# Cardinal Sins? Conspicuous Consumption, Cardinal Status and Inequality

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**Abstract**

This paper analyzes the social dilemma arising when a large population of individuals with differing incomes have cardinal concerns over relative deprivation in terms of conspicuous consumption. This includes inequity aversion, where negative comparisons are more important than positive, rivalrous preferences, and comparison with mean consumption. The resulting Nash equilibrium is inefficient. When preferences are rivalrous, the optimal consumption schedule makes all better off than in the non-cooperative equilibrium, but under inequity aversion it makes the rich worse off. In this setting, inequality has a direct effect on behavior - under rivalrous preferences, increased inequality can lead to lower welfare at all income levels and increased consumption at most.

**Keywords:** relative comparisons, relative deprivation, games, social status, conspicuous consumption.

**JEL Classifications:** C72, D63, D91.

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# 1 Introduction

Why should we be concerned by increasing inequality? Suppose people have relative concerns, so that the incomes or consumption of others affect satisfaction. Then, an increase in incomes and consumption for the very rich could have a direct, negative effect on the rest of the population. This could work through subjective wellbeing - the happiness of the non-rich is reduced. But it could also affect behavior, with an increase in the consumption of the rich leading others to increase consumption. There is considerable evidence both at the individual and at the society level for negative externalities from richer neighbors (Clark et al., 2008), just as there is evidence for social or other-regarding preferences from laboratory studies (Cooper and Kagel, 2016). There is also evidence for the specific hypothesis of consumption spillovers (Bertrand and Morse, 2016).

However, existing theoretical models of relative concerns are not consistent with most of these hypotheses. Ordinal status models, such as in Frank (1985) and Hopkins and Kornienko (2004), and signalling models such as used by Ireland (2001), Charles et al. (2009) and Moav and Neeman (2012), have two important characteristics in common. First, conspicuous consumption expenditure in fact in general *decreases* with income inequality. Second, the equilibrium is constructed bottom up so that changes in the top of the income distribution alone cannot change the behavior of the poor or middle class.[1] Further, in the other main alternative, sometimes known as "Keeping Up with the Joneses" (Galí, 1994), increases in inequality that do not change average income are unlikely to have an effect because the model only considers the average of others' consumption.

Here I show that the effects of inequality depend on the exact form of relative concerns assumed. In particular, suppose such concerns are cardinal and not ordinal, so that people worry about the distance between their consumption and others' and not just their relative position. Further, relative concerns have often been found to be asymmetric, as with loss aversion, negative comparisons weigh more heavily than positive ones. Indeed, if individuals specifically look upwards at their richer neighbours, one might guess that there are different results than in equilibria built from the bottom. However, to get such results is difficult because models with cardinal concerns have not have been solved for the general case of heterogenous agents and indeed require different methods from those with ordinal concerns.

In this paper, I show how to solve large population games between heterogeneous consumers who have cardinal relative concerns. Each must choose how to divide her income between conspicuous and non-conspicuous consumption. Individuals have cardinal preferences over their relative levels of conspicuous consumption, with in particular

[1] More technically, in auction-like and signalling models, the boundary condition for an equilibrium is given by the behavior of the lowest type - here the poorest agent. But this means that each individual is only affected by the income distribution between that poorest agent and herself. Consumption can potentially increase in inequality in a signalling model but only if such consumption in convex in income. Generally, Engel curves, that is consumption as a function of income, are concave. See, for example, Heffetz (2011).

utility decreasing in relative deprivation in terms of conspicuous expenditure, or equivalently the average expenditure of those above them (envy). I allow for attitudes to those below them to be either negative ("rivalrous" or "competitive" preferences) or positive ("inequity aversion"). A special case is when concerns are negative and symmetric, and then utility simply depends on the average consumption of others (known as "Keeping up with Joneses" or external habit). The model has to be solved simultaneously at all income levels, rather than starting at the lowest income level, which requires new techniques. Nonetheless, it is possible to show, under quite general conditions, that there exists a monotone Nash equilibrium in which conspicuous consumption is strictly increasing in income.

