Contagious Illiquidity

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SUMMARY:

market is illiquid $\equiv$ market suffers from severe adverse selection (Akerlof)

illiquidity is contagious
because adverse selection is contagious

adverse selection is contagious

- through time (back from future to present)
- across markets

these two channels of contagion can feed on each other, and lead to financial collapse

ROAD MAP:

A rudimentary static Akerlof example

single item owned by seller;   many buyers

second, divisible, good – “income” (numeraire)

μ^s = seller’s marginal utility of income

μ^b = buyers’ marginal utility of income

where $\mu^s > \mu^b$ (e.g. $\because$ seller has income shortfall)

item has 2 possible qualities: utility H or L

only seller knows quality; $H > L \geq 0$

buyers know item has quality L with probability λ

High-price equilibrium

both qualities are traded

price $\overline{p}$ satisfies

$$\mu^b \overline{p} = (1-\lambda)H + \lambda L$$

seller with H-quality must want to trade:

$$\mu^s \overline{p} > H$$

that is, the high-price equilibrium exists iff

$$\lambda\left(1 - \frac{L}{H}\right) < \frac{\mu^s - \mu^b}{\mu^s}$$

If reverse inequality holds:

$$\lambda\left(1 - \frac{L}{H}\right) > \frac{\mu^s - \mu^b}{\mu^s}$$

then only L-quality is traded:

Low-price equilibrium

price $\underline{p}$ satisfies $\mu^b \underline{p} = L$

In limit case $L = 0$, $\underline{p} = 0$

– effectively there is complete market failure, without any trade

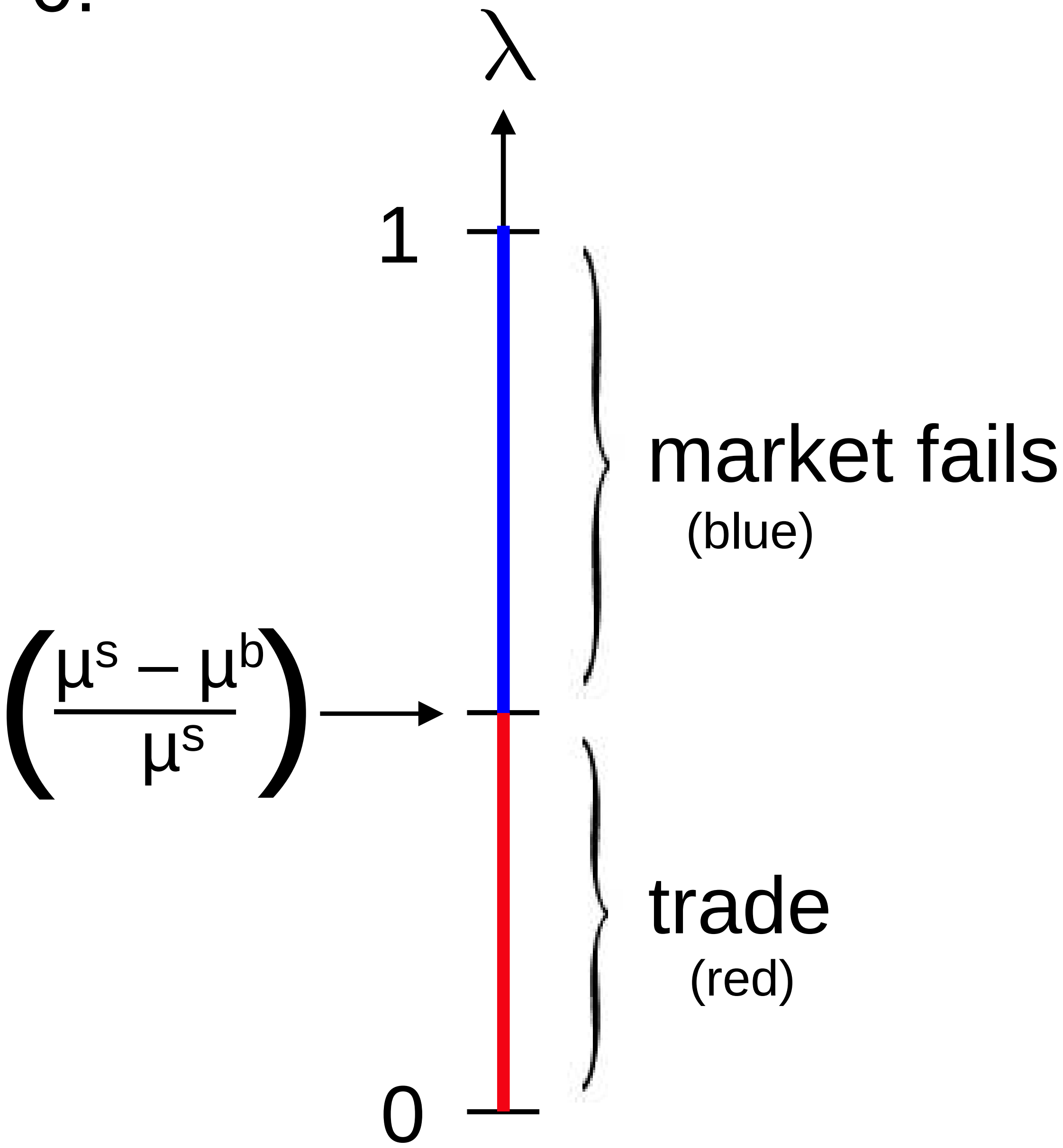
Market “fails” iff

$$\lambda\left(1 - \frac{L}{H}\right) > \frac{\mu^s - \mu^b}{\mu^s}$$

We see that market failure is more likely if there is

- a greater fraction λ of “lemons”
- more % difference between H and L
- less % difference between the seller’s and the buyers’ marginal utilities of income

When L = 0:



λ
1
market fails
(blue)
$\left(\frac{\mu^s - \mu^b}{\mu^s}\right)$
trade
(red)
0

Tempting to think that in red region, complete market failure must also be an equilibrium...

zero price $\Leftrightarrow$ zero quality

Theory of illiquidity? No.

Although in red region $\exists$ 2 “Walrasian” equilibria
– parametric price is either high ($\overline{p}$) or zero –

in fact only the high-price equilibrium is “Nash”, i.e. where agents actively make bids/offers:

a buyer could profitably deviate from zero by bidding $\varepsilon > 0$ below $\overline{p}$

Contagion Through Time

discrete time: “days” $t = 1, 2, \ldots, T, T+1$

- single consumption good (divisible, storable)
- assets: pay off on day T+1

common overnight discount factor $\beta < 1$

normalise everyone’s marginal utility of day T+1 consumption to equal 1 on previous days, agents alternate their marginal utilities ($\mu^s > \mu^b$):

today's seller:	μ^s	μ^b	μ^s	…	
	$\uparrow$	$\uparrow$	$\uparrow$		
	today	tomorrow	next day	…	day T
	$\downarrow$	$\downarrow$	$\downarrow$		
today's buyer:	μ^b	μ^s	μ^b	…	

assume: $\mu^b > \beta\mu^s \Rightarrow$ no storage

& that intertemporal contracts cannot be written

$\Rightarrow$ only means of intertemporal redistribution is by trading the assets on days 1, 2, …, T

2 types of asset; fraction λ of type L:

type H: pays $V > 0$ on day $T+1$
– no payoff from asset before then

type L: pays zero – a “lemon”

at the end of each day t (after the market closes) current holder of an asset privately learns its type

to simplify, assume “anonymity of assets”:

- the trading history of an asset isn’t observed
- no-one can identify an asset he previously sold

if an asset is a lemon then, again at the end of each day t (after the market closes), with probability α_t there is a public announcement:

"This Asset is a Lemon"

(on the days after such an announcement,
the market price of the asset is zero)

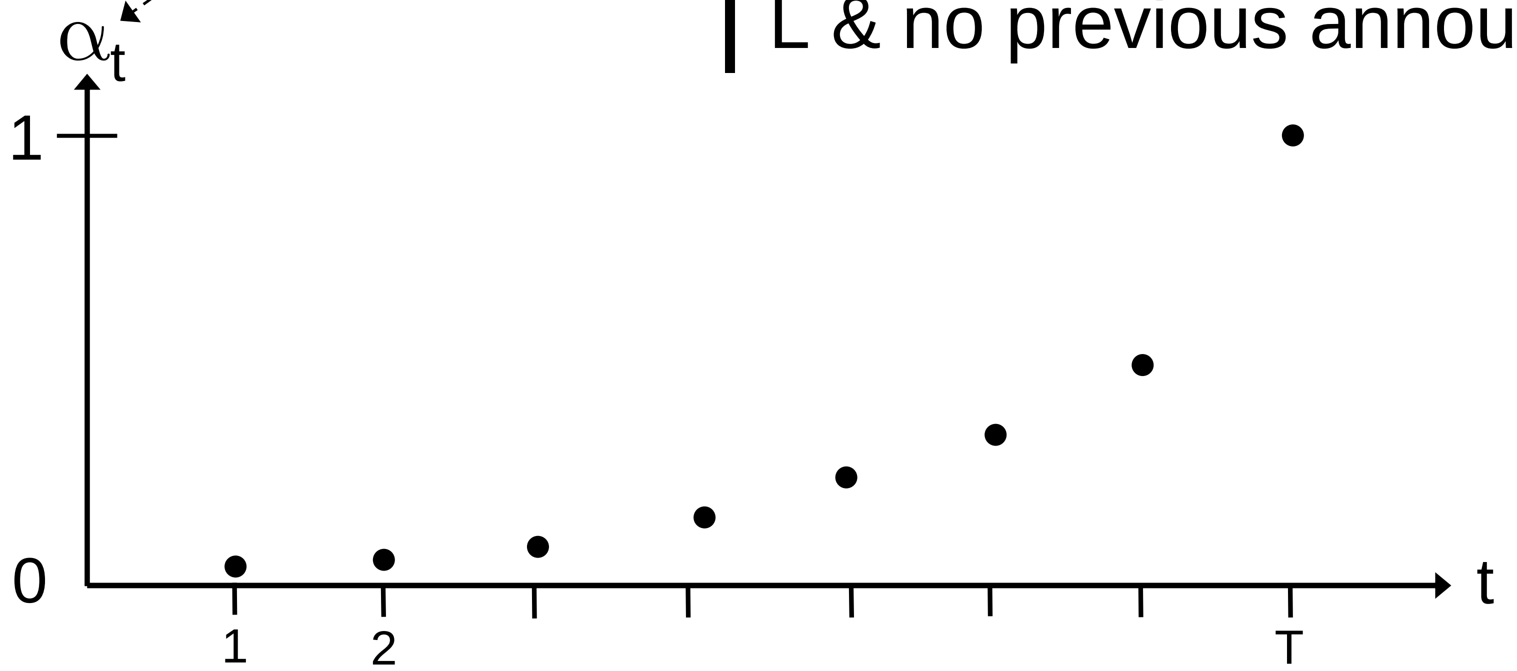
if asset is not a lemon then no announcements

