Leverage Stacks and the Financial System

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A (very) brief history of how modern macrotheory has treated financial markets

Phase 1. RBC model: Robinson B. Crusoe

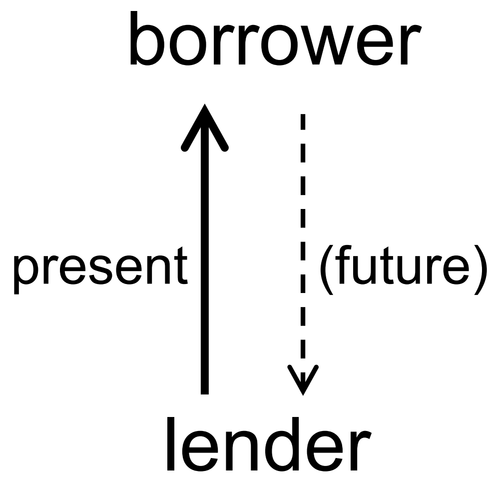
(aka Adam)

no financial markets

in fact, no markets of any kind

Phase 2. Adam and Eve

Frictions in financial market:
(between Adam and Eve)



$\Rightarrow$ level of aggregate activity (investment)
is affected by distribution of net worth

Simple example

borrower has net worth w

and has constant-returns investment opportunity:

$$\text{net rate of return on investment} = r$$

lender has lower opportunity cost of funds:

$$\text{net rate of interest on loans} = r^* < r$$

but only lends against $\theta^* < \dfrac{1+r^*}{1+r}$ of gross return

$$\text{e.g. } r = 3\%,\ \ r^* = 2\%,\ \ \theta^* = 9/10$$

borrower's flow-of-funds:

$$\underset{\text{investment}}{i} \quad \leq \quad w \quad + \quad \underbrace{\left(\frac{1}{1 + r^*}\right)d}_{\text{borrowing}}$$

$$\text{s.t.} \qquad \underset{\text{debt}}{d} \quad \leq \quad \underset{\text{pledgable return}}{\theta^*(1 + r)i}$$

with maximal levered investment:

$$i \quad = \quad \frac{w}{\left(1 - \frac{\theta^*(1+r)}{1+r^*}\right)}$$

net rate of return on *levered* investment equals

$$\frac{(1-\theta^*)(1+r)i \; - \; w}{w}$$

$$= \quad r \quad + \quad \frac{\dfrac{\theta^*(1+r)}{1+r^*}}{\left(1 - \dfrac{\theta^*(1+r)}{1+r^*}\right)}(r - r^*)$$

$\approx$ 12% when $r = 3\%$, $r^* = 2\%$, $\theta^* = 9/10$

12% seems awfully high (cf 3%). Double check:

Suppose net worth $w = 100$

$\theta^* = 9/10 \Rightarrow$ borrow $b = 900$ approx

$\Rightarrow$ invest $i = 1000$

$r = 3\% \Rightarrow$ gross return = 1030

$r^* = 2\% \Rightarrow$ gross debt repayment = 918

$\Rightarrow$ net return = 112

ie. net rate of return on levered investment = 12%

Phase 3. Adam, Eve and the Serpent

Financial intermediation:

borrower

$\uparrow$

bank

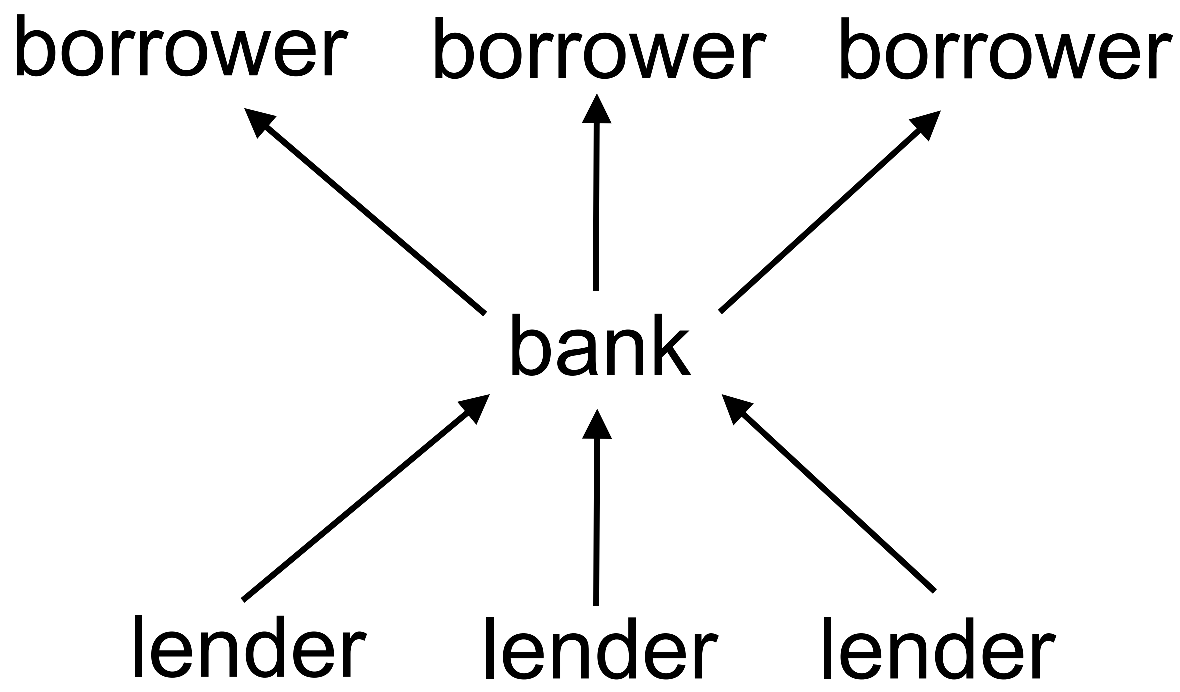
$\uparrow$

lender

In this “double-decker” model, the distribution of net worth across all three types of agent matters

Why not direct lending?

Many possible reasons why not. Individual lenders may not know enough about (or have enough control over) individual borrowers to make direct loans, and instead rely on bank's expertise



Key idea: debt secures debt

lender's loan to bank is secured against bank's loan to borrower

– *not* secured directly against the underlying investment project of the borrower

cf repo markets

Phase 4. Adam, Eve and two Serpents

borrower

↑

bank

↑

bank

↑

lender

Isn't a "triple-decker" model rather OTT?