This Nash equilibrium is not efficient, with consumption mostly higher than in the absence of status concerns. That is, as originally argued in Frank (1985), the pursuit of relative position is a social dilemma. However, the nature of that dilemma differs from the ordinal case. Suppose a planner could choose consumption for each agent to maximize total welfare. I show that the resulting socially optimal consumption allocation is not always a Pareto improvement on the Nash equilibrium outcome. Instead the rich can be worse off than in the non-cooperative equilibrium. In effect, the social planner would make the rich pay for the negative externalities they cause which affect those beneath them. The situation is like that of an industrial plant that pollutes a river, only affecting those who are downstream, and suffers no ill effects itself. Thus, being made to pay for this pollution must make the polluter worse off. However, when there are also negative downward concerns, which generate a social dilemma of excessive consumption at all income levels, there is a possible Pareto improvement as in the ordinal case.

Further, I find that in this setting the distribution of income has a direct effect on behavior and hence on welfare. An increase in relative deprivation, caused by an increase in the incomes of the rich, under rivalrous preferences can lead to increased consumption and lower utility for the middle classes - even though their incomes are unchanged. Further, an increase in inequality under upward looking comparisons can lead to increased consumption by many and a reduction of welfare at all income levels. That is, the effect of greater inequality under cardinal relative concerns is quite different from that under standard assumptions and is directly opposed to that under ordinal relative concerns. This shows that the effect of greater inequality is not obvious and depends heavily on which relative concerns are assumed.

Many others have looked at the question of conspicuous consumption. Frank (1985), Hopkins and Kornienko (2004, 2009) and Becker et al. (2005) analyse the case of ordinal preferences, where individuals care about their rank in the distribution of conspicuous consumption. Turning to cardinal preferences, the most common approach has been to look at the case where individuals care about the difference between their consumption and average consumption in society, a formulation known as "Keeping Up with the Joneses" (KUJ) preferences. But prominent papers in this literature, for example Galí (1994), assume identical agents. Clark and Oswald (1998) and Barnett et al. (2019) look at comparative statics when individuals have KUJ-like preferences. Bilancini and Boncinelli (2012) also contrast ordinal and cardinal status concerns with inequality, but only consider two levels of income. There is a further branch of literature including Ireland (2001), Charles et al. (2009), Heffetz (2011), Moav and Neeman (2012) and Jinkins (2016) that use signalling models. In these signalling models, inequality would normally have a similar effect as in the ordinal status model: signalling/consumption decreases with greater inequality.

The only other papers, to my knowledge, to analyse conspicuous consumption with asymmetric cardinal preferences are Friedman and Ostrov (2008) and Bellet and Colson-Sihra (2018). Friedman and Ostrov consider the case where agents are ex ante identical, rather than the heterogenous case considered here. Bellet and Colson-Sihra (2018) show that conspicuous consumption will be increasing in relative deprivation (similar to Lemma 1 here), and test this result with data from India. Frank et al. (2014) consider upward-looking relative concerns but directly assume effects on consumption behavior. Thus, this is the first paper that contains analytic results on a game of status with either KUJ or asymmetric preferences under full heterogeneity and thus is able to address the effect of inequality.

Relative deprivation, crucial to this paper, was first formalized by Yitzhaki (1979). Preferences in which individuals care about relative deprivation have been extensively analysed in the context of laboratory experiments (Fehr and Schmidt, 1999, and an enormous subsequent literature). The clear difference is that this literature on social preferences defines utility over money outcomes of an individual and those she compares herself with, while here the preferences are over consumption. The justification for this is simply that consumption is more visible than income and more likely to be the cause of invidious comparisons.

