}\left(1-\delta\right)\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right)\Delta + v'\left(w_{t,i}(x^t)\right)\eta \simeq 0. \qquad (44)$$

This frontloading satisfies all constraints: worker utility falls at $t+1$, and so from this point on the no undercutting constraints are satisfied; similarly, the no undercutting constraint is also satisfied for the cohort both at $x^t$ and earlier because utility is held constant over the two periods; likewise the initial utility offered to this cohort is unchanged so hiring remains constant.

The change in profits (viewed from $x^t$) per worker in this cohort is $\Delta P = \pi_{x_t x}\beta_f$ $(1-\delta)\Delta - \eta$. Using (44) to eliminate $\eta$ gives the change in profits as

$$\Delta P \simeq \pi_{x_t x}\beta_f\left(1-\delta\right)\Delta - \frac{\pi_{x_t x}\left(1-\delta\right)\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right)\Delta}{v'\left(w_{t,i}(x^t)\right)}. \qquad (45)$$

The change in profits cannot be positive by optimality of the original contract, i.e., $\Delta P \leq 0$, so using (45) (by considering $\Delta$ sufficiently small the approximation in (45) can be made

[65] I.e., by a term $h(\varepsilon) = o(\varepsilon)$ so that $h(\varepsilon)/\varepsilon \to 0$ as $\varepsilon \to 0$. This follows as the derivative of the R.H.S. of (43) with respect to $x$ at the limit contract, i.e., the initial ($\varepsilon = 0$) contract, equals zero. (Recall that by continuity $n_2'$, $n_2''$, converge to the same value, etc.)

arbitrarily precise) we get

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) \geq \beta_f v'\left(w_{t,i}(x^t)\right). \tag{46}$$

2. Next, consider backloading wages, i.e., repeat the above arguments but for an increase in the wage at $t+1$ of $\Delta$, offset by a decrease of $\eta$ at $t$. Note that in this case the $t+1$ no undercutting constraint *will* be violated if it is binding initially. By an analogous argument to the above, backloading is profitable if

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) > \beta_f v'\left(w_{t,i}(x^t)\right). \tag{47}$$

In this case the $t+1$ no undercutting constraint must bind, as otherwise a small backloading would increase profits, and would violate no other constraints by a similar logic to that given above. Thus if the $t+1$ no undercutting constraint is slack, from (46) and the negation of (47),

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) = \beta_f v'\left(w_{t,i}(x^t)\right). \tag{48}$$

3. Suppose that

$$\beta_w v'\left(w_{t+1,1}(x^t,x)\right) > \beta_f v'\left(w_{t,i}(x^t)\right), \tag{49}$$

or equivalently, $\tilde{w}_{t+1,i+1} > w_{t+1,1}$. Then there are two possibilities.

A. The no-undercutting constraint is binding for this cohort at $t+1$ (i.e., at $t+1$, the cohort with tenure $i+1$):

$$E[\sum_{\tau=t+1}^{T}(\beta_f(1-\delta))^{\tau-t-1}w_{\tau,\tau-t+i} \mid (x^t,x)] = E[\sum_{\tau=t+1}^{T}(\beta_f(1-\delta))^{\tau-t-1}w_{\tau,\tau-t} \mid (x^t,x)].$$

The continuation utilities offered by the two contracts at $t+1$ must be the same: If

$$V_{t+1,i+1}(\sigma;Z,x^{t+1}) < V_{t+1,1}(\sigma;Z,x^{t+1}),$$

then it would be optimal for the firm to replace the incumbent continuation by the new hire one since this cannot violate any constraint (both satisfy all no undercutting constraints from $t+2$), and would allow the firm to adjust $w_{t+1,i+1}(x^t,x)$ downwards to equalize continuation utilities, slackening the $t+1$ constraint which is therefore satisfied, and saving costs. If on the other hand

$$V_{t+1,i+1}(\sigma;Z,x^{t+1}) > V_{t+1,1}(\sigma;Z,x^{t+1}),$$

then replacing the new hire by the incumbent continuation will not violate any constraints, which are satisfied by the incumbent continuation. The higher utility offered to new hires allows the firm to reduce vacancies while hiring the same number, reducing costs, again a contradiction. So

$$V_{t+1,i+1}(\sigma; Z, x^{t+1}) = V_{t+1,1}(\sigma; Z, x^{t+1}).$$

It follows that $w_{t+1,i+1}(x^t, x) = w_{t+1,1}(x^t, x)$. To see this, suppose not and consider taking a convex combination of the two continuation contracts,

$$w^c_\tau(x^\tau) := 0.5 w_{\tau,\tau-t+i}(x^\tau) + 0.5 w_{\tau,\tau-t}(x^\tau),$$