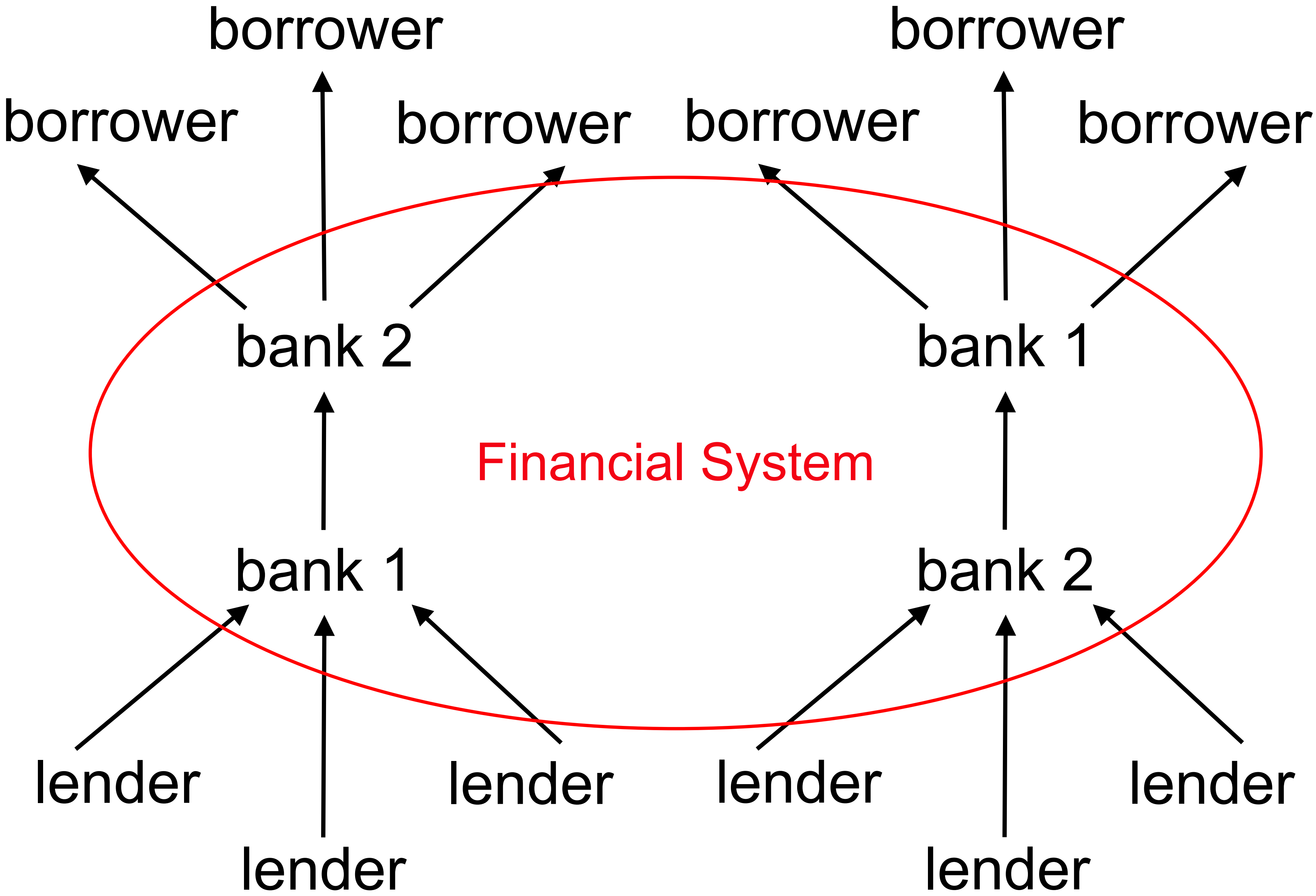
Shouldn’t we be applying Occam’s Razor?

> “Numquam ponenda est pluralites
> sine necessitate”

William of Ockham (1285-1349),
English logician and theologian

But a triple-decker model is needed to understand the financial system

– in particular, to understand systemic risk


borrower
borrower
borrower
borrower
borrower
borrower
bank 2
bank 1
Financial System
bank 1
bank 2
lender
lender
lender
lender
lender
lender

Two questions:

Q1 “Why hold mutual gross positions?”

Why should a bank borrow from another bank and simultaneously lend to that other bank (or to a third bank), even at the same rate of interest?

Q2 “Do gross positions create systemic risk?”

Is a financial system without netting – where banks lend to and borrow from each other (as well as to and from outsiders) – more fragile than a financial system with netting?

Leverage Stacks

entrepreneurs

$\left\{ \begin{array}{l} \text{interest rate } r^{**} \\ \text{credit limit } \theta^{**} \end{array} \right.$

bank

$\left\{ \begin{array}{l} \text{interest rate } r \\ \text{credit limit } \theta \end{array} \right.$

bank

$\left\{ \begin{array}{l} \text{interest rate } r^{*} \\ \text{credit limit } \theta^{*} \end{array} \right.$

households

where $r^{**} > r > r^{*}$ eg. $r^{**} = 5\%$, $r = 3\%$, $r^{*} = 2\%$

$\theta^{**} = \theta = \theta^{*} = 9/10$

A bank has two feasible strategies:

"Outside lending" (to entrepreneurs),
levered by "inside borrowing" (from another bank)

e.g. outside lending at $r^{**} = 5\%$,
9/10 levered by inside borrowing at $r = 3\%$,
yields a net return of $\approx$ 23%.

"Inside lending" (to another bank),
levered by "outside borrowing" (from households)

e.g. inside lending at $r = 3\%$,
9/10 levered by outside borrowing at $r^* = 2\%$,
yields a net return of $\approx$ 12%.

levered outside lending (@ 23%)

$>$ levered inside lending (@ 12%)

$\Rightarrow$ all banks should adopt 23% strategy

But, in formal model, not all banks can do so:

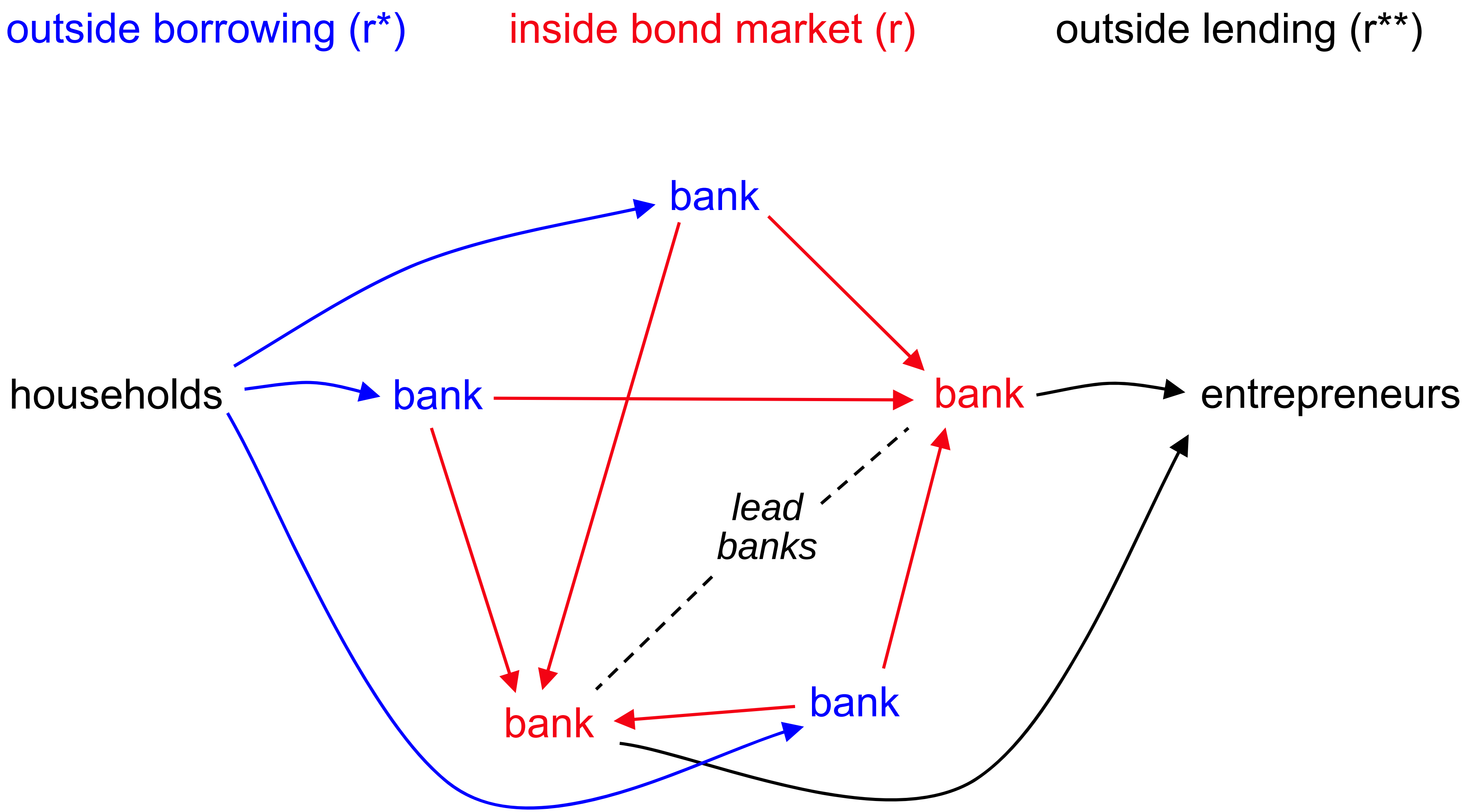
outside lending opportunities are periodic