Turning to recent empirical studies, Charles et al. (2009), Heffetz (2011), Jinkins (2016) and Bellet and Colson-Sihra (2018) have very different methodologies but all find evidence for conspicuous consumption being an important phenomenon. Frank et al. (2014), Drechsel-Grau and Schmid (2014), Alvarez-Cuadrado et al. (2016) and Bertrand and Morse (2016) find evidence for relative consumption effects. Perhaps the only paper that tests directly between ordinal and cardinal preferences is Brown et al. (2008) which finds that a range-frequency model that incorporates both cardinal and ordinal measures is the best fit to their data. However, note that Drechsel-Grau and Schmid's (2014) and Bertrand and Morse's (2016) findings that consumption of the rich affects the consumption of the non-rich are in contradiction to the ordinal model which in effect assumes that people only look downwards.

Finally, both relative concerns and negative externalities can be found in contexts outside conspicuous consumption. Azmat and Iriberri (2010) and Tincani (2018) investigate relative concerns as an incentive for educational performance. Gitmez et al. (2020) note that the negative externalities from risky behavior during the coronavirus pandemic have a similar form to those from conspicuous consumption. Therefore, obtaining a better understanding of negative externalities and how they interact with inequality may be timely.

# 2 The Model

Standard economic theory treats choice of consumption as a single-agent decision problem. However, if individuals compare their consumption with that of others, the decision becomes strategic because the actions of others affect the outcome of the individual. This strategic approach is taken in Frank (1985), Hopkins and Kornienko (2004) and Becker et al. (2005), but with a crucial difference. This earlier work assumed ordinal status concerns - satisfaction depends on how an individual ranks in consumption. Here, relative concerns are cardinal, depending on the difference between own consumption and that of others. Further, concerns can be asymmetric with greater weight placed on upwards comparisons.

I consider a large population of individuals who all possess similar relative concerns but who differ in income $z$. Income is distributed according to the exogenous distribution $G(z)$ on $[\underline{z}, \bar{z}]$, where $\underline{z} > 0$, with continuous non-zero density $g(z)$ and mean $\mu$. As in standard Bayesian games, nature moves first and informs each player of her income level which is her private information, whereas the distribution of income is common knowledge. Then, all simultaneously decide how much income $z$ to spend on visible consumption $x$, with the remainder $z - x$ spent on other non-visible consumption $y$.[2] The alternative interpretations, under slightly different assumptions, are that $x$ is consumption and $y$ is leisure or savings. In any case, let the resulting consumption choices $x$ be aggregated into the distribution $F(x)$, which thus is endogenous.

The next step is to construct a cardinal measure of relative position. Let relative deprivation in consumption of an individual who has visible consumption $x$ facing visible consumption of others $x_{-i}$ be,

$$D(x; x_{-i}) \equiv \int_x^{\infty} (t - x)\ dF(t) = d(x_{-i}) - x(1 - F(x)), \tag{1}$$

where

$$d(x_{-i}) = \int_x^{\infty} t\ dF(t). \tag{2}$$

That is, $d(x_{-i})$ is the total expenditure of those having greater consumption than $x$. Thus the relative deprivation of an individual consuming $x$ is equal to the average distance between $x$ and the consumption levels higher than $x$. Similarly, define relative advantage as,

$$A(x; x_{-i}) \equiv \int_0^{x} (x - t)\ dF(t) = xF(x) - a(x_{-i}) \tag{3}$$

where

$$a(x_{-i}) = \int_0^{x} t\ dF(t). \tag{4}$$

[2] Heffetz (2011) studies empirically the visibility of different categories of consumption and finds that cigarettes, cars and clothes are the most visible, while insurance and underwear are the least. Bellet and Colson-Sihra (2018) find that in India relative concerns increase demand for goods such as clothing, dairy products, meat, fuel and lighting, packaged products and drinks, and shift demand away from cheap nutritious goods such as cereals, pulses and vegetables.

That is, $a(x_{-i})$ is the total expenditure of others who have consumption lower than $x$. So, relative advantage is equal to the average distance between $x$ and the consumption levels lower than $x$. This formalization of relative deprivation was introduced, in terms of incomes, by Yitzhaki (1979).[3] The current formulation of relative deprivation and advantage is inspired by Fehr and Schmidt (1999) (extended to a continuum population in Deaton, 2003).