$\tau = t+1, t+2, \ldots$, where $x^\tau$ is any history that starts with $(x^t, x)$. By strict concavity of $v$ this contract offers a higher discounted utility at $t+1$ than the initial contracts, costs the same at $t+1$, and must satisfy the no-undercutting constraint after $x^{\tau'}$ since

$$\begin{aligned} &E[\sum_{\tau=\tau'}^{T} (\beta_f(1-\delta))^{\tau-\tau'} w^c_\tau \mid x^{\tau'}] = \\ &0.5E[\sum_{\tau=\tau'}^{T} (\beta_f(1-\delta))^{\tau-\tau'} w_{\tau,\tau-t+i} \mid x^{\tau'}] + 0.5E[\sum_{\tau=\tau'}^{T} (\beta_f(1-\delta))^{\tau-\tau'} w_{\tau,\tau-t} \mid x^{\tau'}] \\ &\qquad \leq E[\sum_{\tau=\tau'}^{T} (\beta_f(1-\delta))^{\tau-\tau'} w_{\tau,\tau-\tau'+1} \mid x^{\tau'}], \end{aligned}$$

using equation (18). As above, using the convex contract would allow the firm to cut costs, a contradiction.

B. The no-undercutting constraint is slack for this cohort at $t+1$. Then by the arguments in 1. and 2. above we have

$$\beta_w v'\left(w_{t+1,i+1}(x^t, x)\right) = \beta_f v'\left(w_{t,i}(x^t)\right).$$

From (49) $\beta_w v'\left(w_{t+1,1}(x^t, x)\right) > \beta_w v'\left(w_{t+1,i+1}(x^t, x)\right)$, so we get

$$w_{t+1,1}(x^t, x) < w_{t+1,i+1}(x^t, x). \tag{50}$$

So the incumbent contract is cheaper, from $t+1$. To be cheaper, in view of (50), there must be a date $\tau \geq t+1$ and a continuation history $(\tilde{x}^\tau, x')$ with the property that the wage ranking reverses in the next period, i.e.,

$$w_{\tau,\tau-t}\left(\tilde{x}^\tau, x'\right) < w_{\tau,\tau-t+i}\left(\tilde{x}^\tau, x'\right) \tag{51}$$

and

$$w_{\tau+1,\tau-t+1}\left(\tilde{x}^\tau, x'\right) > w_{\tau+1,\tau-t+i+1}\left(\tilde{x}^\tau, x'\right), \tag{52}$$

and discounted costs for the incumbent contract are lower from $\tau+1$. However this implies that the constraint for the incumbent contract is slack at $\tau+1$, so from 2. above, (48) holds for the incumbent cohort (with $t = \tau$):

$$\beta_w v'\left(w_{\tau+1,\tau-t+i+1}\left(\tilde{x}^\tau, x'\right)\right) = \beta_f v'\left(w_{\tau,\tau-t+i}\left(\tilde{x}^\tau, x'\right)\right).$$

Then from (51) and (52),

$$\beta_w v'\left(w_{\tau+1,\tau-t+1}\left(\tilde{x}^\tau, x'\right)\right) < \beta_f v'\left(w_{\tau,\tau-t}\left(\tilde{x}^\tau, x'\right)\right),$$

which violates (46) and we have a contradiction. Thus only case A is possible, and $w_{t+1,i+1}(x^t, x) = w_{t+1,1}(x^t, x) < \tilde{w}_{t+1,i+1}\left(x^t\right)$.