specifically, we assume:

at each date, a bank has an outside lending opportunity with probability $\pi < 1$

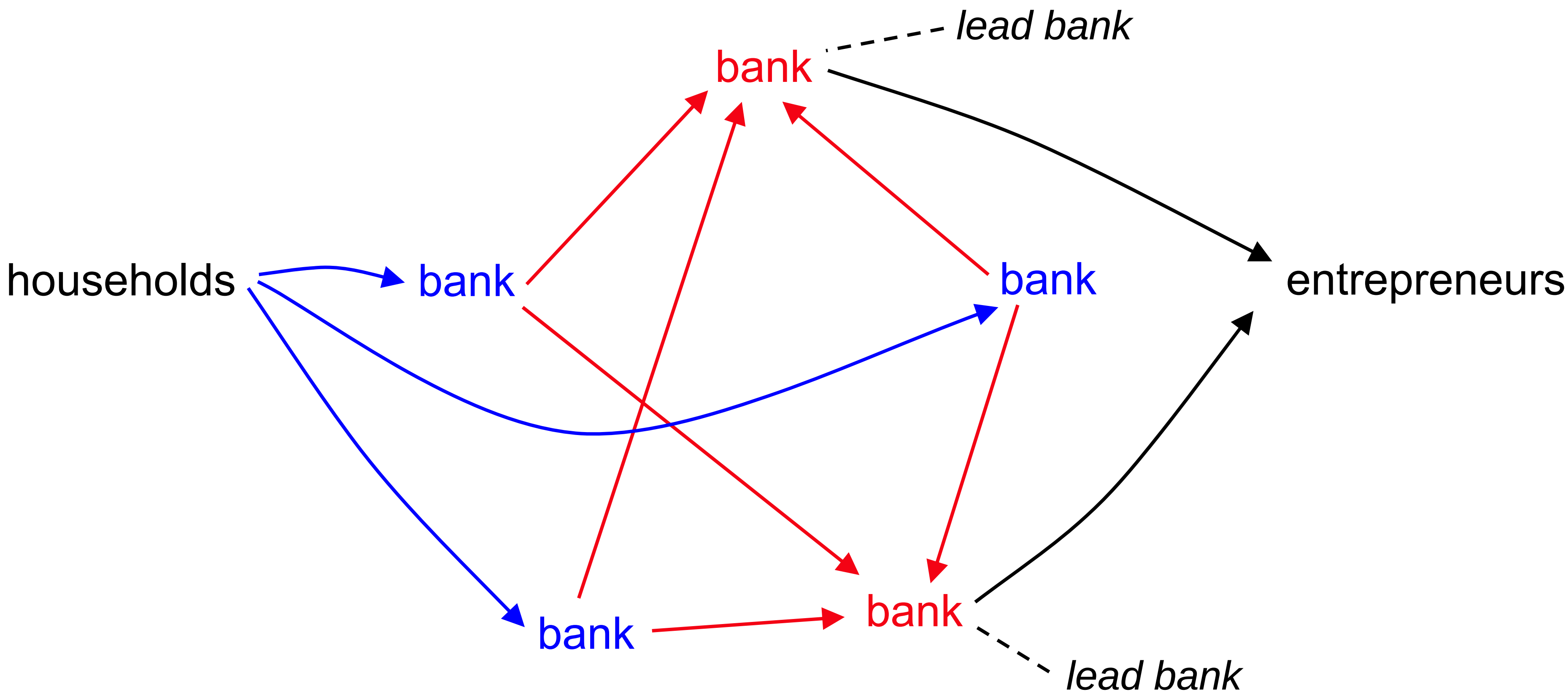
In effect, banks take turns to be "lead banks":

e.g. five banks and $\pi = 2/5$:



at next date, identity of lead banks changes, eg:

outside borrowing (r^*) inside bond market (r) outside lending (r^{**})