Let status in consumption for an individual $i$ with visible consumption $x$ facing others' visible consumption $x_{-i}$ be

$$S(x; x_{-i}) \equiv -\alpha D(x; x_{-i}) - \beta A(x; x_{-i}). \tag{5}$$

The idea is that, in assessing her own consumption, the individual places weight $\alpha$ on upward or negative comparisons and weight $\beta$ on downward or positive comparisons.[4] Assume that $1 > \alpha \geq 0$ and $1 \geq \beta > -1$. That is, status is decreasing in relative deprivation but may be decreasing or increasing in relative advantage. There are four important cases.

1. Inequity aversion: $\alpha \geq \beta > 0$. Status is decreasing in relative deprivation and relative advantage.
2. Neoclassical baseline: $\alpha = \beta = 0$. No relative concerns.
3. Upward comparisons only: $\alpha > \beta = 0$. Status is decreasing in relative disadvantage only.
4. Rivalrous or competitive comparisons: $\alpha > 0 > \beta$. Status decreases in relative deprivation but is increasing in relative advantage.

Where $\beta$ is positive so that an agent dislikes advantage, or has "compassion" for those lower than her or "guilt", this is inequity aversion as in Fehr and Schmidt (1999). Note that usual additional assumption $\alpha \geq \beta$ implies social loss aversion - negative comparisons are felt more strongly than positive. In contrast, when $\beta$ is negative, so that an individual has "pride" in being higher up than the poor, then I refer to this as competitive or rivalrous concerns.[5] Here status depends on relative deprivation and relative advantage in terms of visible consumption and not income. The justification for this is that, first, visible consumption is literally more visible than income and, second,

[3]However, Yitzhaki defines a different counterpart to relative deprivation called relative satisfaction, in current notation, $\int_0^x 1 - F(t)\ dt = x - A(x, x_{-i})$.

[4]Thus, relative satisfaction might be a better descriptor for $S$. I use "status" because of its long association with conspicuous consumption.

[5]Experimental studies (Engelmann and Strobel, 2004; Iriberri and Rey-Biel, 2013) find a range of social or distributional preferences to be present in subject populations. Drechsel-Grau and Schmid (2014) find that, in an empirical study on consumption behavior, comparisons are only upward looking. Experimental research also explores dynamic aspects, such as reciprocity, which in turn can be modelled using psychological game theory (see, for example, Battigalli and Dufwenberg, 2020). Here the model is static and so it abstracts away from such issues.

plausibly negative comparisons follow from seeing the consumption of the rich, not just from them having income.

A special case is when, under rivalrous preferences, one sets $\beta = -\alpha < 0$. Note that

$$A(x; x_{-i}) - D(x; x_{-i}) = \int_0^\infty (x - t)\ dF(t) = x - \int_0^\infty t\ dF(t) = x - \mu_X, \tag{6}$$

where $\mu_X$ is average expenditure on $x$. Further, $a(z; x_{-i}) + d(z; x_{-i}) = \mu_X$. Thus, when $\beta = -\alpha < 0$, status (5) becomes

$$S = \alpha(x - \mu_X). \tag{7}$$

This kind of relative concern that depends on the average consumption of others is known as "Keeping Up With the Joneses" (KUJ) preferences (Galí, 1994). The ERC model of relative concerns due to Bolton and Ockenfels (2000) is also based on the average of others.