α_T in effect equals 1,
since type is revealed on day T+1

e.g. "<u>uniformity</u>": announcements are uniformly timed between day 1 and T – i.e. ex ante probability (1/T) of occurring on any given day

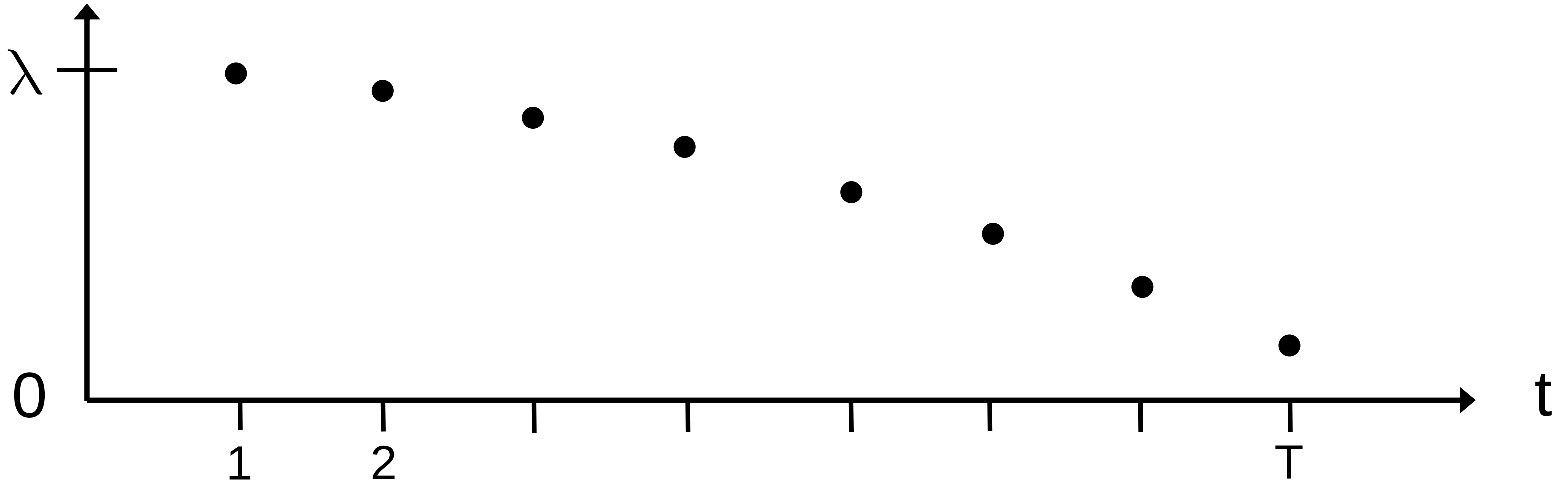
$$\Rightarrow \quad \alpha_t = \frac{1}{T - t + 1} \quad \text{for } t = 1, 2, \ldots, T$$

prob (announcement after market closes on day t | L & no previous announcement)



from these exogenous parameters $\{\alpha_t\}$, use Bayes' Rule to derive posterior probabilities that prevail in the market on day t = 1, 2, …, T:

$$\pi_t = \text{prob}\left(L \,\middle|\, \text{no announcement yet} \right)$$



with "uniformity", $\pi_t = \dfrac{\lambda(T - t + 1)/T}{1 - \lambda + \lambda(T - t + 1)/T}$

$\alpha_t \pi_t$ = prob (announcement after market closes on day t | no announcement yet)

make assumption about parameters $\{\alpha_t\}$:

$\alpha_t \pi_t$ is greatest on day t = T <u>Assumption (A1)</u>

i.e. $\alpha_t \pi_t \leq \pi_T$ for all t (since $\alpha_T = 1$)

with "uniformity", $\alpha_t \pi_t = \dfrac{\lambda/T}{1 - \lambda + \lambda(T - t + 1)/T}$

which is increasing in t

to find overall equilibrium, start at T (assuming no announcement yet):

high-price $\overline{p}_T$ equilibrium iff

$$\mu^b \, \overline{p}_T = \beta(1 - \pi_T)V \qquad \text{(buyers indifferent)}$$

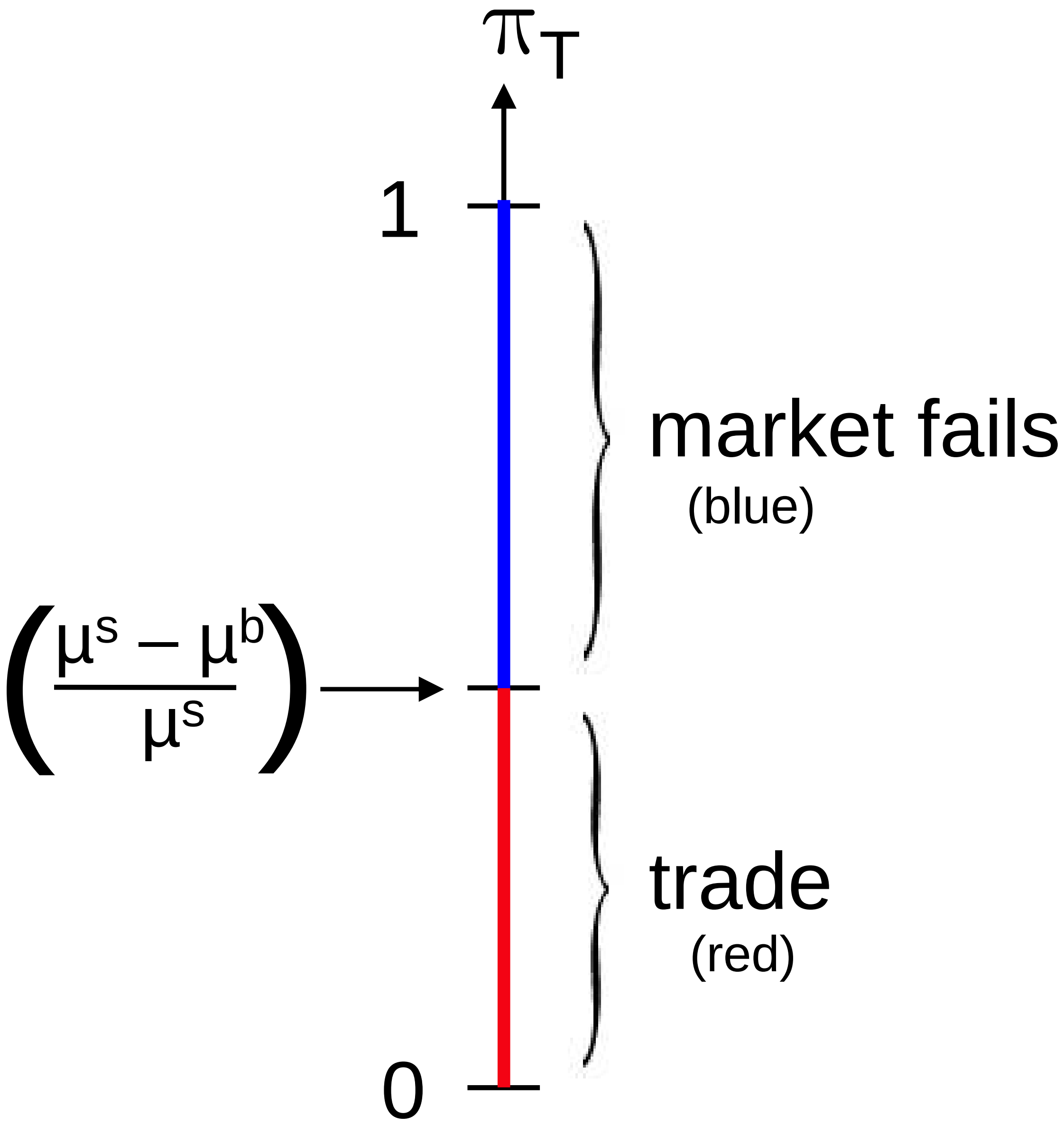
and

$$\mu^s \, \overline{p}_T > \beta V \quad \text{(holder of H-type wants to sell)}$$

that is,

$$\text{trade iff} \quad \pi_T < \frac{\mu^s - \mu^b}{\mu^s}$$

on day T:



π_T
1
market fails
(blue)
$\left(\frac{\mu^s - \mu^b}{\mu^s}\right)$
trade
(red)
0

Proposition 1:

If market fails on day T (blue region),

it also fails on the earlier days $t \leq T-1$.