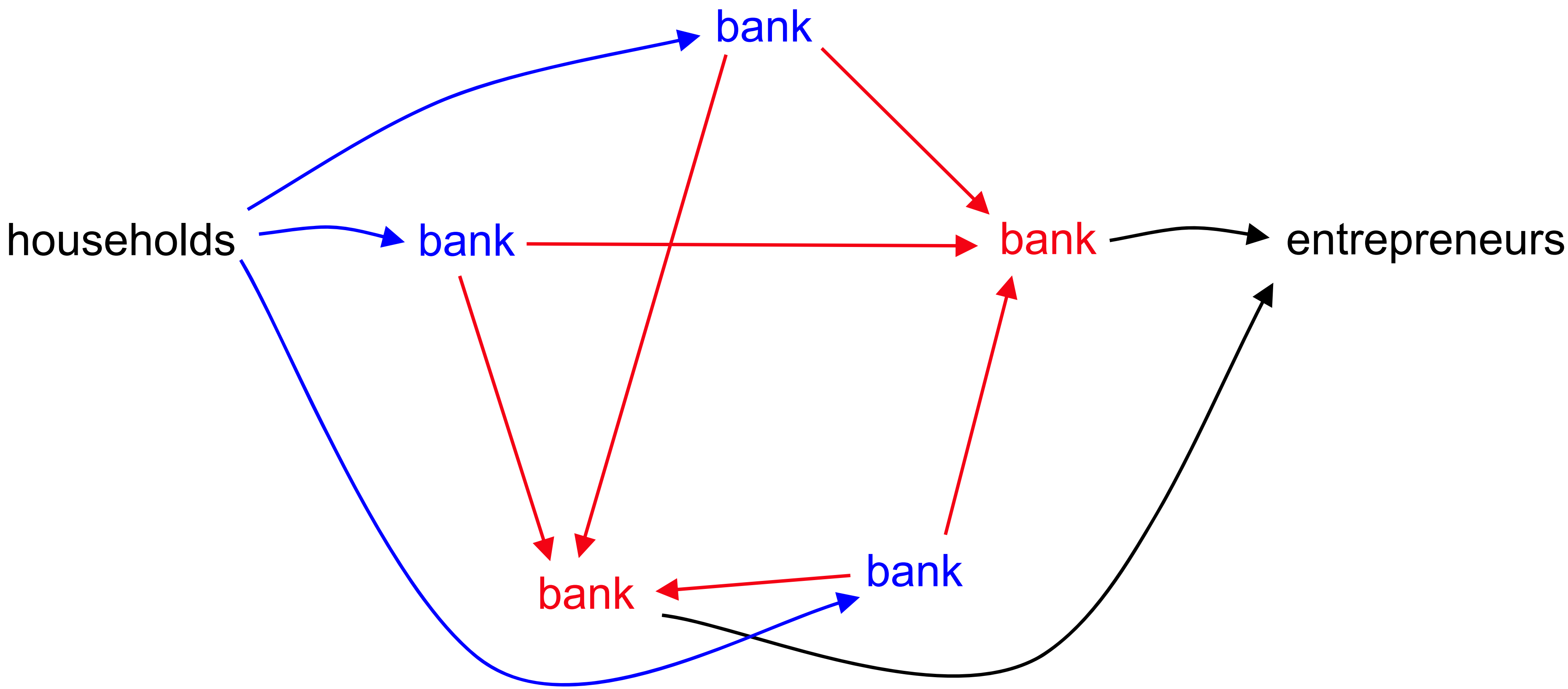


Crucial assumption: it is *not* feasible to lend outside (to entrepreneurs), levered by outside borrowing (from households)

e.g. outside lending at $r^{**} = 5\%$,
9/10 levered by inside borrowing at $r^{*} = 2\%$,
would yield a net return of $\approx$ 32%!!

Why not? When lending to bank 1, say, a householder can't rely on entrepreneurs' bonds as security, because she does not know enough to judge them. But she can rely on a bond sold to bank 1 by bank 2 that is itself secured against entrepreneurs' bonds *which bank 1 is able to judge* (and bank 1 has "skin in the game").

We have a pretty diagram:



but we’ve made no progress answering Q1 & Q2, because banks don’t hold mutual gross positions

To make progress, we need to introduce long-term assets and liabilities.

We suppose:

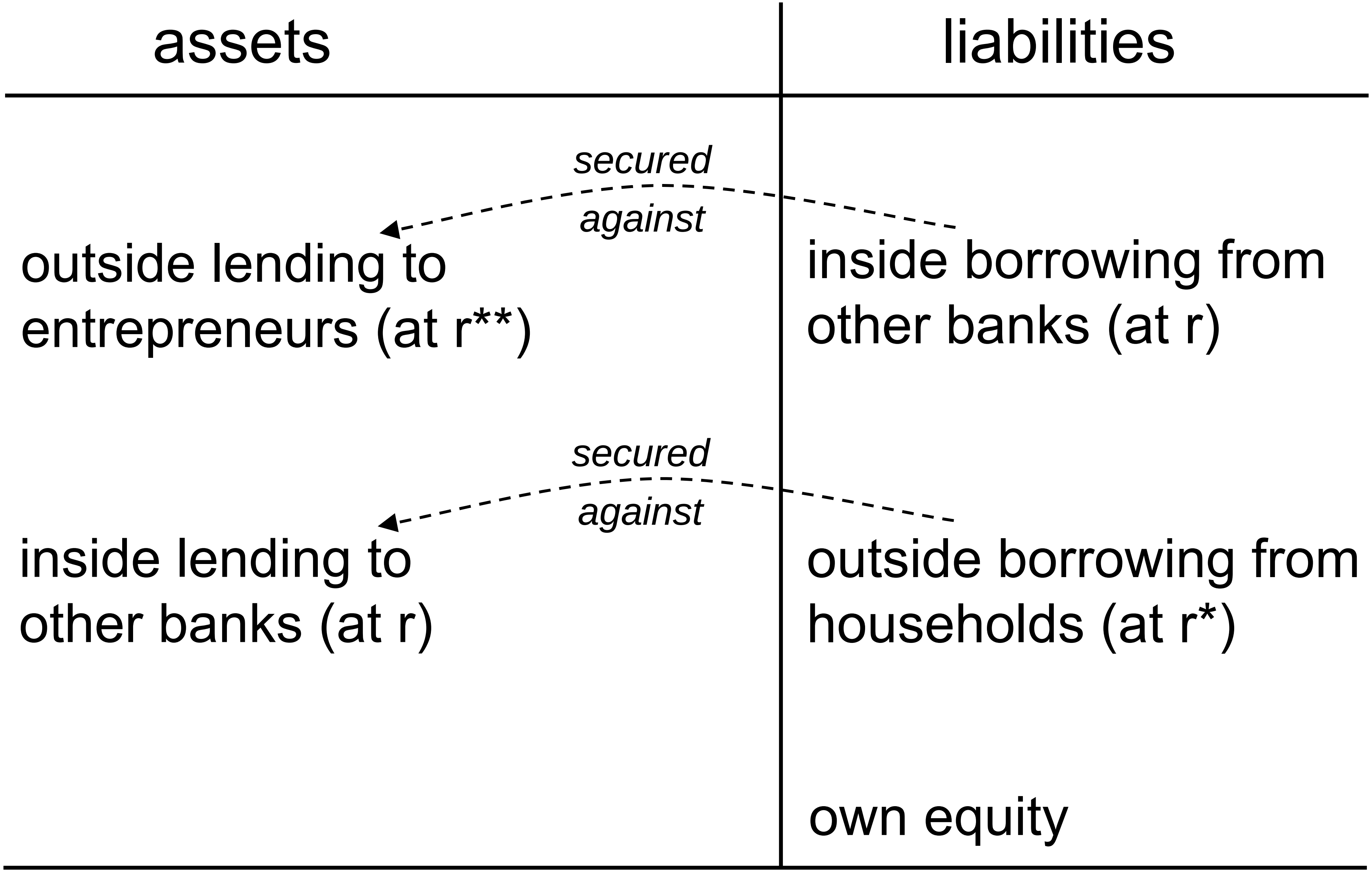
- outside lending is long term
  – yields a stream of returns

- inside borrowing is shorter term
  (i.e. inside bonds mature earlier than the outside lending that secures them)

$\Rightarrow$ inside bonds are periodically rolled over

Over time, banks accumulate assets & liabilities…

typical bank's balance sheet



Rollover

= a bank issues new inside bonds, against its current holding of outside assets

– in part, to make the terminal payments due on the inside bonds that are now maturing

Rollover happens when inside borrowing has a shorter maturity than outside lending

New inside borrowing (rollover) is at rate r (3%)

$\Rightarrow$ lead banks should clearly roll their borrowing over, to fund outside lending at r** (5%) – which in turn can be levered by further inside borrowing

Critical issue is the behaviour of the other banks, the “non-lead banks”

Should they roll their borrowing over too

– given that they cannot lend outside at r**, and only have the option to lend at r?

In effect we are reposing our earlier Q1:

should non-lead banks roll over their own borrowing at r, merely in order to lend at r?