However, individuals have preferences over more than just status. Specifically, take utility to be $U(x, y, S)$, utility is increasing in visible consumption $x$, non-visible consumption $y$ and status, $S$. Note that the good $x$ is valued both in terms of its absolute consumption as well its contribution to status $S$. For example, a car is useful for transport as well as possibly giving prestige. Further, I assume a series of conditions on the utility function, similar to those in Hopkins and Kornienko (2010), that will enable the derivation of a monotone equilibrium and clear welfare results. (i) $U$ is twice continuously differentiable (smoothness); (ii) $U_x(x, y, S) > 0$, $U_y(x, y, S) > 0$, $U_S(x, y, S) > 0$ (monotonicity); (iii) $U_{xy}(x, y, S)$, $U_{ys}(x, y, S) \geq 0$ (complementarity); (iv) $U_{ii}(x, y, S) \leq 0$ for $i = x, y, S$ (own concavity); (v) $U_{xy} - U_{yy} > 0$ (strict normality); (vi) $U_x(x, z - x, S) - U_y(x, z - x, S) = 0$ has a unique solution $x = \gamma(z, S) \in (0, z)$ and whenever $x > \gamma(z, S)$ it holds that $U_{xS}(x, z - x, S) - U_{yS}(x, z - x, S) < 0$. Finally, define the "privately optimal" consumption as $\hat{x}(z) = \gamma(z, 0)$, the level of consumption an individual would chose in the absence of status, for example when $\alpha = \beta = 0$. Condition (v) ensures that $\gamma$ is strictly increasing in income $z$, so that good $x$ is strictly normal in the absence of relative concerns. The last condition (vi) seems somewhat complicated but it is automatically satisfied if utility is multiplicatively separable in $S$. It ensures (see Lemma 1 below) that optimal conspicuous consumption is increasing in the consumption of richer others.

The first result is a characterization of how individuals respond to changes in others' consumption. Without any assumptions on the strategies of others, one can differentiate $U(x, y, S)$ with respect to own consumption $x$ to obtain,

$$U_x(x, z - x, S) - U_y(x, z - x, S) + (\alpha(1 - F(x)) - \beta F(x))U_S(x, z - x, S) = 0. \tag{8}$$

If further $F(x)$ is differentiable (this would be the case, for example, if consumption is strictly increasing in income), then the first order condition implicitly defines a reaction function

$$x(z; x_{-i}) = R(z; F(x); \alpha d(x_{-i}) - \beta a(x_{-i})). \tag{9}$$

I show that, under rivalrous preferences, others' consumption is a strategic complement, but under inequity aversion, changes in consumption by the rich and the poor generate different reactions.[6] Under inequity aversion, only expenditure by the rich is a complement, while expenditure by the poor is a substitute.[7] In contrast, in the well-known model of Fehr and Schmidt (1999) utility is linear and separable in $D$ and $A$ and hence in $S$. In this case, in contrast to the above result, changes in others' consumption would not change consumption choices - see Section 2.2 below.

**Lemma 1.** *Let $F(x)$ be differentiable at own consumption $x_i$. If $\alpha > 0 > \beta$ (rivalrous preferences), then others' consumption is a **strategic complement**, $\partial R/\partial d > 0$ and $\partial R/\partial a > 0$, own consumption $x_i$ is increasing in the consumption of richer others and poorer others. If $\alpha > \beta > 0$ (inequity aversion), then, for an individual whose consumption satisfies $F(x_i) < \alpha/(\alpha+\beta)$, $\partial R/\partial d > 0$, own consumption $x_i$ increases in richer others (**strategic complement**), but $\partial R/\partial a < 0$, own consumption $x_i$ decreases in the consumption of poorer others (**strategic substitute**).*

I now look for an equilibrium in which all individuals use a strictly monotone equilibrium strategy $x(z) : [\underline{z}, \bar{z}] \to [0, \bar{z}]$ so that conspicuous consumption is strictly increasing in income $z$. When $x(z)$ is strictly increasing, it holds that $G(z) = F(x(z))$. That is, an individual consuming $x(z)$ has the same rank in the distribution of consumption as in the distribution of income. Later, Proposition 1 shows such an equilibrium exists and there is only one such strictly monotone equilibrium. Other, not strictly monotone, equilibria are considered later in Section 2.3. The reasons for initially concentrating on monotone equilibria are simplicity and plausibility. As we will see the other equilibria involve some degree of pooling - people with different incomes choosing the same consumption. This would be in conflict with the standard empirical finding that, on average, consumption is strictly increasing in income.