Proposition 2:

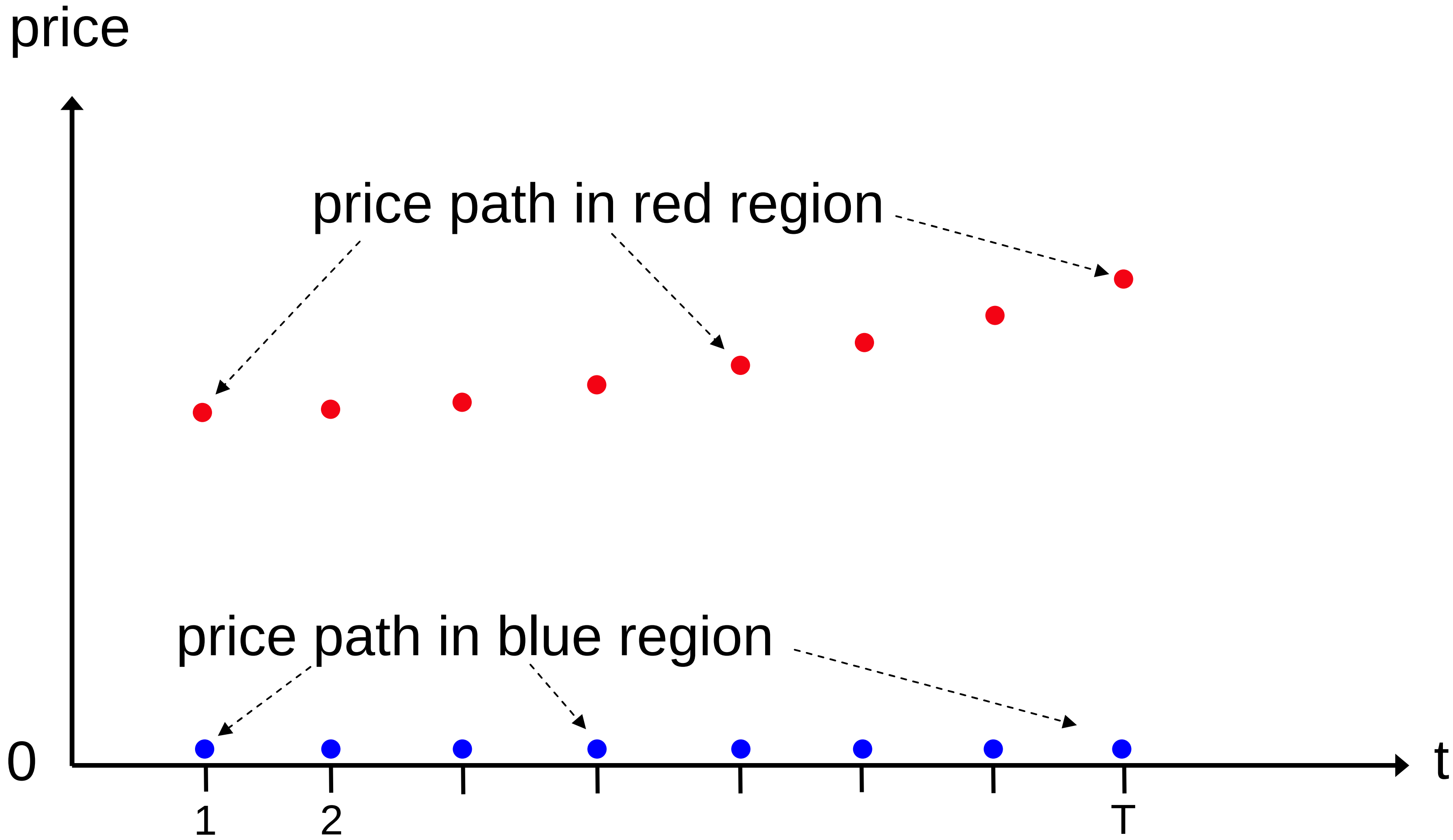
If trade occurs on day T (red region),

it also occurs on the earlier days $t \leq T-1$.

Moreover the price path is increasing,

but at a rate no faster than $1/\beta$.

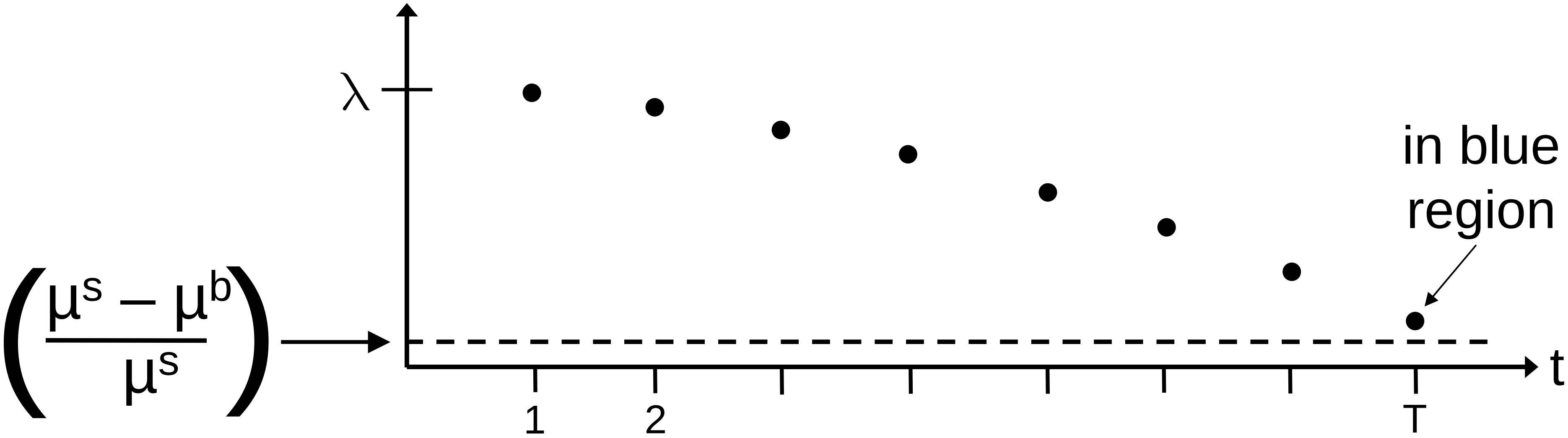
In short: tail wags dog
(liquidity of market on day T determines market liquidity on all previous days)



Proof of Proposition 1 (blue region):

if market fails on days t+1, …, T, then in effect day t looks like day T except that $\pi_t > \pi_T$:

$$\pi_t = \text{prob}\left(L \,\middle|\, \text{no announcement yet}\right)$$



$$\text{for all } t \leq T-1,\ \pi_t > \frac{\mu^s - \mu^b}{\mu^s}$$

$$\Rightarrow \text{ market failure on day } t$$

Q.E.D.

Proof of Proposition 2 (red region):

backward induction: $t = T-1, T-2, \ldots, 1$

suppose: trade on days $t+1, \ldots, T$

price path $\{\bar{p}_{t+1}, \ldots, \bar{p}_T\}$ increasing at rate $< 1/\beta$

on day t, high-price ($\bar{p}_t$) equilibrium must satisfy:

$$\mu^b \, \bar{p}_t = \beta \, \underbrace{(1 - \alpha_t \pi_t)}_{\text{probability of no announcement after market closes on day } t} \, \underbrace{\mu^s \, \bar{p}_{t+1}}_{\text{a buyer sells on day } t+1}$$

probability of no
announcement after
market closes on day t

a buyer sells on day t+1
– no matter which type of asset
he learnt that he purchased on day t

the price ratio $\bar{p}_{t+1} \,/\, \bar{p}_t$ equals $\dfrac{(\mu^b/\mu^s)}{\beta(1 - \alpha_t \pi_t)}$

– which lies strictly between 1 and $1/\beta$,

because for $t \leq T-1$,

$$\beta \;<\; \mu^b/\mu^s \;<\; 1 - \pi_T \;\leq\; 1 - \alpha_t \pi_t \;<\; 1$$

in red region by Assumption (A1)

thus price path is increasing at rate $< 1/\beta$

from day t onwards

But will seller with asset H want to trade at $\bar{p}_t$?

Yes if $\mu^s \bar{p}_{T-1} > \beta^2 V$ (for $t = T-1$)

and $\mu^s \bar{p}_t > \underbrace{\beta^2 \mu^s \bar{p}_{t+2}}$ (for $t \leq T-2$)

if didn't sell on day t,
then would sell on day t+2

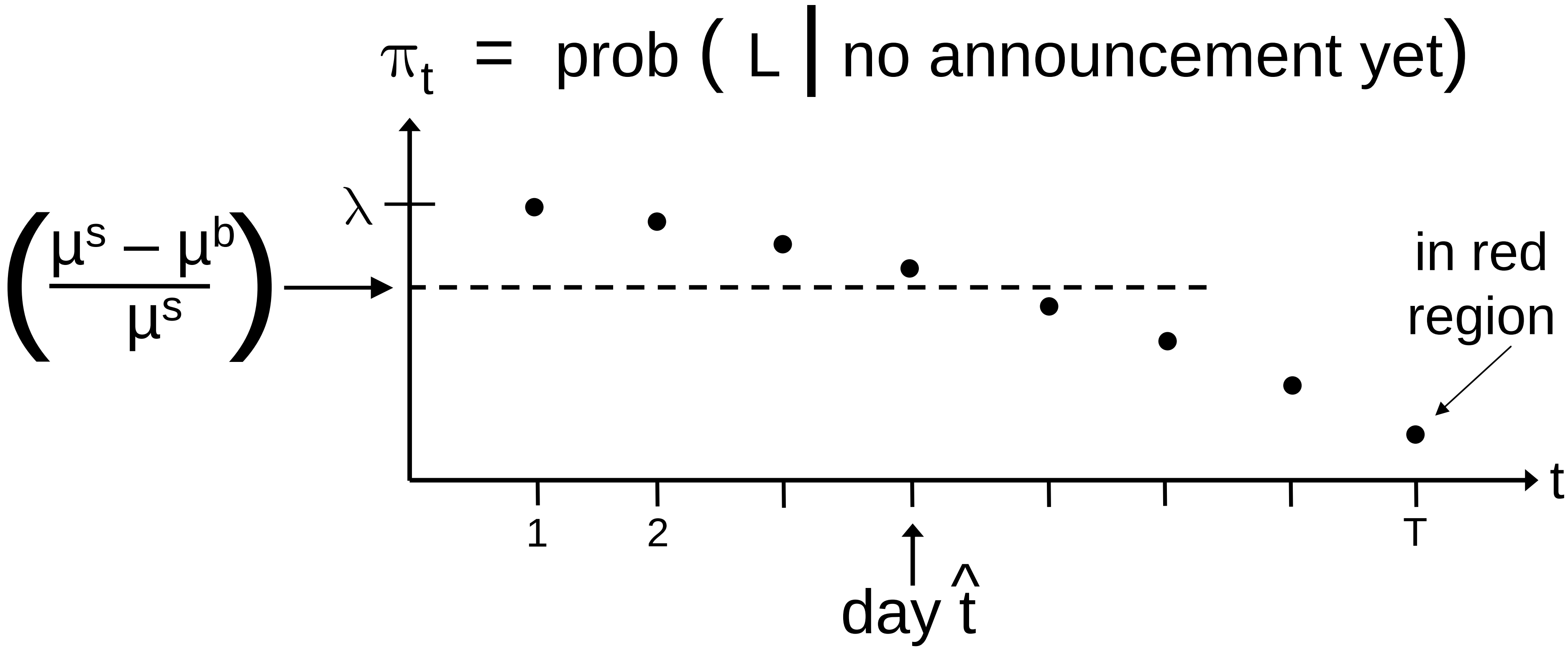
these two inequalities hold, given

$$\frac{\bar{p}_t}{\beta^{T-t}} > \cdots > \frac{\bar{p}_{T-1}}{\beta} > \bar{p}_T = \frac{\beta(1-\pi_T)V}{\mu^b} > \frac{\beta V}{\mu^s}$$

in red region

Q.E.D.