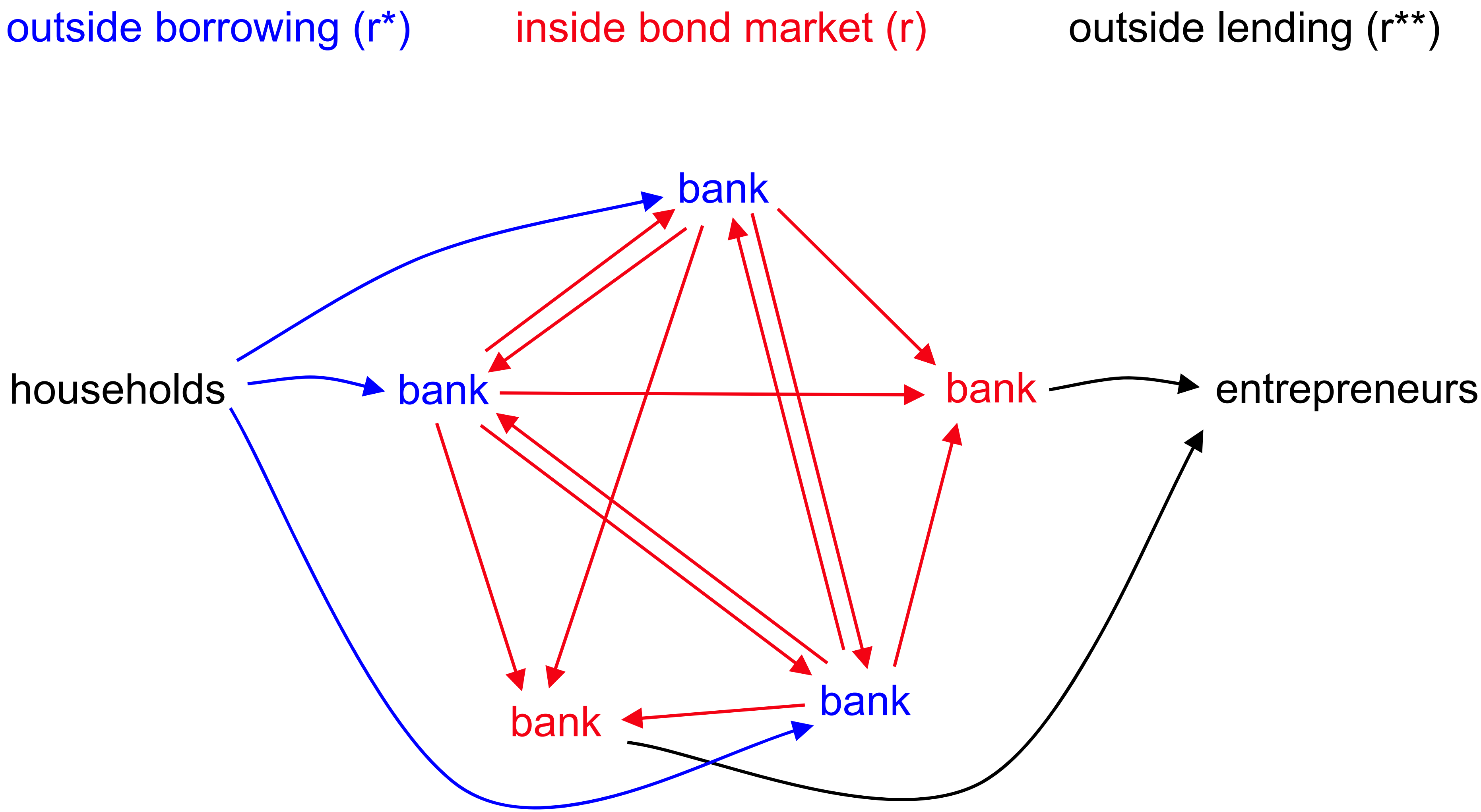
Answer: Yes! e.g., with our numbers,

although inside borrowing is at r = 3%,

inside lending is effectively at $\approx$ 12%,

because it can be levered by outside borrowing at r* = 2% (with θ = 9/10)

$\Rightarrow$ there are mutual gross positions among the non-lead banks:



banks' mutual gross positions offer security to households

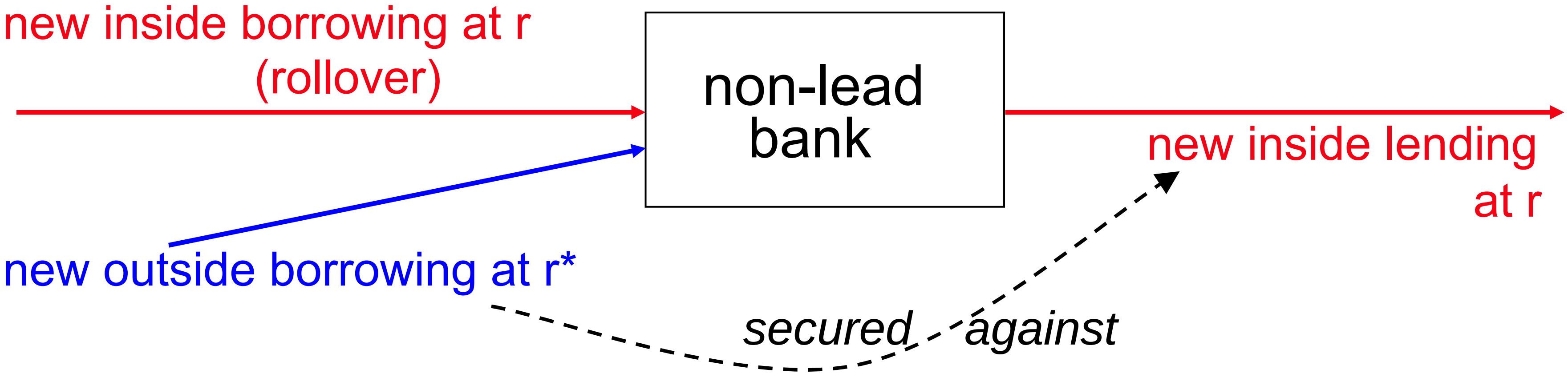
$\Rightarrow$ funds flow in to the banking system, from households

$\Rightarrow$ funds flow out of the banking system, to entrepreneurs

$\Rightarrow$ greater investment

BUT although steady-state economy operates at a higher level, it is more vulnerable:

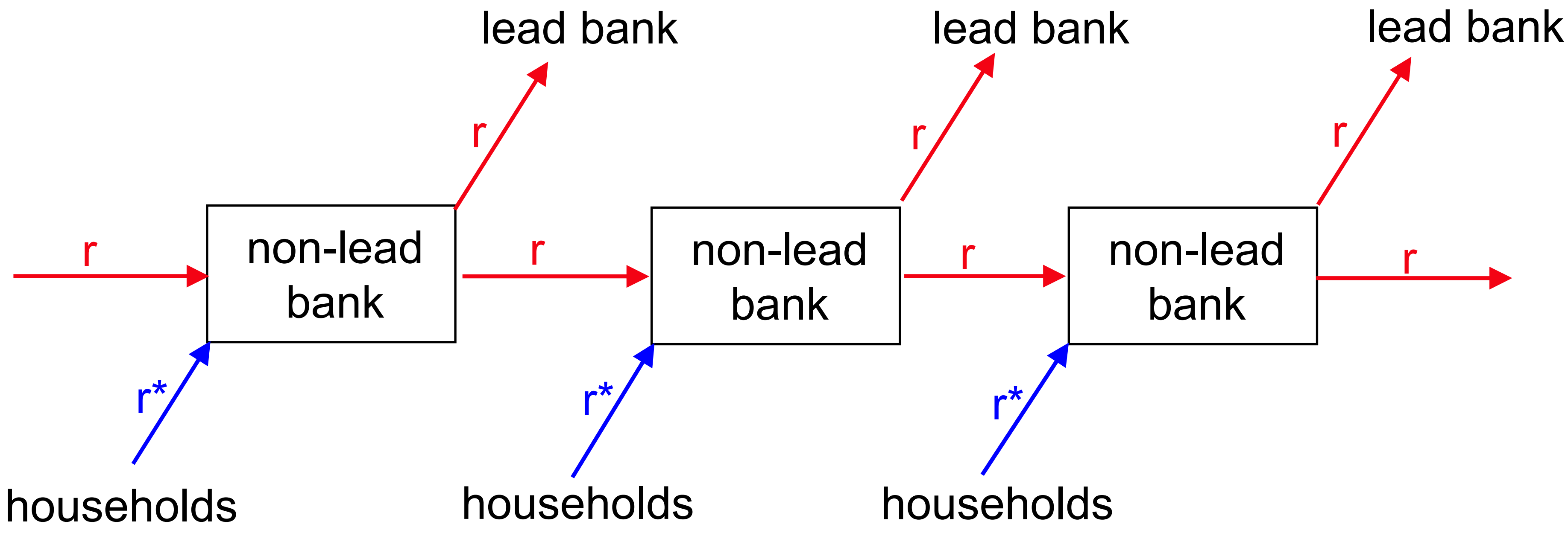
key point: non-lead banks are both borrowers and lenders in the interbank market