4. Finally, suppose that

$$\beta_w v'\left(w_{t+1,1}(x^t, x)\right) \leq \beta_f v'\left(w_{t,i}(x^t)\right), \tag{53}$$

or equivalently, $\tilde{w}_{t+1,i+1} \leq w_{t+1,1}$. If

$$\beta_w v'\left(w_{t+1,i+1}(x^t, x)\right) > \beta_f v'\left(w_{t,i}(x^t)\right), \tag{54}$$

by part 2. the $t+1$ no undercutting constraint is binding, and using (53),

$$w_{t+1,i+1}(x^t, x) < w_{t+1,1}(x^t, x). \tag{55}$$

We have two contracts costing the same; repeating the argument of part 3 case A, we can again show

$$w_{t+1,i+1}(x^t, x) = w_{t+1,1}(x^t, x),$$

which contradicts (55). Thus (54) cannot hold, and so, given (46), (48) holds.

(b) If $\beta_w v'\left(w_{t+1,1}\right) > \beta_f v'\left(w_{t,1}\right)$, consider backloading the wages of cohort $t$ (i.e., the cohort hired at $t$) and any other cohorts with $w_{t,i} = w_{t,1}$, with an increase in the wage at $t+1$ of $\Delta$, offset by a decrease of $\eta$ at $t$ in the new state, so utility at $t$ is unchanged, and also increase $w_{t+1,1}$ by $\Delta$. Choose $\Delta$ sufficiently small that $w_{t,1} - \eta > w_{t,i}$ for all $i$ such that $w_{t,i} < w_{t,1}$. Otherwise hold contracts and hiring constant. This change does not violate any constraints: at $t$ and $t+1$ the cost of all the affected cohorts and new hires change by the same amount, while at $t$ the cost of other cohorts remains less than that of new hires by part a) and $w_{t,1} - \eta > w_{t,i}$, and at $t+1$ the cost of new hires rises; so no undercutting is satisfied. Moreover $V_{t,1}$ is unchanged so hiring is unaffected at $t$ with an unchanged number of vacancies. By part (a), $w_{t+1,2} = w_{t+1,1}$, so $\beta_w v'\left(w_{t+1,2}\right) = \beta_w v'\left(w_{t+1,1}\right) > \beta_f v'\left(w_{t,1}\right)$, and so following the logic of part 2. of (a) the firm's costs of employing cohort $t$ and any other affected cohorts are reduced by the backloading. At $t+1$ however wage costs of new hires increase, and we need to show that the net hiring costs do not increase by more than the backloading savings. Suppose contrary to the proposition that $w_{t+1,1}$ lies on or below the unconstrained quasi-supply curve at $n_{t+1}$, that is, $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} \leq k$. The cost per new hire incurred at $t+1$ is $w_{t+1} + k/q$, which changes by $\eta\left(1 - k\tilde{q}'/q^2\right) \leq 0$ to a first-order approximation; the backloading produces a first-order reduction in costs (see part 2. of (a)) hence for $\Delta$ small enough, profits are increased, contradicting the assumed optimality of the original contract.

If $\beta_w v'\left(w_{t+1,1}\right) \leq \beta_f v'\left(w_{t,1}\right)$, then by part (a) $w_{t+1,1} \geq w_{t+1,i}$ for all $i$. If $w_{t+1,1}$ lies below the unconstrained quasi-supply curve at $n_{t+1}$, that is, $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} < k$, then holding hiring constant at $t+1$, the cost per new hire incurred at $t+1$, $w_{t+1,1} + k/q$, is decreasing in $w_{t+1,1}$ (taking the first derivative). Raising $w_{t+1,1}$ cannot violate no undercutting and would increase profits. If $w_{t+1,1}$ lies above the unconstrained quasi-supply curve at $n_{t+1}$, that is, $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} > k$, and if $\beta_w v'\left(w_{t+1,1}\right) < \beta_f v'\left(w_{t,1}\right)$, then cutting $w_{t+1,1}$ would increase profits (as $d(w_{t+1,1} + k/q)/dw_{t+1,1} > 0$) and since $w_{t+1,1} > w_{t+1,i}$ all $i$, no undercutting is not violated for a sufficiently small cut; if $\beta_w v'\left(w_{t+1,1}\right) = \beta_f v'\left(w_{t,1}\right)$ then there is a first-order reduction in hiring costs if $w_{t+1,1}$ is cut, and to avoid no undercutting being violated at $t+1$, cut the wages at $t+1$ of all cohorts $i$ with $w_{t,i} = w_{t,1}$ by the same amount, increasing wages at $t$ to leave utilities unchanged; by initial optimality this frontloading will only have a second-order effect on costs so there is overall an increase in profits for a small enough change. Again this contradicts optimality of the original contract. ■