Proposition 2 may at first appear surprising:



Why doesn't market fail on day $\hat{t}$?

Because of option-to-sell lemon on day $\hat{t}$+1.

Intuition for contagion through time:

If tomorrow's market is <u>not</u> expected to fail, then today's buyer of an unknown asset will sell tomorrow, whether or not he buys a lemon.

Thus, the only downside to buying a lemon is the (small) risk of a public announcement after the close of today's market.

In other words, the % difference in future utility between the good asset and a lemon is small.

But, as in 5 finger exercise, small % difference

$\Rightarrow$ market doesn't fail today either

Conversely, if markets in the future <u>are</u> expected to fail, then today's buyer of an unknown asset will be stuck with it for a long time.

Thus, the % difference in future utility between the good asset and a lemon is big.

But big % difference $\Rightarrow$ market fails today too

Scope for multiple Nash equilibria in stationary (infinite horizon) environments?

Yes. See Appendix

Summary of our dynamic Akerlof example:

- market fails completely iff $\pi_T > \dfrac{\mu^s - \mu^b}{\mu^s}$

- otherwise, trade on days 1, 2, …, T

  – unless there is an announcement

with "uniformity", market fails iff

$$\frac{1}{1 + \dfrac{(1-\lambda)T}{\lambda}} > \frac{\mu^s - \mu^b}{\mu^s}$$

market failure more likely as $\begin{cases} \lambda \text{ rises} \\ \mu^s/\mu^b \text{ falls} \\ T \text{ falls} \quad \text{(IMPORTANT)} \end{cases}$

Primitive Model of a Bank

discrete time: day $t = 0, 1, 2, \ldots$ discount factor β

single consumption good

single capital good

bank's objective function:

$$E \sum_t \beta^t u(c_t) \qquad \text{“dividend stream”}\{c_t\}$$

$u(\cdot)$ increasing, weakly concave

at start of each day t, bank has capital stock K_t

overnight depreciation factor δ

bank may invest i_t:

$$K_{t+1} = (1 - \delta)K_t + i_t$$

but investment opportunities are stochastic

capital can be liquidated at price $\ell_t < 1$

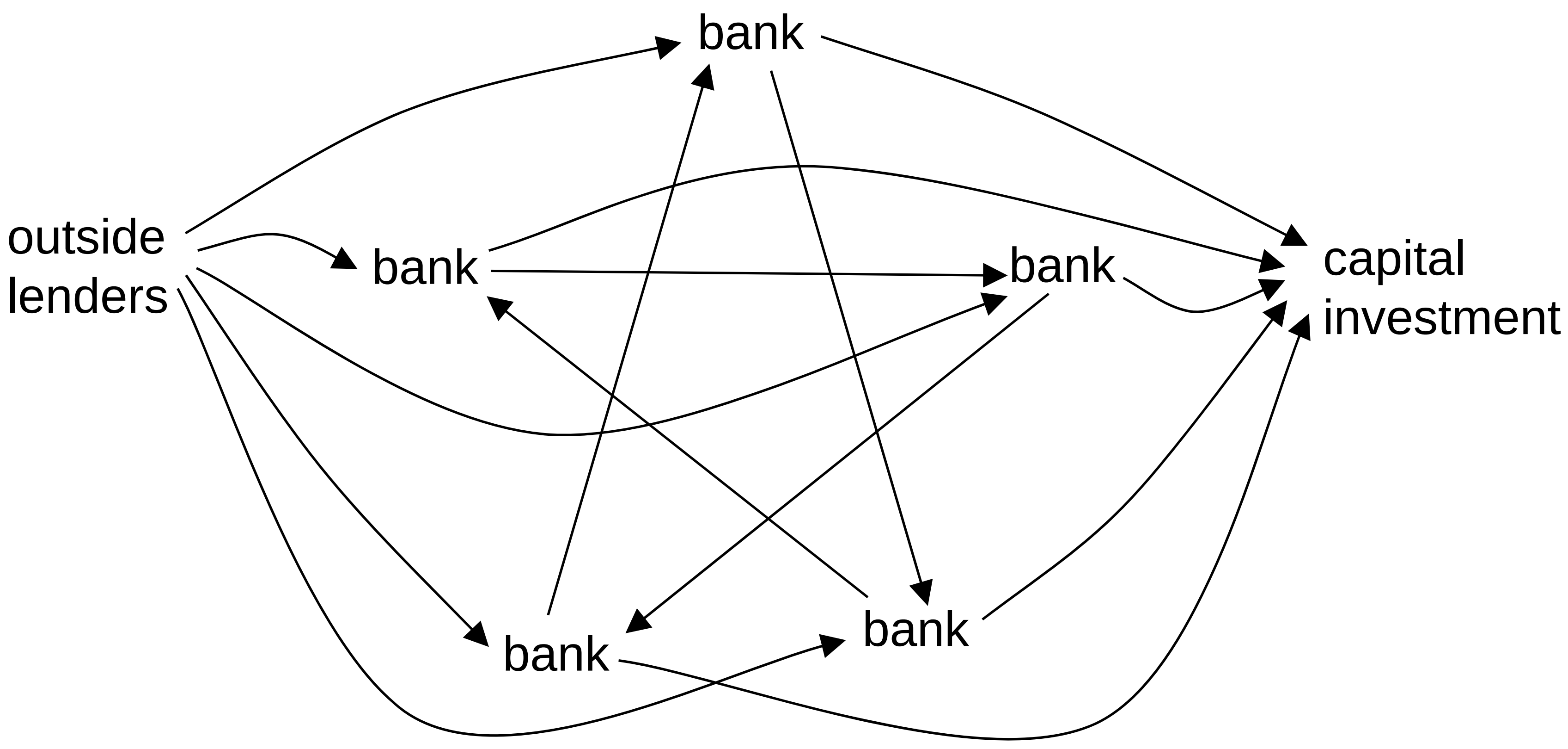
(collateral price – exogenous to model)

capital stock yields return $\tilde{y}_t$:

$$\tilde{y}_t = \tilde{F}(K_t) \geq -\ell_t K_t$$

returns are stochastic (and may be negative, e.g. because of maintenance costs of capital) banks borrow and lend to each other

banks also borrow from “outside lenders”,
who determine marginal interest rate



let us sidestep choice of maturity structure,
by assuming that only available credit instrument
is a consol:

promise {1, 1, 1, …} from tomorrow onwards

refer to consol as "paper"

A_t = accumulated stock of other banks' paper (a vector)

B_t = outstanding borrowing to outside lenders & other banks (own paper issued up to end of day $t – 1$)

NB consols have two key features:

– discipline of a short-term debt obligation

– long-term debt instrument whose resaleability may be subject to adverse selection

bank's balance sheet at start of day t

assets	liabilities
other banks' paper held, A_t	own paper issued, B_t
capital stock, K_t	equity (dividends c_t, c_{t+1}, …)

flow-of-funds on day t (for $i_t \geq 0$):

price vector of other banks' debt

$$i_t \quad + \quad B_t \quad + \quad c_t \quad + \quad p_t^* \cdot a_t$$

new investment (stochastic opportunity)

payment due on outstanding borrowing (without default)

dividend

purchases of others' debt

$$\leq \quad \tilde{y}_t \quad + \quad 1 \cdot \tilde{A}_t \quad + \quad p_t\, b_t$$

return on capital

receipts from others' debt

new borrowing

others may default: $\tilde{A}_t \leq A_t$

price of own paper

daily shocks to bank's flow-of-funds:

- investment opportunity?
- return on old investment, $\tilde{y}_t$
- receipts from other banks, $\tilde{A}_t$
- prices, p_t^* and p_t

bank's choices in response to shocks:

- investment and dividends, i_t and c_t
- asset purchase/sale ($a_t > 0/a_t < 0 \longleftarrow$ vector)
- borrowing/repurchase ($b_t > 0/b_t < 0$)

agency problem: bank's choices cannot be governed by ex ante contract

aside from the discipline of short-term debt, only other constraint applies to new borrowing ($b_t > 0$):

$$B_t + b_t \leq \text{function}\left[(K_t + i_t), \ell_{t+\tau}, (A_t + a_t), p^*_{t+\tau} \right]$$

$\tau = 0,1,2,\ldots$

weakly increasing in all arguments

inter alia, this constraint rules out Ponzi schemes

IMPORTANT: on days when bank does not borrow ($b_t \leq 0$), constraint may not hold on days when bank suffers negative shocks, and cannot borrow, it sells off assets

it may sell off capital stock (at price ℓ_t)

or other banks' paper (at price p_t^*)

order of sell-off dictated by price

if bank has exhausted its assets,

it declares itself bankrupt – no terminal value

but social loss of F-technology

NB no renegotiation with creditors en route to bankruptcy

Information: it is not publicly known if a bank
may be en route to bankruptcy

asymmetric information re bank’s balance sheet

$\Rightarrow$ adverse selection in resale markets
for the bank’s paper

$\exists$ two kinds of seller in resale market:

- information sellers, who have privately learnt that the bank may be en route to bankruptcy
- liquidity sellers, who need funds either to invest or to meet their own debt obligations

price p_t of bank’s paper determined by marginal outside lender (who has no private information)

p_t reflects the mix of information/liquidity sellers

a second bank – an “insider” – may privately learn that the first bank is <u>not</u> en route to bankruptcy, and hence be eager to buy at p_t

effectively, $\exists$ two forms of investment:

- expanding own capital stock
- buying another bank’s paper

A description of steady state:

- banks invest in capital stock and in each other
- investment in these new assets is funded by retained earnings and/or sales of old assets
- investment may be levered by issuing new paper, but subject to a borrowing constraint
- in troubled periods, a bank meets its debt obligations by selling assets
- if a bank has exhausted its assets, it declares itself bankrupt

information about banks' balance sheets is privately dispersed, which leads to adverse selection in the resale market for paper

on the one hand …

a fall in the price of one bank's paper

- harms that bank (it has to pay a higher interest rate on any new borrowing)
- harms other banks who wish to sell (in order to raise funds)

on the other hand …

a fall in the price of the first bank’s paper

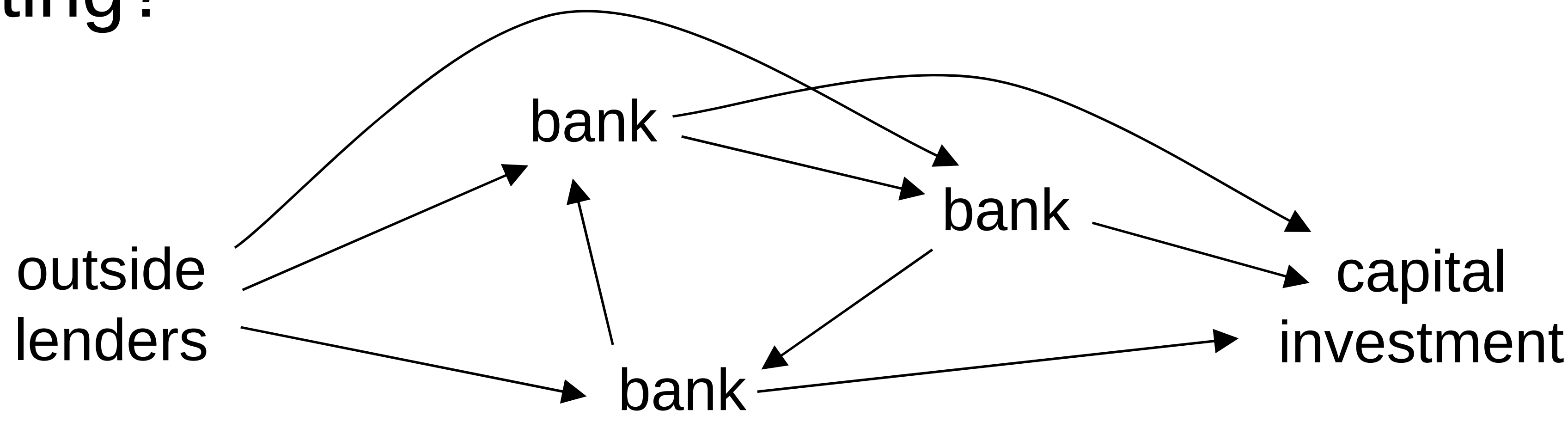
- benefits a second bank who wishes to buy (because it has learnt that the first bank is <u>not</u> en route to bankruptcy, and so knows that the price is “too low”)

the second bank may lever this investment (because the price of its own paper is higher)

$\Rightarrow$ chains of debt:

outside lenders $\longrightarrow$ second bank $\longrightarrow$ first bank $\longrightarrow$ capital investment

Netting?



assume multilateral netting is infeasible

(bilateral netting may be feasible
- although, given the possibility of bankruptcy, bilateral netting may not be "dollar for dollar")

we will see that financial collapse is fostered by multilateral chains of debt that aren't netted

Contagion Through Time (revisited)

consider a bank whose paper pays, from day 1,

either, with probability $1 - \lambda$,

$$(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, \ldots) \quad \text{H-paper (safe)}$$

or, with probability λ,

$$(1, 1, \ldots, 1, 1, \underset{\substack{\uparrow \\ \text{day } T+1 \\ \text{(bankruptcy)}}}{0}, 0, 0, 0, 0, 0, \ldots) \quad \text{L-paper (unsafe)}$$

lemon

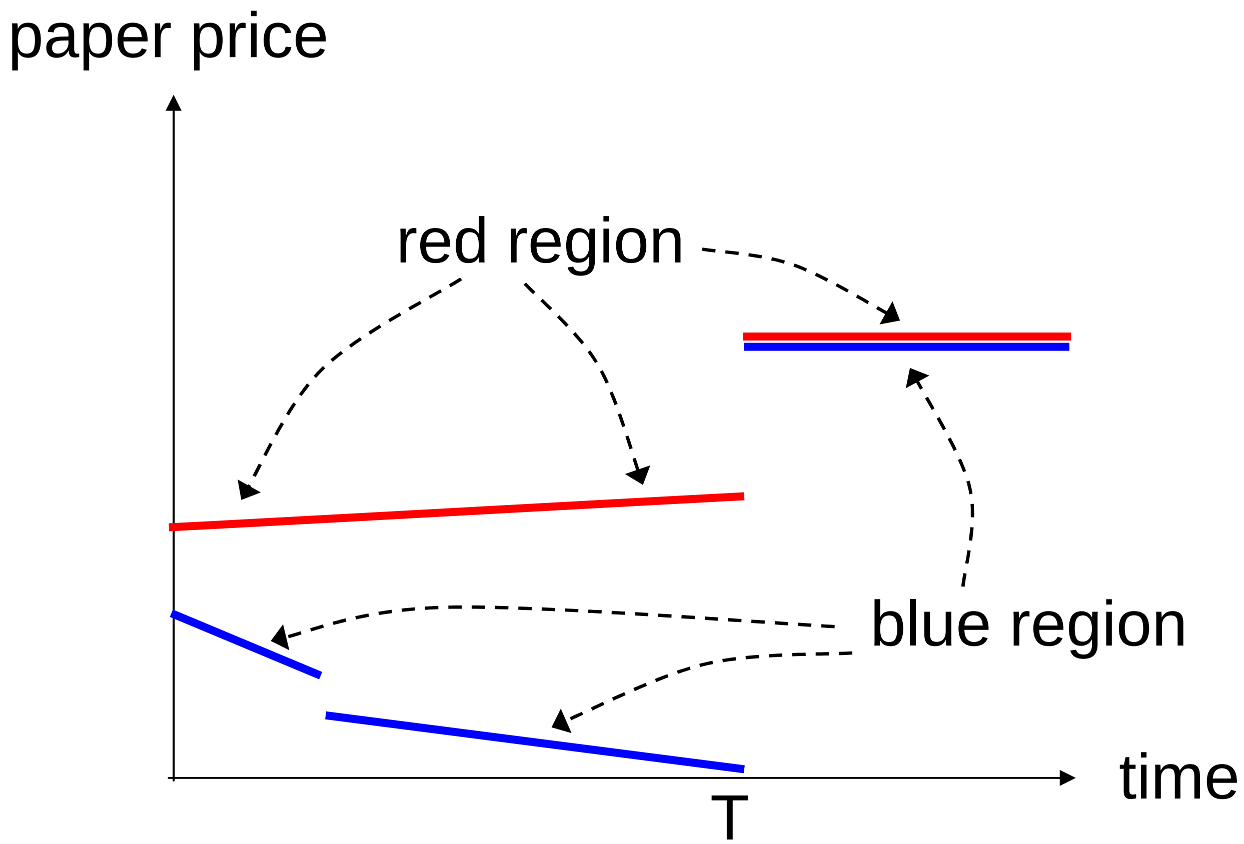
however, type of paper is not publicly known

assume that on each day t, $0 \leq t \leq T$:

- anyone holding paper privately learns whether or not it is a lemon
- a fraction *m* of paper holders have liquidity needs (they may have an investment opportunity, or they may have debt obligations to meet): their marginal utilities are scaled up by $\mu^s / \mu^b > 1$