notice multiplier effect: if for some reason bank's value of new inside borrowing ↓ (by x dollars, say)

⇒ bank's value of new inside lending ↓↓ (by >> x dollars, because of outside leverage)

⇒ bank's *net* lending ↓



if the “outside-leverage multiplier”

exceeds the “leakage” to lead banks

then we get amplification along the chain collateral-value multiplier:

interbank bond prices $\downarrow$

$\downarrow$

collateral values $\downarrow$

$\downarrow$

outside borrowing $\downarrow$

$\downarrow$

net lending by non-lead banks $\downarrow$

$\downarrow$

interbank interest rates $\uparrow$

In extremis we can have systemic failure:

shortfall in new inside borrowing so great

$\Rightarrow$ bank unable to meet existing inside & outside debt obligations

$\Rightarrow$ bank defaults against other banks

$\Rightarrow$ other banks unable to meet their obligations ...

The systemic failure here arises from the fact that banks hold gross mutual positions (Q2)

MODEL

discrete time, dates $t = 0, 1, 2, \ldots$

at each date, single good (numeraire)

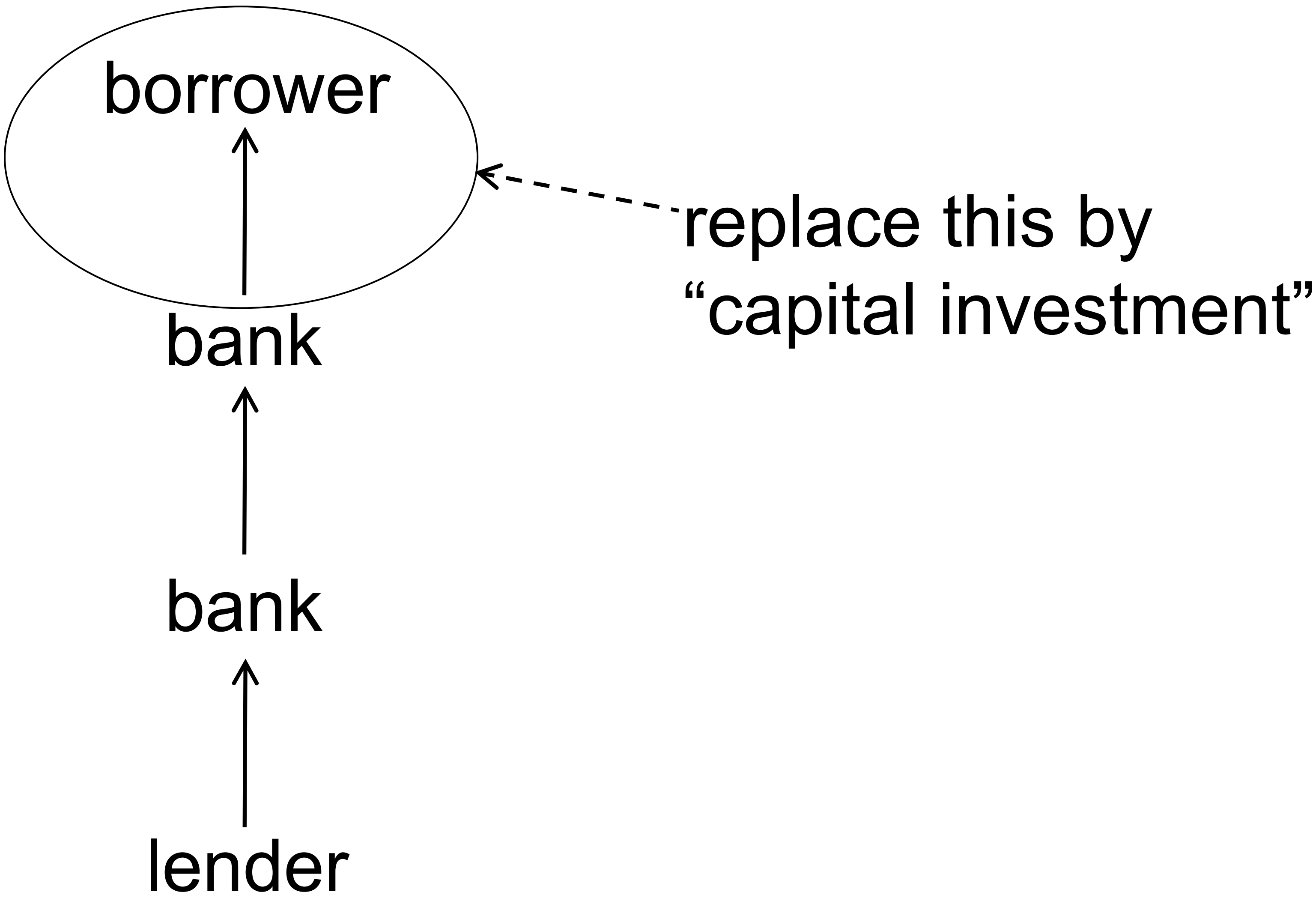
aside from an initial (unexpected) shock at $t = 0$ taking economy away from steady state, there is no further aggregate uncertainty

– perfect foresight path

fixed set of agents (“banks”)

in background: outside suppliers of loans at r^*

Apply Occam’s Razor to top of leverage stack:



Capital investment

constant returns to scale; per unit of project:

date t	date $t+1$	date $t+2$	date $t+3$	…
-1	a	λa	$\lambda^2 a$	…

unit cost

depreciation factor $\lambda < 1$

to simplify the presentation, let's suppose banks derive utility from their scale of investment

$\Rightarrow$ a bank invests maximally if opportunity arises

Investment opportunities arise with probability π
(i.i.d. across banks and through time)

A bank can issue inside bonds (i.e. borrow from other banks) against capital investment

per unit of project, bank can issue

$\theta < 1$ inside bonds

price path of inside bonds: $\{q_0, q_1, q_2, \dots\}$

with steady-state price $\overline{q}$

an inside bond issued at date t:

matures at date t+s with probability $\mu^{s-1}(1-\mu)$

where $0 \leq \mu < 1$ and s = 1, 2, 3, …

& promises to pay:	a	at date t+1
	λa	at date t+2
	$\lambda^2 a$	at date t+3
	$\vdots$	
	$\lambda^{s-2} a$	at date t+s–1
and	$\lambda^{s-1} a + \lambda^s E_t q_{t+s}$	at date t+s