If visible consumption $x(z)$ is strictly increasing then there is a one-to-one relation between income and consumption. Let $d(z)$ and $a(z)$ be the expenditure of those richer and poorer respectively than the income $z$,

$$d(z) \equiv \int_{z}^{\bar{z}} x(t)g(t)\, dt; \;\; a(z) \equiv \int_{\underline{z}}^{z} x(t)g(t)\, dt. \tag{10}$$

This notation emphasizes that the solution is different from the earlier $d(x_{-i})$ and $a(x_{-i})$ where others' expenditure is arbitrary and not necessarily ordered by income. Similarly, let

$$S(x; z) = x(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z), \tag{11}$$

and $S_x(x; z) = \alpha(1 - G(z)) - \beta G(z)$. Thus, in the inequity averse case, $S$ reaches an interior maximum at $z^* = G^{-1}(\alpha/(\alpha + \beta))$. In contrast, when $\beta \leq 0$, $S$ is always increasing (see Figure 1). Utility becomes,

$$U(x, y, S) = U\left(x, z - x, x(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z)\right), \tag{12}$$

[6] Inequity aversion also induces technical complications such that it is not possible to sign exactly the competitive responses of high ranked agents.

[7] Clark and Oswald (1998), Barnett et al. (2019) alternatively derive differing strategic responses from differences in concavity or convexity of preferences.

using the budget constraint $y = z - x$.

One now can differentiate the utility function (12) to obtain the following first order condition,

$$U_x(x, z - x, S(x; z)) - U_y(x, z - x, S(x; z)) + S_x(x; z)U_S(x, z - x, S(x; z)) = 0. \quad (13)$$

The first term is the intrinsic marginal return to visible consumption $x$, the second is the marginal return to other consumption $y$ and the third represents an additional marginal return to consumption from relative concerns. When $S_x$ is positive (negative), this additional wedge is positive (negative) and so conspicuous consumption $x$ is greater (smaller) than in the absence of relative concerns.

Next, from (10), one can derive the following system of differential equations and boundary conditions,

$$d'(z) = -x(z)g(z), \;\; a'(z) = x(z)g(z); d(\bar{z}) = 0, \;\; a(\underline{z}) = 0. \quad (14)$$

where $x(z)$ solves the first order condition (13). The equations (13) and (14) together form a differential-algebraic system, the solution to which is the equilibrium of the game.

The first main result shows the above defines a strictly monotone equilibrium and derives some of its qualitative properties. In particular, while there is only one monotone equilibrium, it is the unique equilibrium only in the KUJ case ($\beta = -\alpha < 0$). In other cases, there can exist partial pooling weakly monotone equilibria that I detail later in Section 2.3. Importantly, the equilibrium individual consumption depends not only on own income $z$ but also on the distribution of income $G(z)$, both because of its direct presence in the first order condition (13) and because it affects the consumption of others which determines $d(z)$ and $a(z)$. Nonetheless, given an explicit utility function and income distribution $G(z)$, one can solve for closed form solutions to (13) and (14) and hence derive an explicit result for $x(z)$ (see, for example, Section 2.1).

**Proposition 1.** *For $\alpha$ and $\beta$ sufficiently small, there exists a unique symmetric strictly monotone equilibrium $x(z)$ that solves (13) and (14). It is the unique equilibrium in the KUJ case ($\beta = -\alpha < 0$). Equilibrium status $S(x(z); z)$ is strictly increasing on $(\underline{z}, \bar{z})$ if $\beta \leq 0$, but for $\alpha \geq \beta > 0$ it is increasing on $(\underline{z}, z^*)$ and decreasing on $(z^*, \bar{z})$, where $z^* = G^{-1}(\alpha/(\alpha + \beta))$. Further, comparing equilibrium consumption $x(z)$ to privately optimal consumption $\hat{x}(z)$, (i) if $\alpha > \beta > 0$ then there is a $\tilde{z} \in [z^*, \bar{z})$ such that $x(z) > \hat{x}(z)$ on $[\underline{z}, \tilde{z})$ with $x(z) < \hat{x}(z)$ on $(\tilde{z}, \bar{z}]$; (ii) if $\alpha > \beta = 0$ then $x(z) > \hat{x}(z)$ on $[\underline{z}, \bar{z})$ with equality at $\bar{z}$; (iii) if $\alpha > 0 > \beta$ then $x(z) > \hat{x}(z)$ on $[\underline{z}, \bar{z}]$.*

