# Indexation and Insurance: A Stochastic Model of Credit Cycles

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## Introduction

- How do credit constraints propagate the effects of shocks on aggregate output and asset price?

$$\begin{tikzcd} \text{shock} \arrow[r] & \text{net worth}\downarrow\downarrow \arrow[rr, "\text{credit constraint}"] & & \text{investment}\downarrow \arrow[dl, "\text{market}"] \\ & & \text{asset price}\downarrow \arrow[ul, "\text{leverage}"] & \end{tikzcd}$$

- Net worth is vulnerable to changes in asset price:

B/S

| assets | liabilities 90 |
|---|---|
| 100 → 95 | net worth 10 → 5 |

liabilities not indexed
insurance not used

- Question: Why not?

# Explanation

- Consider an economy in which intertemporal exchange faces commitment problem

$$\begin{tikzcd} \text{lender} \arrow[r, bend left, "\text{present goods}"] & \text{entrepreneur} \arrow[l, bend left, "\text{future goods ?}"] \end{tikzcd}$$

⇒ use of collateral

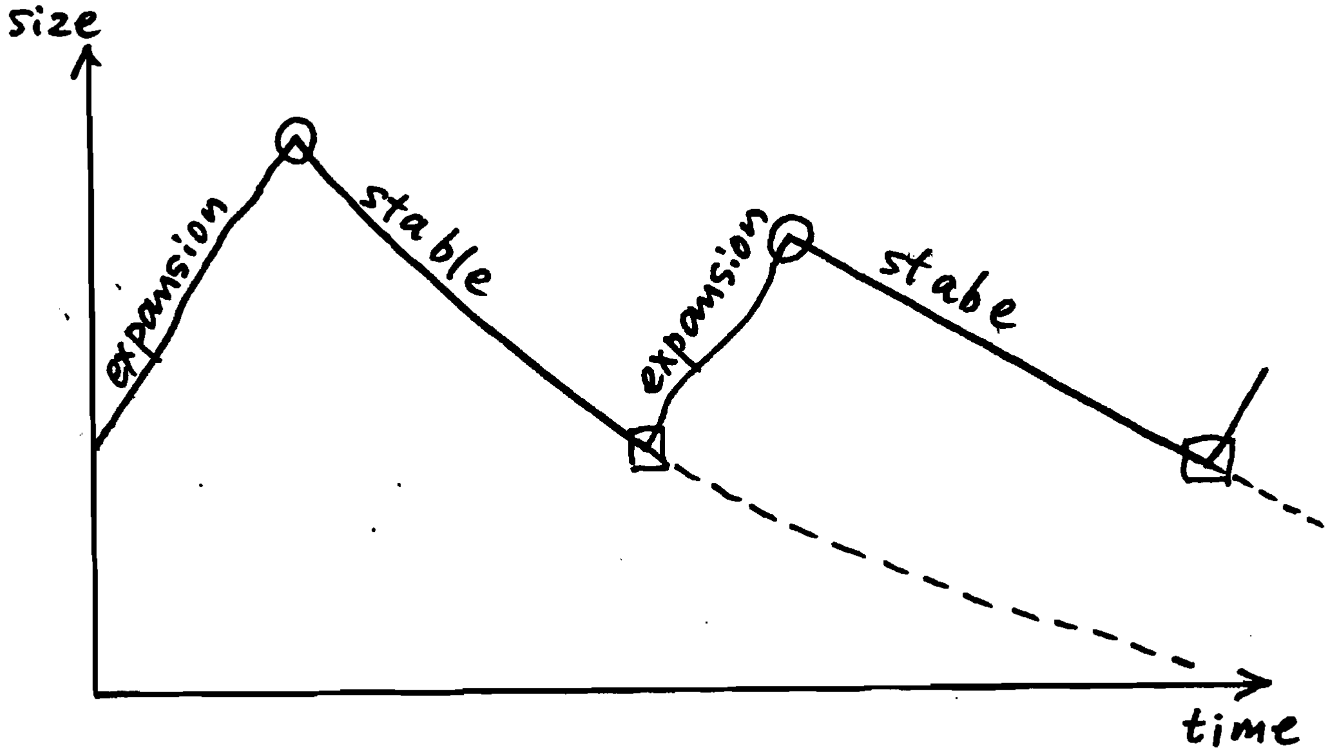
- Two forms of collateral

| Two forms of collateral | indexed to asset price? |
|---|---|
| PV of future cash flow | no |
| Liquidation value of assets | yes |