This form of stochastic inside bond is equivalent to a bundle of deterministic bonds issued at date t maturing at dates t+s,   s = 1, 2, 3, …, ∞:

a fraction $1-\mu$ of one-period bonds

that pay $a + \lambda E_t q_{t+1}$ at date t+1

a fraction $\mu(1-\mu)$ of two-period bonds

that pay a at date t+1

and $\lambda a + \lambda^2 E_t q_{t+2}$ at date t+2 a fraction $\mu^2(1-\mu)$ of three-period bonds

that pay	a	at date t+1
	λa	at date t+2
and	$\lambda^2 a + \lambda^3 E_t q_{t+3}$	at date t+3

$\vdots$

etc

the probabilities $(1-\mu)$, $\mu(1-\mu)$, $\mu^2(1-\mu)$, … have been chosen so that, at any date $t+s > t$:

"second-hand" debt (issued at t),
that has not yet matured,
looks identical to new debt issued at $t+s$
– *provided expected prices haven't changed*

parameter μ indexes the maturity of the debt:

$\mu = 0$	$\mu = 1$
short-term debt (full rollover)	equity (no rollover)

Key idea behind bond structure:

creditor is promised (a fraction θ of) the flow of project returns

$$a, \quad \lambda a, \quad \lambda^2 a, \quad \ldots$$

until maturity – at which point he also receives the *expected* price of a new bond issued at that date against the residual flow of returns

i.e. the collateral that secures existing bond

= project returns

+ expected sale price of new bond

Outside borrowing

A bank can issue outside bonds (i.e. borrow from households) against its holding of inside bonds

outside bonds exactly mimic inside bonds
– same maturity & payment structure

per inside bond, bank can issue

$$\theta^* < 1 \quad \text{outside bonds}$$

price path of outside bonds: $\{q_0^*, q_1^*, q_2^*, \dots\}$
with steady-state price $\bar{q}^*$

Critical assumption:

These promised payments – on inside & outside bonds – are *fixed* at issue, date t, using that date's expectation (E_t) of future bond prices

$\Rightarrow$ bonds are unconditional,
without any state-dependence

In the event of, say, a fall in bond prices, or
a fall in project returns,

the debtor bank must honour its fixed payment obligations, or risk default & bankruptcy

typical bank's balance sheet at start of date t

assets		liabilities
capital investment holdings (k_t)	← *secured against*	inside bonds issued ($\leq \theta k_t$)
inside bond holdings (b_t)	← *secured against*	outside bonds issued ($\leq \theta^* b_t$)
		own equity

lead bank's flow-of-funds

(along perfect foresight path)

rollover

$i_t \leq ak_t - [\,a + (1-\mu)\lambda q_t\,]\,\theta k_t$

capital investment — returns — payments to other banks

$+\ [\,a + (1-\mu)\lambda q_t\,]\,b_t - \theta^*[\,a + (1-\mu)\lambda q_t\,]\,b_t$

payments from other banks — payments to households

$+\ q_t\mu\lambda b_t - q_t{}^*\theta^*\mu\lambda b_t + q_t\theta\{(1-\mu)\lambda k_t + i_t\}$

resale of other banks' bonds — repurchase of outside bonds — sale of new inside bonds

Hence, for a lead bank starting date t with (k_t, b_t),

$$b_{t+1} = 0$$

$$\text{and} \quad k_{t+1} = \lambda k_t + i_t$$

where i_t is given by

$$\frac{a(1-\theta)k_t + (1-\theta^*)[a + (1-\mu)\lambda q_t]b_t + (q_t - \theta^* q_t^*)\mu\lambda b_t}{1 - \theta q_t}$$

non-lead bank's flow-of-funds

(along perfect foresight path)

rollover

$$q_t \{ b_{t+1} - \mu\lambda b_t \} \leq ak_t - [a + (1-\mu)\lambda q_t] \theta k_t$$

purchase of other banks' bonds — returns — payments to other banks

$$+ [a + (1-\mu)\lambda q_t] b_t - \theta^*[a + (1-\mu)\lambda q_t] b_t$$

payments from other banks — payments to households

$$+ q_t \theta(1-\mu)\lambda k_t + q_t^* \theta^* \{ b_{t+1} - \mu\lambda b_t \}$$

sale of new inside bonds — sale of new outside bonds

Hence, for a non-lead bank starting date t with (k_t, b_t),

$$k_{t+1} = \lambda k_t$$

and b_{t+1} is given by

$$\mu\lambda b_t + \frac{a(1-\theta)k_t + (1-\theta^*)[a + (1-\mu)\lambda q_t]b_t}{q_t - \theta^* q_t^*}$$

each bank has its personal history of, at each past date, being either a lead or a non-lead bank

$\Rightarrow$ in principle we should keep track of how the distribution of $\{k_t, b_t\}$'s evolves (hard)

however, the great virtue of our expressions for k_{t+1} and b_{t+1} is that they are linear in k_t and b_t

$\Rightarrow$ aggregation is easy

At the start of date t, let

K_t = aggregate stock of capital investment

B_t = aggregate stock of inside bonds

We know that from market-clearing at date t–1,

$$B_t = \theta K_t$$

Now $K_{t+1} = \lambda K_t + I_t$

where I_t = aggregate capital investment along a perfect foresight path, I_t is given by

$$\frac{\pi\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t + \mu\lambda\theta(q_t-\theta^*q_t^*)\right\}K_t}{1-\theta q_t}$$

and B_{t+1} is given by

$$(1-\pi)\mu\lambda\theta K_t \quad + \quad \frac{(1-\pi)\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t\right\}K_t}{q_t-\theta^*q_t^*}$$

Market clearing

the price sequence $\{q_t, q_{t+1}, q_{t+2}, \ldots\}$ clears the market for inside bonds at each date t, t+1, t+2, ..

at date t,

aggregate bond demand $= B_{t+1}$

aggregate bond supply $= \theta K_{t+1} = \theta(\lambda K_t + I_t)$

these are homogeneous in K_t

$\Rightarrow$ along a perfect foresight path,
market-clearing at date t requires:

$$(1-\pi)\mu\lambda\theta \quad + \quad \frac{(1-\pi)\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t\right\}}{q_t - \theta^* q_t^*}$$

aggregate demand for inside bonds

$$= \quad \lambda\theta \quad +$$

$$\frac{\pi\theta\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t + \mu\lambda\theta(q_t - \theta^* q_t^*)\right\}}{1 - \theta q_t}$$

aggregate supply of inside bonds

since outside bonds exactly mimic inside bonds (same maturity & payment structure), the prices q_t^*, q_{t+1}^*, q_{t+2}^*,… are functions of the market-clearing prices q_t, q_{t+1}, q_{t+2},…

defining iteratively: for $s \geq 0$,

$$q_{t+s}^* = \frac{1}{1+r^*}\left\{ a + (1-\mu)\lambda q_{t+s+1} + \mu\lambda q_{t+s+1}^* \right\}$$

households lend at r*

Steady State

Note that, to simplify the presentation, we have not allowed for any curvature in

- households' demand for outside bonds
  (r* fixed)
- capital investment technology
  (a fixed)

As a result, the model in this handout is over-determined (witness the fact that our market-clearing condition is homogeneous in K_t).

In full model, r* & a together satisfy a no-growth condition, but curvature plays little role.