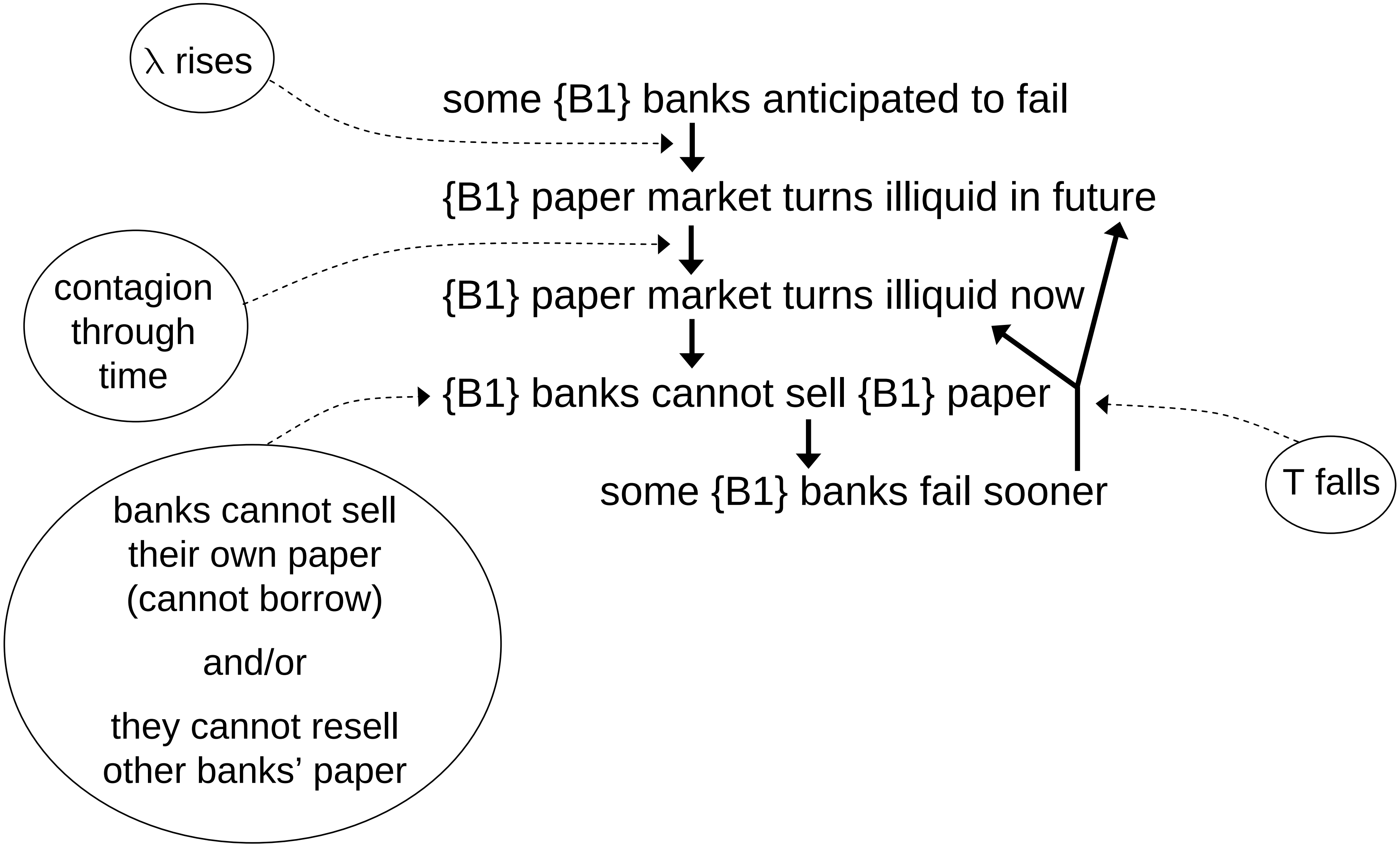
$$\Rightarrow \quad \frac{\#\text{ information sellers}}{\#\text{ liquidity sellers}} = \frac{\lambda}{m(1-\lambda)}$$

as in earlier analysis, ∃ two possible regions:



IMPORTANT: blue region more likely as T falls

Anatomy of a Financial Collapse (1)



Direct Contagion Across Markets

{B1} paper market turns illiquid

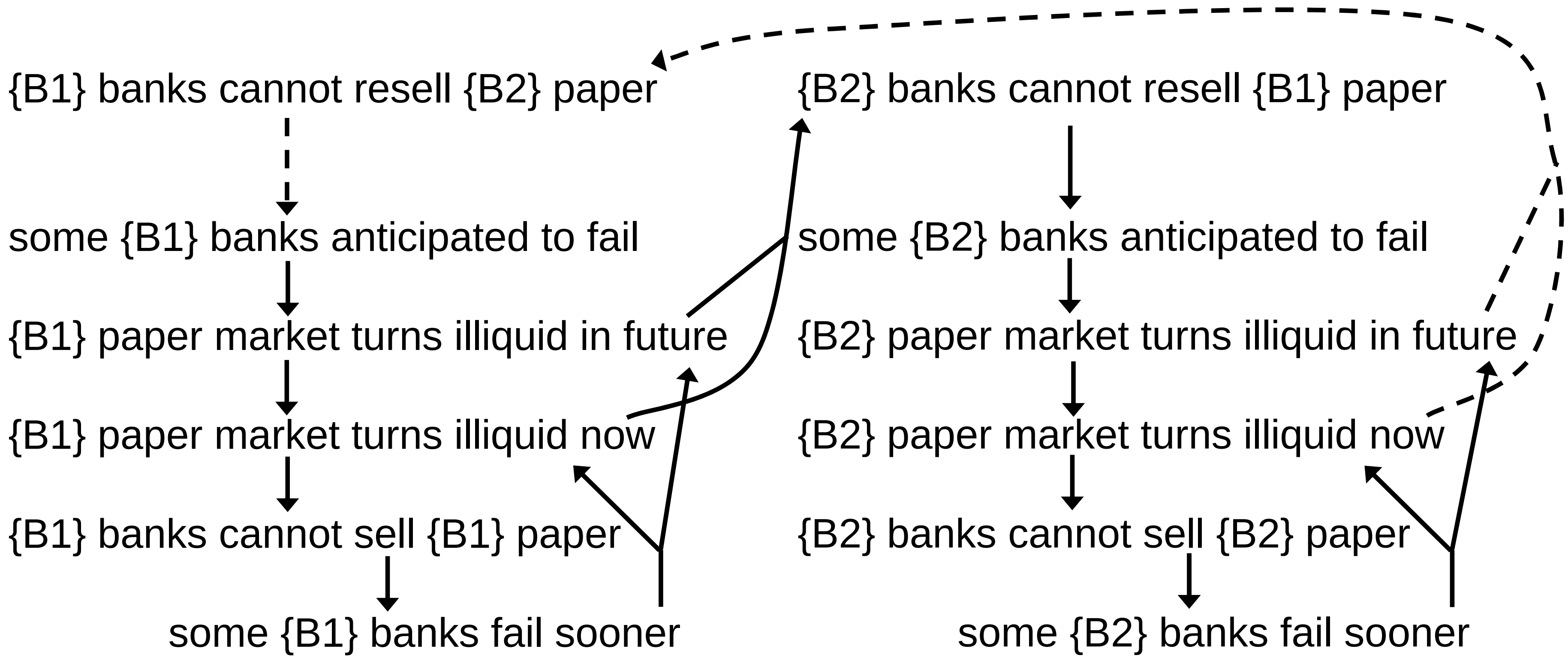
{B2} banks cannot resell {B1} paper

some {B2} banks anticipated to fail

{B2} paper market turns illiquid in future

{B2} paper market turns illiquid now

Anatomy of a Financial Collapse (2)



Indirect Contagion Across Markets

consider a bank making a levered investment:

suppose

- bank's rate of return on unlevered investment is 6%
- bank can borrow at rate 5%
- on each dollar of own money that bank invests, it can borrow 9 dollars
- bank has 100 dollars of own money to invest

with maximum leverage,

gross investment $= (100 + 900) = 1000$

gross payoff $= 1.06 \times 1000 = 1060$

of which $1.05 \times 900 = 945$ is owed

net payoff $= (1060 - 945) = 115$

$\Rightarrow$ rate of return on levered investment $= 15\%$

cf rate of return on unlevered investment $= 6\%$

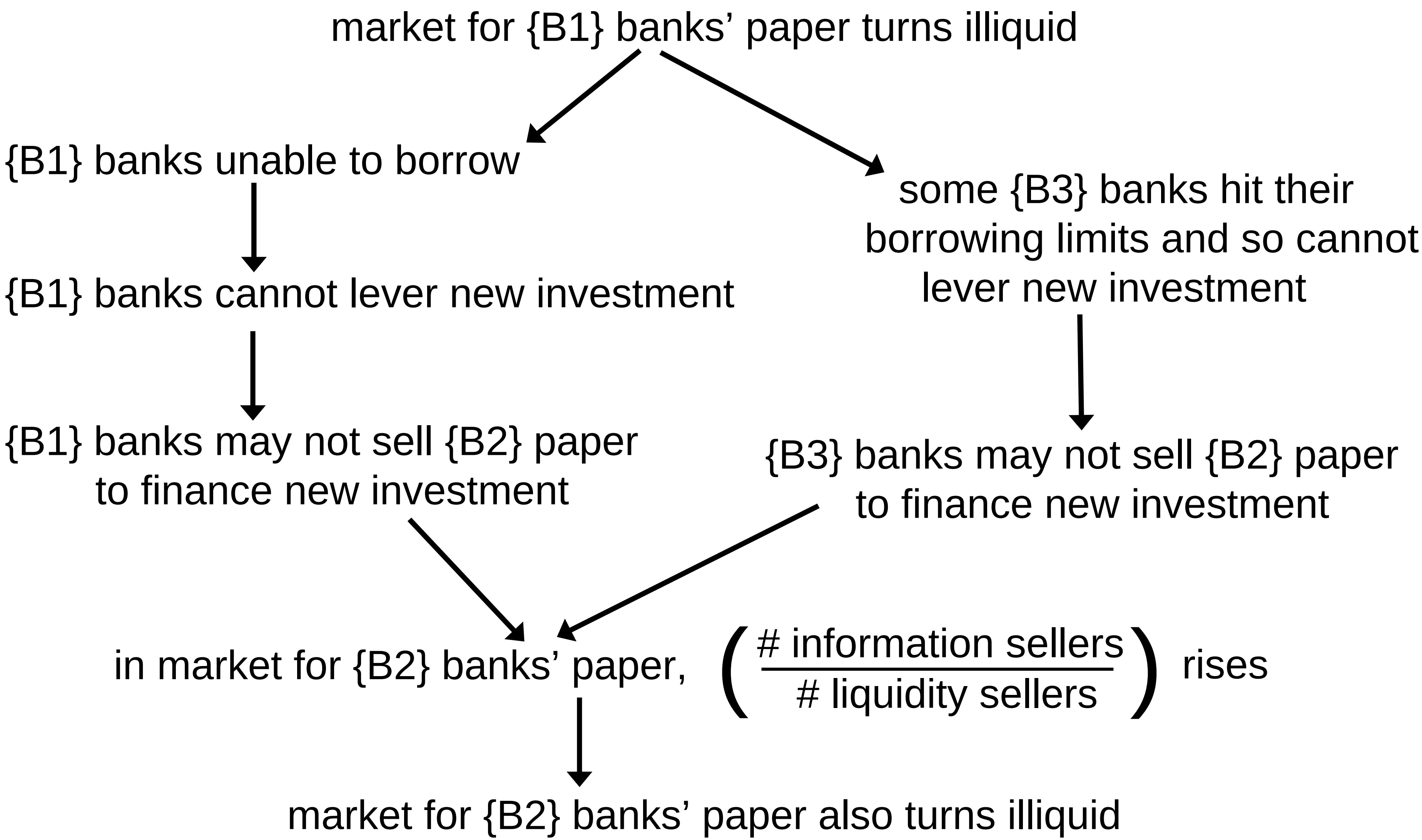
now suppose our bank currently owns another bank's paper (purchased previously), whose implied rate of return – taking into account the paper's current resale price – is 12%

given the alternative rate of return (15%), our bank will sell this other bank's paper to finance additional levered investment

but if our bank is denied borrowing and cannot lever additional investment, it will not sell the other bank's paper

$\Rightarrow$ fewer liquidity sellers in other bank's paper market, and hence adverse selection there?