- If the entire PV of future cash flow becomes collateral, then the investment is first best.

- If entrepreneur E's human capital is essential, then the lender relies on liquidation value of collateral assets.

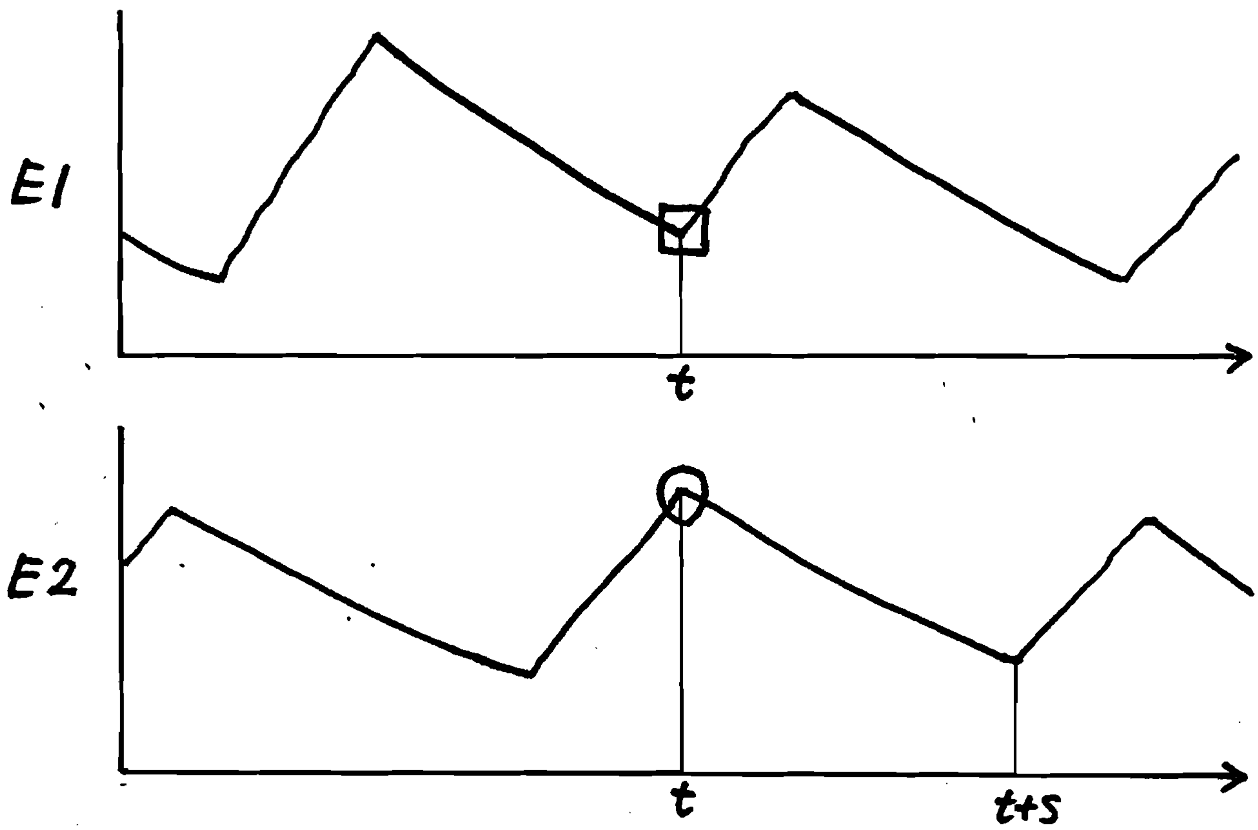
· E's firm goes through phases




| · Common View | forms of collateral |
|---|---|
| E is essential in expansion | liquidation value |
| E is replaceable in stable phase | PV of cash flow |

| | old debt | new debt |
|---|---|---|
| O : end expansion | indexed | not indexed |
| □ : start expansion | not indexed | indexed |