### B.5 Proof of Proposition 5

**Proof.** If replacement occurs, as in Section 2.1, the firm must locally maximize profits plus weighted incumbent utility:

$$f\left((1-\delta)n_1 + n_2; x\right) - w_{2,2}(1-\delta)(1-q)n_1 - w_{2,1}\left(q(1-\delta)n_1 + n_2\right) - k\overline{n}_2 \\ + n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\left(1-q\right)v\left(w_{2,2}\right) + \delta Z_2 + \left(1-\delta\right)qv\left(b\right)\right),$$

where $\overline{n}_2$ is again the number of *new* jobs created, and $n_2 = q\left(\theta\left(w_{2,1}, Z_2\left(x\right)\right)\right)\overline{n}_2$. This situation differs from (4) in that the probability of replacement $q$ is accounted for in the composition of period-2 workers and workers' period-1 utility. Then, differentiating with respect to $w_{2,2}$,

$$(1-\delta)(1-q)n_1 = n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\left(1-q\right)v'\left(w_{2,2}\right)\right),$$

so that $w_1 = w_{2,2}$, as expected. Differentiating with respect to $w_{2,1}$, we obtain

$$f'\left(n; x\right)\tilde{q}'\overline{n} + (1-\delta)n_1(w_{2,2} - w_{2,1})\tilde{q}' - w_{2,1}\tilde{q}'\overline{n}_2 - q\left((1-\delta)n_1 + \overline{n}_2\right) + \\ n_1\left(1/v'\left(w_1\right)\right)(1-\delta)\left(q'\right)\left(v\left(b\right) - v\left(w_{2,2}\right)\right) = 0$$

where the latter term is the extra cost of compensating period-1 hires for their increased likelihood of replacement (defining $\tilde{q}'$ as before). Differentiating with respect to $\overline{n}_2$,

$$f'(n;x)\,q = w_{2,1}q + k. \tag{56}$$

Thus, employment is on the labour demand curve, as in (7). We can combine these latter two equations to obtain

$$(k/q)\,\tilde{q}'\overline{n}_2 + (1-\delta)n_1\tilde{q}'\left((w_{2,2}-w_{2,1}) + \left(1/v'(w_1)\right)(v(b) - v(w_{2,2}))\right) = \\ q\left((1-\delta)n_1 + \overline{n}_2\right)$$

or

$$k\tilde{q}'/q^2 = 1 + (1-\delta)n_1\tilde{q}'\left((w_{2,1}-w_{2,2}) + \left(1/v'(w_1)\right)(v(w_{2,2}) - v(b))\right)/q\overline{n}_2 + \\ (1-\delta)n_1/\overline{n}_2 \tag{57}$$

Both the second and third terms on the RHS of (57) are positive, the second as $v$ is concave, $w_{2,2} = w_1$ from the above, $w_{2,2} > w_{2,1}$ (as replacement occurs) and $b \leq w_{2,1}$. Recall from the proof of Proposition 1 that $\tilde{q}'/q^2$ is decreasing in $\theta$ and $w_{2,1}$.Thus, in comparison to the unconstrained quasi-supply given by (8), at fixed $\theta$, or equivalently fixed $n_2$ given $n_2 = p(\theta_2)\,S_2$ as in Figure 2, $w_{2,1}$ must be lower to satisfy (57). Thus, the intersection with the downward sloping (7) must occur at a lower wage and higher employment than in the unconstrained (commitment) solution.