Indirect Contagion Across Markets



Intuition for indirect contagion across markets:

Consider a bank with an investment opportunity
– so that, in terms of our earlier analysis,
it has a marginal utility of income $\mu^s > \mu^b$.

If the market for the bank's own paper turns illiquid, or if its asset holdings turn illiquid, then it may be unable to lever the new investment.

This significantly depresses the effective rate of return on its investment – which is tantamount to a (paradoxical*) REDUCTION in μ^s/μ^b

$\Rightarrow$ further market failure, as in 5 finger exercise

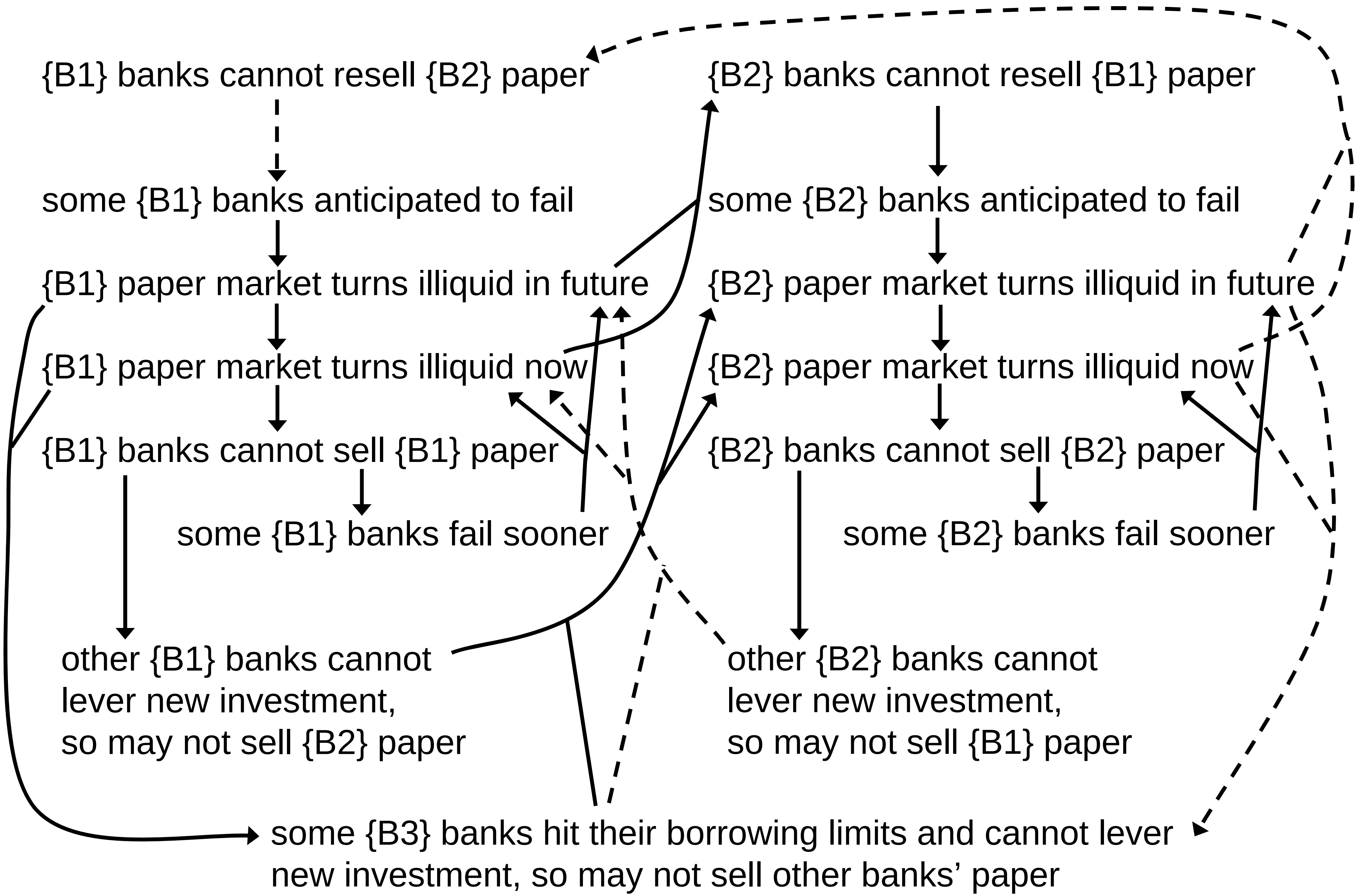
* Why “paradoxical”?

Because in times of financial distress, an agent (a bank) will typically both curtail consumption (reduce dividends) – INCREASE marginal utility of income – and sell off assets. That is, ceteris paribus, financial distress increases “liquidity selling” – which ameliorates adverse selection.

But we have identified a countervailing effect:

The inability to lever new investment depresses the rate of return, and inhibits the sale of assets to raise funds to finance new investment.

Anatomy of a Financial Collapse (3)



Two final remarks:

Remark 1

notice the two-way feedback:

“vertical” contagion (illiquidity through time)

$\downarrow \quad \uparrow$

“horizontal” contagion (illiquidity across markets)

Remark 2

- this is a model of contagious illiquidity,
  not a model of chains of default

debt markets turn illiquid

↓ ↑

risk of default rises & debt becomes riskier

- it is a model of financial collapse
  <u>prior</u> to actual default

Appendix: example of multiple Nash equilibria in a stationary infinite horizon environment

discrete “days”; single consumption good

agents alternate their marginal utilities:

today’s seller:	μ^s	μ^b	μ^s	…
	$\uparrow$	$\uparrow$	$\uparrow$	
	today	tomorrow	next day	…
	$\downarrow$	$\downarrow$	$\downarrow$	
today’s buyer:	μ^b	μ^s	μ^b	…

daily discount factor $\beta < \mu^b / \mu^s < 1$

2 types of asset (fraction λ of type L):

type H: pays 1, at the start of each day

type L: pays zero

current owner privately receives payment (1 or 0) thus privately learns the type of asset he holds

to simplify assume: no-one can identify an asset he previously sold

Stationary high-price equilibrium

each day, the price $\bar{p}$ is dictated by a typical buyer's indifference condition:

$$\mu^b \bar{p} \quad = \quad \beta\mu^s [\,(1 - \lambda) \;+\; \bar{p}\,]$$

he sells tomorrow – no matter which type of asset he learns that he has bought today

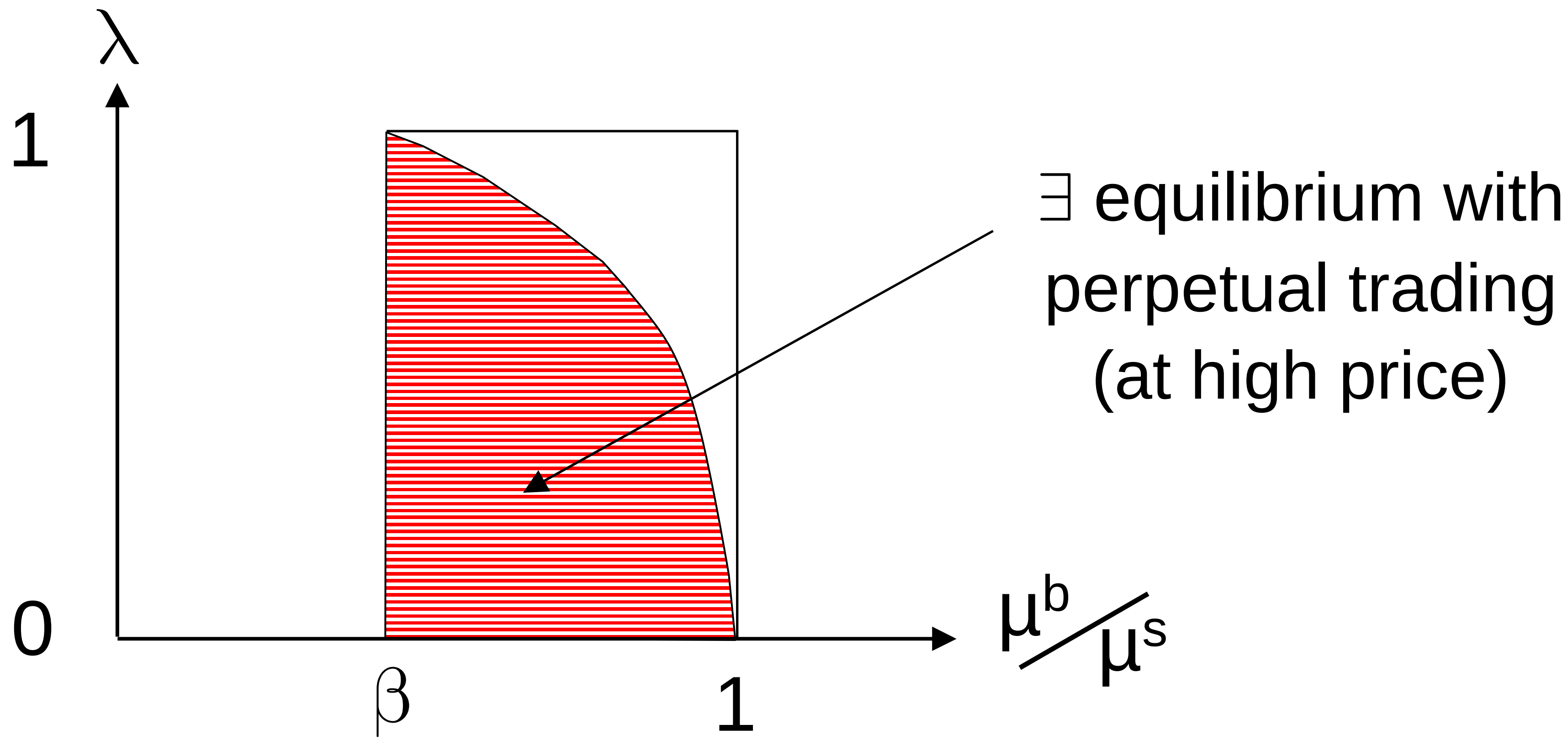
seller with the H-type asset must want to trade:

$$\mu^s \bar{p} \;>\; \beta\mu^b \;+\; \beta^2\mu^s [\,1 \;+\; \bar{p}\,]$$

$$> \; \beta\mu^b + \beta^2\mu^s + \beta^3\mu^b + \beta^4\mu^s + \ldots$$

that is, a “perpetual trading” equilibrium exists iff

$$\left(\frac{\mu^b}{\mu^s}\right)^2 < 1 - \lambda(1-\beta^2)$$



Does a "perpetual no-trading" equilibrium exist?