The restriction on $\alpha$ and $\beta$ is necessary to ensure a monotone equilibrium for the following fundamental reason: from (11) one can calculate that $S_{xz} = -(\alpha + \beta)g(z)$ and thus is strictly negative. Given utility is a function of status $S$, the standard single crossing condition between action $x$ and type $z$ can fail to hold for $\alpha$ and $\beta$ sufficiently large. Intuitively, an increase in income could, in the inequity averse case, increase guilt so much that an individual would spend less not more. These issues are not present in the rivalrous case where $\beta$ is negative so that $\alpha + \beta$ is close or equal to zero.

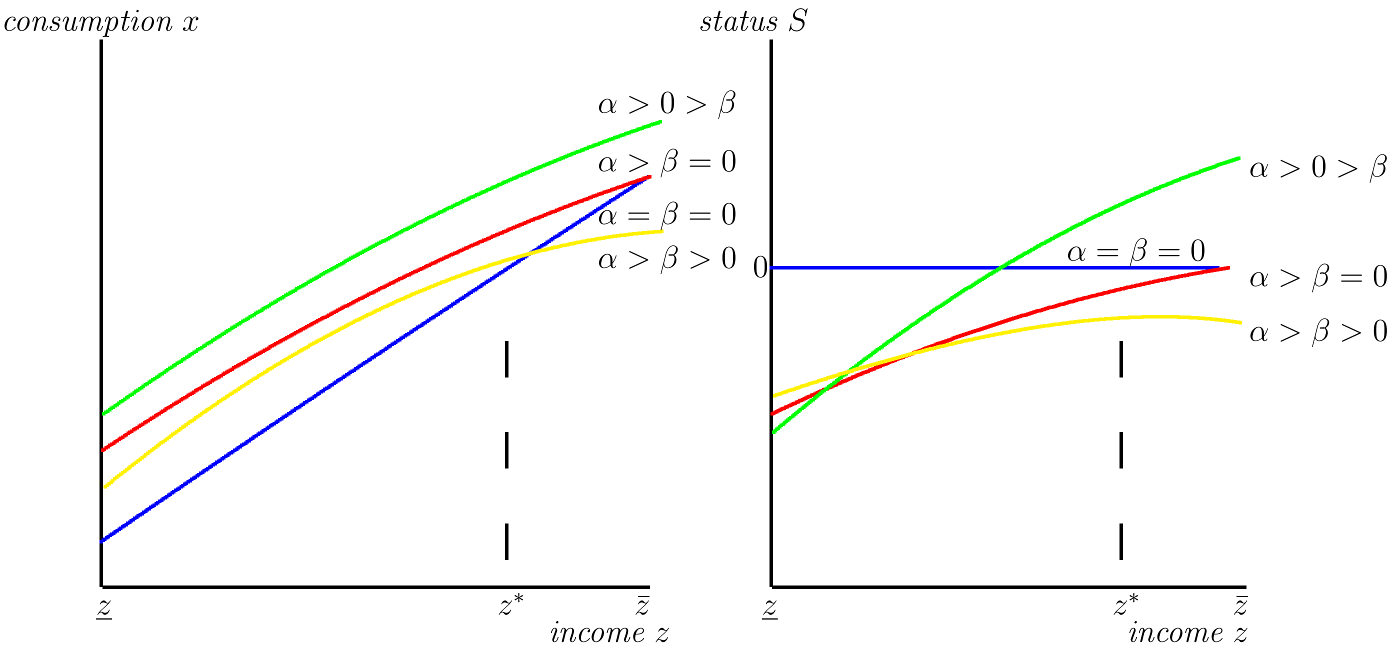


Figure 1: Equilibrium consumption choices (left) and equilibrium status (right) for different parameter values. $\alpha > \beta > 0$ inequity aversion, $\alpha = \beta = 0$ baseline neoclassical or "privately optimal", $\alpha > \beta = 0$ only upward comparisons, $\alpha > 0 > \beta$ rivalrous preferences.