· Many firms : Asynchronized




· Asset price falls at date t

$\Rightarrow$ E1's available funds $\downarrow\downarrow$ $\Rightarrow$ E1's investment $\downarrow$ at t

E2's available funds $\uparrow\uparrow$ $\Rightarrow$ E2's investment $\uparrow$ at t+s

· Note fall in asset price shifts available funds
from E's who are investing now
to E's who will invest later

· We show in rational expectations equilibrium

asset price $\downarrow$ $\rightleftarrows$ aggregate investment $\downarrow$

# Model

- infinite horizon $\cdots, t-1, t, t+1, t+2, \cdots$

- 2 goods
  nondurable goods, $m$, numeraire
  durable capital, $k$, price $q_t$
  depreciates by factor $\lambda$

- 2 types of agents
  entrepreneurs $E$
  deep pockets $D$

- deep pockets
  risk neutral with discount factor $1/R$
  demand for capital depends on the user cost

$$u_t = q_t - \frac{\lambda}{R} E_t(q_{t+1})$$

$\Rightarrow$ residual supply of capital to the entrepreneurs

$$u_t = u(\underset{\oplus}{K_t}),$$

$K_t$ : aggregate capital of E's

E has 2 technologies

- Illiquid technology

$$
\begin{array}{ccccc}
\text{date } t & & \text{date } t+1 & & \text{date } t+2 \\
\left\{\begin{array}{l} k_t \text{ capital} \\ \phi k_t \text{ goods} \end{array}\right. & \longrightarrow & \left\{\begin{array}{l} \lambda k_t \text{ capital} \\ a k_t \text{ goods} \end{array}\right. & \longrightarrow & \left\{\begin{array}{l} \lambda^2 k_t \text{ capital} \\ \lambda a k_t \text{ goods} \end{array}\right. \longrightarrow \\
\text{input} & & \text{output} & & \text{output}
\end{array}
$$

- Initially, between dates $t$ and $t+1$, E is essential for production (expansion phase)

  $\Rightarrow$ future output cannot be mortgaged

  borrowing limit $= \frac{\lambda}{R} E_t(q_{t+1}) k_t$

- Subsequently, after date $t+1$, E can be replaced (stable phase)

  $\Rightarrow$ future output can be mortgaged

  borrowing limit $= \frac{a}{R-\lambda} k_{t+1} > \frac{\lambda}{R} E_t(q_{t+1}) k_{t+1}$

- New expansion opportunity arrives with probability $\pi$ every period (i.i.d. across time and E's)

  (start of new expansion phase)

- Liquid technology (17)

date $t$ date $t+1$

$m_t$ goods input $\longrightarrow$ $\sigma_{t+1} m_t$ goods output (cannot be collateralized)

$\sigma_{t+1}$ is iid across time
$\sigma_{t+1}$ is common to all E's

- E's preference

illiquid investment $\succ$ liquid investment $\succ$ saving

- Insurance contract

absense of spare collateral

$\Rightarrow$ premium must be paid in advance

$\Rightarrow$ "rate of return" on policy $= R$, which is dominated by liquid technology