From the equilibrium price path $\{q_t, q_{t+1}, q_{t+2}, \ldots\}$ we can compute the effective interbank rates of interest on inside bonds $\{r_t, r_{t+1}, r_{t+2}, \ldots\}$:

for $s \geq 0$, the effective interbank interest rate, r_{t+s} say, between date t+s and date t+s+1 solves

$$q_{t+s} = \frac{1}{1+r_{t+s}} \left\{ a + (1-\mu)\lambda q_{t+s+1} + \mu\lambda q_{t+s+1} \right\}$$

We need to confirm that $r_t > r^*$, so that (non-lead) banks will choose to lever their inside lending with outside borrowing:

Lemma 1

The steady-state interbank interest rate $\bar{r}$ strictly exceeds the outside borrowing rate r^* iff

(A.1):

$$\theta > \pi\theta\theta^* + (1-\pi)(1-\lambda+\lambda\theta) + (1-\pi)(1-\theta\theta^*)r^*$$

Aggregate Shocks

Suppose at date 0, starting from steady state, the economy suffers a one-time, negative, proportional shock to its capital stock – *keeping banks' existing debt obligations intact*.

In this presentation, let's avoid the gory details – which in large part are to do with repricing the old inside bonds. (New and old bonds must deliver the same *levered* rate of return.)

Old debt casts a long shadow, insofar as the debt has a long maturity.

<u>Lemma 2</u> The *proximate* effect of the shock, at each date $t \geq 0$, is to raise the interest rate r_t.

The knock-on effects can be dramatic:

<u>Proposition 1 (interest rate cascades)</u>

If, in addition to Assumption (A.1), we assume

$$\text{(A.2)} \qquad \frac{\theta^*(1-\mu)\pi[1 - \lambda^2\mu(1-\pi)]}{(1 - \lambda + \lambda\pi)^2} > 1$$

then a (ceteris paribus) rise in any future interest rate r_{t+s}, for $s \geq 1$, causes the current interest rate r_t to rise too.

We saw the intuition earlier. For a non-lead bank:

future interest rates $\uparrow$

$\Rightarrow$ current price of inside bonds $\downarrow$

$\Rightarrow$ bank's value of new inside borrowing $\downarrow$
(by x dollars, say)

$\Rightarrow$ bank's value of new inside lending $\downarrow\downarrow$
(by >> x dollars, because of outside leverage)

$\Rightarrow$ bank's *net* lending $\downarrow$

$\Rightarrow$ current interest rate $\uparrow$

$$(1-\pi)\mu\lambda\theta \quad + \quad \frac{(1-\pi)\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t\right\}}{q_t - \theta^* q_t^*}$$

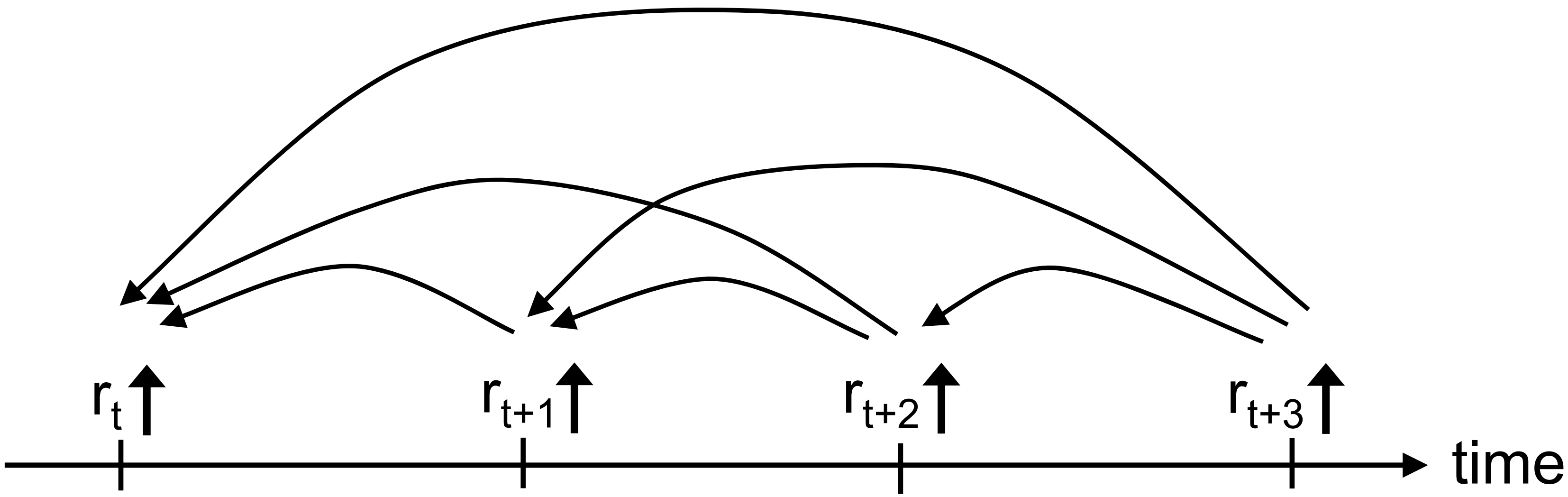
aggregate demand for inside bonds

the falls in these prices do the work in Proposition 1

$$= \quad \lambda\theta \quad + \quad \frac{\pi\theta\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t + \mu\lambda\theta(q_t-\theta^* q_t^*)\right\}}{1-\theta q_t}$$

aggregate supply of inside bonds

effects of interest rate cascades on q_t and I_t:



$\Rightarrow \quad q_t \downarrow\downarrow\downarrow \qquad q_{t+1} \downarrow\downarrow \qquad q_{t+2} \downarrow \qquad q_{t+3}$

$\Rightarrow \quad I_t \downarrow\downarrow\downarrow$

Recall that I_t equals

$$\frac{\pi\left\{(1-\theta\theta^*)a + (1-\mu)\lambda\theta(1-\theta^*)q_t + \mu\lambda\theta(q_t - \theta^* q_t^*)\right\}K_t}{1-\theta q_t}$$

As $q_t\downarrow$ (and $q_t^*\downarrow$ in tandem), $I_t \downarrow\downarrow$

in sum: negative shock to capital stock
+ shadow cast by old debt obligations

$\Rightarrow$ interbank interest rates $\uparrow$ and bond prices $\downarrow$

$\Rightarrow$ banks' outside borrowing limits tighten

$\Rightarrow$ funds are taken *from* banking system, just as they are most needed to rebuild capital stock amplification effect of interest rate cascades

$\Rightarrow$ banks are vulnerable to failure

“most vulnerable” banks:

banks that have just made maximal capital investment (because they hold no cushion of inside bonds that if necessary could be resold)

Failure of these banks can precipitate a failure of the entire banking system:

Proposition 2 (systemic failure)

In addition to Assumptions (A.1) and (A.2), assume

(A.3) $\theta^* > (1-\pi)\,\lambda$

If the aggregate shock is enough to cause the most vulnerable banks to fail, then all banks fail (in the order of the ratio of their capital stock to their holding of other banks' bonds).

NB In proving Proposition 2, use is made of the steady-state (ergodic) distribution of the $\{k_t, b_t\}$'s across banks

Parameter consistency?

Assumptions (A.1), (A.2) and (A.3) are mutually consistent:

e.g.

$$\pi = 0.1$$

$$\lambda = 0.975$$

$$\mu = 0.6$$

$$\theta = \theta^* = 0.9$$

$$r^* = 0.02$$