Finally, $w^U_{2,1}(x; w_1, n_1) < w_1$ because otherwise, the commitment solution could be implemented, which would be superior. ■

# C Further Tables

Table A1: Number of Spells of Incumbent and Newly Hired Workers

| Year | Newly Hired | Incumbents |
|---|---|---|
| 1978 | 536,480 | 860,131 |
| 1979 | 580,482 | 1,070,423 |
| 1980 | 562,231 | 1,254,231 |
| 1981 | 472,966 | 1,423,195 |
| 1982 | 383,748 | 1,535,036 |
| 1983 | 384,038 | 1,607,852 |
| 1984 | 421,761 | 1,650,744 |
| 1985 | 433,296 | 1,703,623 |
| 1986 | 480,197 | 1,829,471 |
| 1987 | 467,208 | 1,925,379 |
| 1988 | 501,192 | 2,008,610 |
| 1989 | 580,223 | 2,080,315 |
| 1990 | 674,453 | 2,164,259 |
| 1991 | 651,557 | 2,284,766 |
| 1992 | 569,494 | 2,394,251 |
| 1993 | 482,607 | 2,431,712 |
| 1994 | 496,822 | 2,428,188 |
| 1995 | 516,571 | 2,416,687 |
| 1996 | 481,872 | 2,408,716 |
| 1997 | 481,019 | 2,405,614 |
| 1998 | 524,318 | 2,392,430 |
| 1999 | 580,765 | 2,385,722 |
| 2000 | 601,915 | 2,445,300 |
| 2001 | 558,655 | 2,454,149 |
| 2002 | 471,745 | 2,444,711 |
| 2003 | 424,415 | 2,505,278 |
| 2004 | 395,014 | 2,473,805 |
| 2005 | 391,361 | 2,443,718 |
| 2006 | 441,206 | 2,449,759 |
| 2007 | 487,477 | 2,465,401 |
| 2008 | 474,157 | 2,506,474 |
| 2009 | 400,230 | 2,502,328 |
| 2010 | 462,299 | 2,502,616 |
| 2011 | 444,522 | 2,409,295 |
| 2012 | 430,893 | 2,480,722 |
| 2013 | 418,203 | 2,519,325 |
| 2014 | 432,368 | 2,521,718 |
| Total | 18,097,160 | 79,785,954 |

Note: Newly hired workers identified using the first employment spell in a firm.

Table A2: GDP, CPI, Population, and Unemployment Rate

| Year | Nominal GDP (in Mill. Euros) | CPI | Population (in 1,000) | Unemployment rate (in %) |
|---|---|---|---|---|
| 1978 | 678,940 | 47.6 | 61,322 | 4.3 |
| 1979 | 737,370 | 49.5 | 61,439 | 3.8 |
| 1980 | 788,520 | 52.2 | 61,658 | 3.8 |
| **1981** | 825,790 | 55.5 | 61,713 | 5.5 |
| **1982** | 860,210 | 58.4 | 61,546 | 7.5 |
| 1983 | 898,270 | 60.3 | 61,307 | 9.1 |
| 1984 | 942,000 | 61.8 | 61,049 | 9.1 |
| 1985 | 984,410 | 63.0 | 61,020 | 9.3 |
| 1986 | 1,037,130 | 63.0 | 61,140 | 9 |
| 1987 | 1,065,130 | 63.1 | 61,238 | 8.9 |
| 1988 | 1,123,290 | 63.9 | 61,715 | 8.7 |
| 1989 | 1,200,660 | 65.7 | 62,679 | 7.9 |
| 1990 | 1,306,680 | 67.5 | 63,726 | 7.2 |
| 1991 | 1,415,800 | 70.2 | 64,485 | 6.2 |
| **1992** | 1,485,759 | 73.8 | 65,289 | 6.4 |
| **1993** | 1,503,858 | 77.1 | 65,740 | 8.0 |
| 1994 | 1,556,575 | 79.1 | 66,007 | 9.0 |
| 1995 | 1,606,164 | 80.5 | 66,342 | 9.1 |
| **1996** | 1,625,847 | 81.6 | 66,583 | 9.9 |
| 1997 | 1,664,512 | 83.2 | 66,688 | 10.8 |
| 1998 | 1,711,722 | 84.0 | 66,747 | 10.3 |
| 1999 | 1,751,665 | 84.5 | 66,946 | 9.6 |
| 2000 | 1,799,706 | 85.7 | 67,140 | 8.4 |
| 2001 | 1,856,557 | 87.4 | 65,323 | 8.0 |
| **2002** | 1,879,896 | 88.6 | 65,527 | 8.5 |
| **2003** | 1,888,205 | 89.6 | 65,619 | 9.3 |
| 2004 | 1,933,051 | 91.0 | 65,680 | 9.4 |
| **2005** | 1,960,396 | 92.5 | 65,698 | 11 |
| 2006 | 2,038,803 | 93.9 | 65,667 | 10.2 |
| 2007 | 2,142,032 | 96.1 | 65,664 | 8.3 |
| **2008** | 2,180,829 | 98.6 | 65,541 | 7.2 |
| **2009** | 2,088,073 | 98.9 | 65,422 | 7.8 |
| 2010 | 2,191,138 | 100.0 | 65,426 | 7.4 |
| 2011 | 2,298,449 | 102.1 | 64,429 | 6.7 |
| **2012** | 2,345,295 | 104.1 | 64,619 | 6.6 |
| 2013 | 2,401,853 | 105.7 | 64,848 | 6.7 |
| 2014 | 2,483,514 | 106.7 | 65,223 | 6.7 |