$\Rightarrow$ E will not buy insurance

liquid investment $\succ$ insurance $\succ$ saving

- E with expansion opportunity

$$\underbrace{(q_t + \phi)(k_t - \lambda k_{t-1})}_{\text{purchase \& customization of new capital}} = \underbrace{\sigma_t m_{t-1}}_{\text{return from liquid inv.}} + \underbrace{a k_{t-1}}_{\text{output of illiquid inv.}}$$

$$+ \underbrace{\frac{\lambda}{R} E_t(q_{t+1}) k_t}_{\text{new borrowing}} - \underbrace{d_t}_{\text{repayment of old debt}}$$

$$m_t = 0$$

- E in stable phase

$$k_t = \lambda k_{t-1}$$

$$m_t = \sigma_t m_{t-1} + a k_{t-1} + \underbrace{\frac{a}{R-\lambda} k_t}_{\text{new borrowing}} - d_t$$

where $d_t$ = repayment of old debt

$$= \begin{cases} R \dfrac{a}{R-\lambda} k_{t-1}, & \text{if } t\text{-}1 \text{ was stable phase} \\ \lambda q_t k_{t-1}, & \text{if } t\text{-}1 \text{ was expansion phase} \end{cases}$$

· Rational Expectations Equilibrium $\{K_t, M_t, q_t\}$

E's aggregate capital $K_t$
E's liquid investment $M_t$
capital price $q_t$

$$(1)\quad K_t = (1-\pi)\lambda K_{t-1} + \frac{\pi}{u_t+\phi}\Big\{ \sigma_t M_{t-1} + (a+\lambda\phi)K_{t-1} + \Big(\lambda q_t - R\frac{a}{R-\lambda}\Big)(1-\pi)\lambda K_{t-2}\Big\}$$

$$(2)\quad M_t = (1-\pi)\Big\{\sigma_t M_{t-1} + \Big(R\frac{a}{R-\lambda} - \lambda q_t\Big)\big[K_{t-1} - (1-\pi)\lambda K_{t-2}\big]\Big\}$$

$$(3)\quad q_t - \frac{\lambda}{R}E_t(q_{t+1}) = u_t = u(K_t)$$

- Linearizing around the unique steady state, equilibrium proportional deviations, $\hat{x}_t \equiv \frac{x_t - x^*}{x^*}$, follows stochastic process

$$\begin{pmatrix} \hat{K}_t \\ \hat{K}_{t-1} \\ \hat{M}_t \\ \hat{g}_t \end{pmatrix} = A \begin{pmatrix} \hat{K}_{t-1} \\ \hat{K}_{t-2} \\ \hat{M}_{t-1} \\ \hat{g}_{t-1} \end{pmatrix} + B\ \hat{\sigma}_t$$

In particular

$$\hat{K}_t = \underset{\text{multiplier}}{\frac{1}{1-\mu}} \Big\{ \underset{\text{direct effect}}{\alpha_1 \hat{\sigma}_t} + \underbrace{\alpha_2 \hat{K}_{t-1} + \alpha_3 \hat{K}_{t-2} + \alpha_4 \hat{M}_{t-1}}_{\text{persistence}} \Big\}$$

Analytical convenience, we consider $E_t(\hat{\sigma}_{t+1}) = R$

As $\lambda \to 1$, $R \to 1$, the multiplier converges to $\frac{1}{\pi}$.

ex) $\pi = 0.1$

Indirect effect = 9 times as large as direct effect !

## Summary

- asset price ↓ → available funds of E's starting expansion ↓; available funds of E's ending expansion ↑ → aggregate investment ↓ → asset price ↓

- <u>dynamic multiplier</u>

  decrease in investment persists $\xrightarrow{\text{forward looking}}$ fall in asset price amplified

  fall in asset price amplified $\xrightarrow{\text{leverage}}$ decrease in investment persists

- <u>Indexation</u>

  some debt is indexed, because it is secured against liquidation value of assets
  (expansion phase)

  some debt is not indexed, because it is secured against PV of future cash flow
  (stable phase)

## Insurance

In the absence of spare collateral, premium must be paid ex ante.

⇒ dominated by investment

(The very firm who needs insurance is the firm who chooses not to buy insurance)

We believe that this explanation for the limited use of indexation and insurance may apply more generally.