Suppose the market is expected to fail from tomorrow onwards. Will it fail today too?

A buyer looking to deviate from a zero-price today, wanting to attract "H-type sellers", would have to bid a price p satisfying:

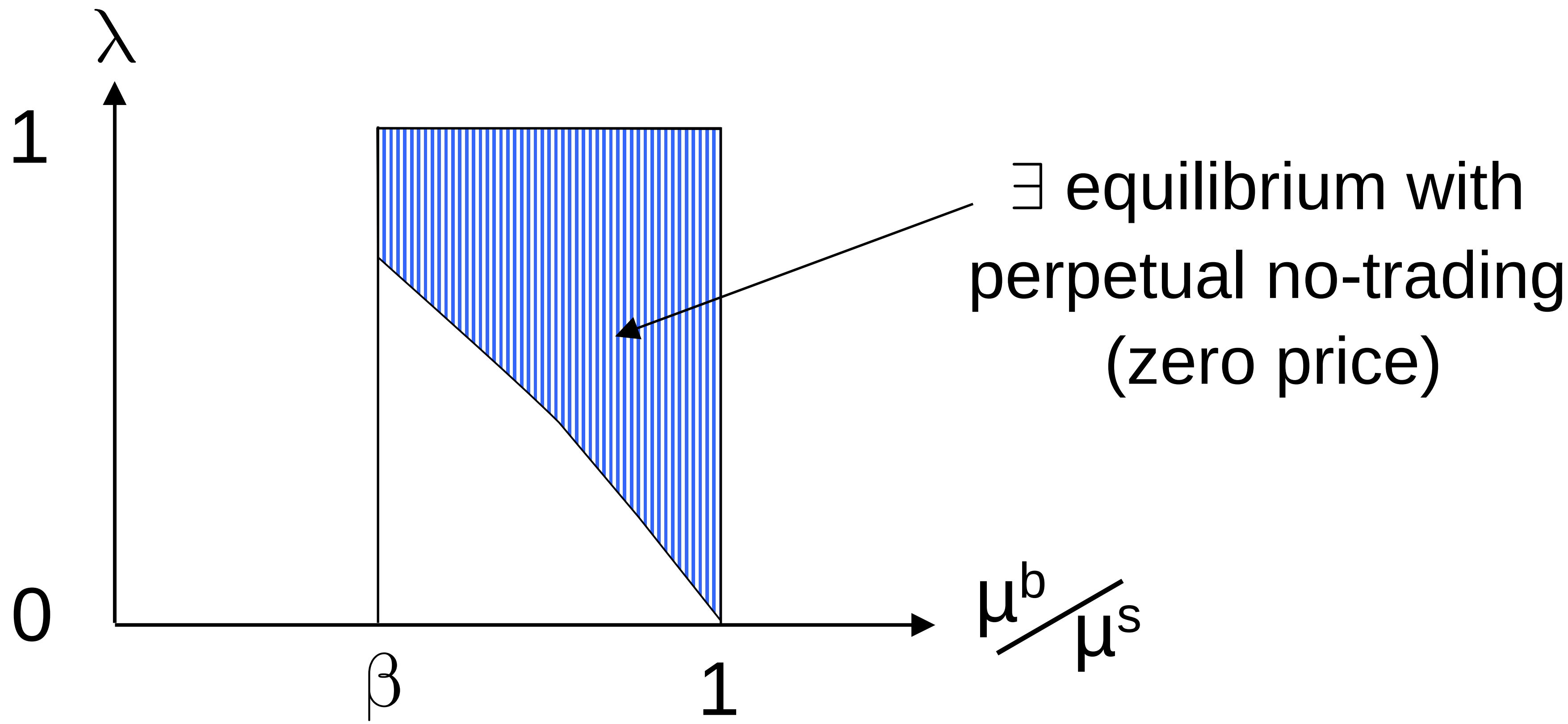
$$\mu^s p \geq \beta\mu^b + \beta^2\mu^s + \beta^3\mu^b + \beta^4\mu^s + \dots$$

Deviating to this bid p would be profitable only if

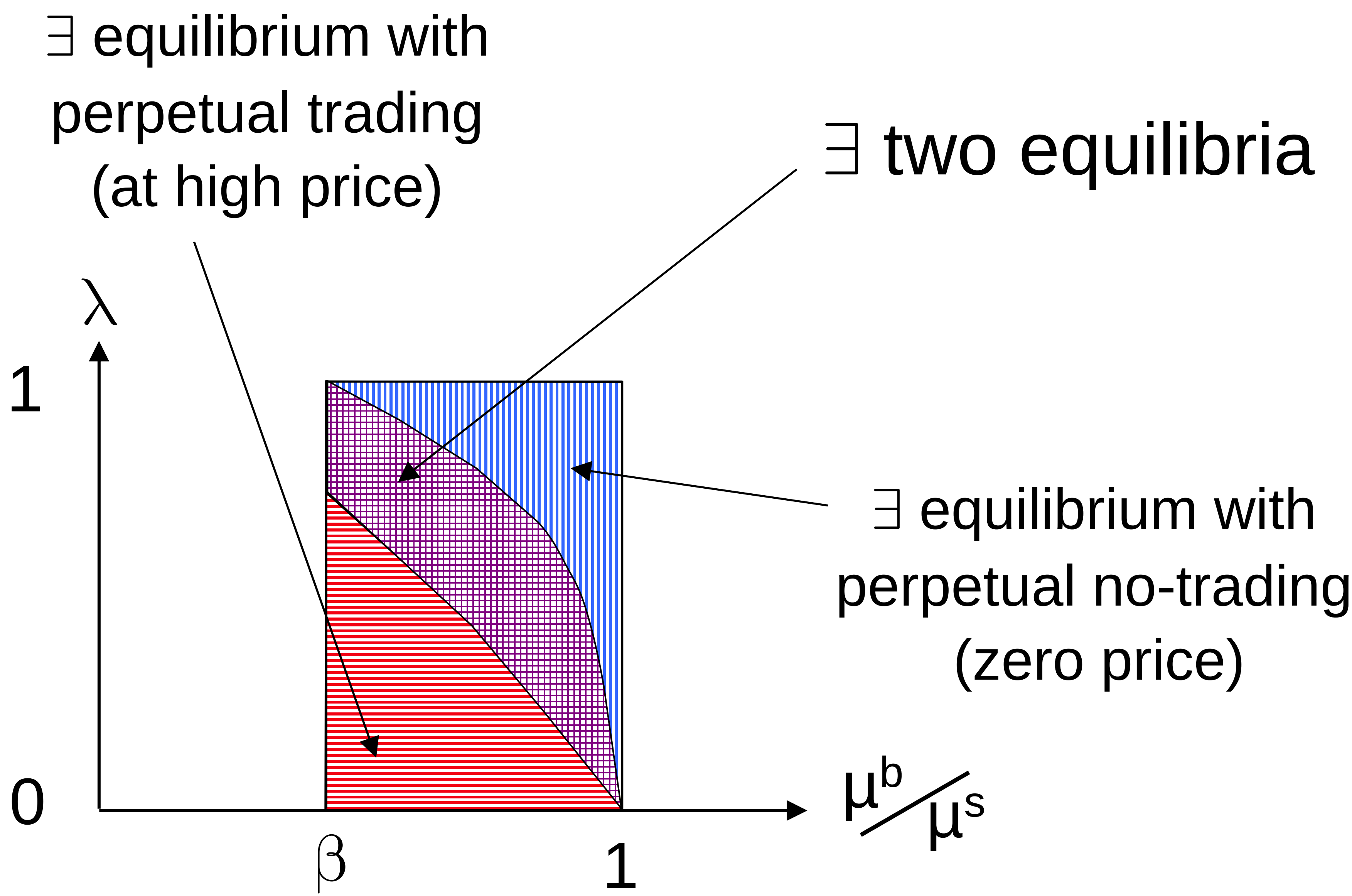
$$\mu^b p < \beta\mu^s(1-\lambda) + \beta^2\mu^b(1-\lambda) + \beta^3\mu^s(1-\lambda) + \beta^4\mu^b(1-\lambda) + \dots$$

that is, a "perpetual no-trading" equilibrium exists iff

$$1 - \lambda \quad < \quad \left(\frac{\mu^b}{\mu^s} + \beta\right) \Big/ \left(\frac{\mu^s}{\mu^b} + \beta\right)$$



Interesting … the two regions overlap:



In overlap region, $\exists$ two stationary equilibria:

- one equilibrium where both types of asset are always traded and prices are positive

- another equilibrium with perpetual market failure (zero prices and no trading)

Both equilibria are “Nash”

i.e. robust to agents actively making bids/offers