Note: Identified downswing years are indicated in bold year numbers. Real GDP per capita calculated using nominal GDP, CPI, and population. Sources for the nominal GDP for West Germany: German Federal Statistical Office & the Federal Statistical Offices of the Federal States. Source German CPI: Federal Reserve Bank of St. Louis (FRED Economic Data). Source West German Population: German Federal Statistical Office. Source West German unemployment rate (in % of total civilian workforce): Sachverständigenrat.

Table A3: Classification of Economic Activities, Edition 2008 (WZ 2008)

| Industry | WZ 2008 | |
|---|---|---|
| | Section | Description |
| 1 | A | AGRICULTURE, FORESTRY AND FISHING |
| 2 | B | MINING AND QUARRYING |
| - | C | MANUFACTURING |
| 3 | C 10-12 | Manufacture of: food products / beverages / tobacco products |
| 4 | C13-15 | Manufacture of: textiles / wearing apparel / leather and related products |
| 5 | C16+31 | Manufacture of: wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials / furniture |
| 6 | C17-18 | Manufacture of paper and paper products / Printing and reproduction of recorded media |
| 7 | C19-25 | Manufacture of: coke and refined petroleum products / chemicals and chemical products / basic pharmaceutical products and pharmaceutical preparations / rubber and plastic products / other non-metallic mineral products / basic metals / fabricated metal products, except machinery and equipment |
| 8 | C26-28 | Manufacture of: computer, electronic and optical products / electrical equipment / machinery and equipment n.e.c. |
| 9 | C29-30 | Manufacture of: motor vehicles, trailers and semi-trailers / other transport equipment |
| 10 | C32 | Other manufacturing |
| 11 | C33 | Repair and installation of machinery and equipment |
| 12 | D | ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY |
| 13 | E | WATER SUPPLY; SEWERAGE, WASTE MANAGEMENT AND REMEDIATION ACTIVITIES |
| 14 | F | CONSTRUCTION |
| 15 | G | WHOLESALE AND RETAIL TRADE; REPAIR OF MOTOR VEHICLES AND MOTORCYCLES |
| 16 | H | TRANSPORTATION AND STORAGE |
| 17 | I | ACCOMMODATION AND FOOD SERVICE ACTIVITIES |
| 18 | J | INFORMATION AND COMMUNICATION |
| 19 | K | FINANCIAL AND INSURANCE ACTIVITIES |
| 20 | L | REAL ESTATE ACTIVITIES |
| 21 | M | PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACTIVITIES |
| 22 | N | ADMINISTRATIVE AND SUPPORT SERVICE ACTIVITIES |
| 23 | O | PUBLIC ADMINISTRATION AND DEFENCE; COMPULSORY SOCIAL SECURITY |
| 24 | P | EDUCATION |
| 25 | Q | HUMAN HEALTH AND SOCIAL WORK ACTIVITIES |
| 26 | R | ARTS, ENTERTAINMENT AND RECREATION |
| 27 | S | OTHER SERVICE ACTIVITIES |
| 28 | T | ACTIVITIES OF HOUSEHOLDS AS EMPLOYERS; UNDIFFERENTIATED GOODS- AND SERVICES-PRODUCING ACTIVITIES OF HOUSEHOLDS FOR OWN USE |
| 29 | U | ACTIVITIES OF EXTRATERRITORIAL ORGANIZATIONS AND BODIES |

Note: n.e.c. = not elsewhere classified. Source: Destatis.