The qualitative nature of equilibrium is illustrated in Figure 1. Under inequity aversion so that $\alpha > \beta > 0$, equilibrium status $S(x(z); z)$ is increasing for low income levels but achieves a maximum at $z^*$ ($z^* = G^{-1}(\alpha/(\alpha+\beta))$), as guilt becomes the dominant factor. The resulting equilibrium expenditure exceeds the level without relative concerns but at incomes above $z^*$ guilt leads consumption below the privately optimal level. With rivalrous preferences $\alpha > 0 > \beta$, status is always increasing and so equilibrium consumption is always greater than without relative concerns.

## 2.1 Log Preferences

This section introduces a particular form of log utility that leads to a closed form solution that is useful for some applications. Indeed, this functional form is similar to that used in some of the applied literature on relative consumption effects (Drechsel-Grau and Schmid, 2014; Alvarez-Cuadrado et al., 2016; Bellet and Colson-Sihra, 2018).

Assume

$$U(x, y, S) = \ln[x + S] + \ln[y]. \tag{15}$$

Note that this specific function is strictly monotonic and also satisfies $U_{xx} < 0, U_{yy} < 0, U_{SS} < 0$, $U_{xy} = 0$ and $U_{xS} - U_{yS} < 0$ and therefore fits the earlier general framework (despite the fact that $U_{xS} < 0$ which is otherwise somewhat unusual in status models). For example, the reaction function can be explicitly derived and is strictly increasing in $d$. Thus, own consumption is always increasing in richer others' consumption (strategic complements) and increasing (decreasing) in poorer others' consumption if $\beta < 0$ ($\beta >$

0). One has

$$x(z) = R(z; G(z); \alpha d(z) - \beta a(z)) = \frac{z}{2} + \frac{\alpha d(z) - \beta a(z)}{2(1 + \alpha(1 - G(z)) - \beta G(z))}. \quad (16)$$

This reaction function can be combined with the differential equation system (14) to solve for an explicit solution. A special case of this ($\alpha = \beta = 0$) is the privately optimal consumption $\hat{x}(z) = z/2$. Equilibrium utility will be

$$U(z) = \ln[1 + \alpha(1 - G(z)) - \beta G(z)] + 2\ln[\frac{z}{2} + \frac{\beta a(z) - \alpha d(z)}{2(1 + \alpha(1 - G(z)) - \beta G(z))}], \quad (17)$$

where $d(z)$ and $a(z)$ are the solutions to the equation system (14).

The "Keeping Up With the Joneses" (KUJ) case is particularly tractable, with a complete explicit solution obtainable. Combining (7) with the reaction function (16) gives,

$$x(z; \mu_X) = \frac{z}{2} + \frac{\alpha \mu_X}{2(1 + \alpha)}, \quad (18)$$

where $\mu_X$ is mean expenditure on $x$. Integrating this with respect to the income distribution $G(z)$ results in

$$\mu_X = \frac{\mu}{2} + \frac{\alpha \mu_X}{2(1 + \alpha)} \Rightarrow \mu_X = \frac{(\alpha + 1)\mu}{\alpha + 2},$$

where $\mu$ is mean income. Thus, the Nash equilibrium strategy is

$$x(z) = \frac{z}{2} + \frac{\alpha \mu}{4 + 2\alpha}, \quad (19)$$

(in this special case, the shape of the income distribution does not matter). It is thus easy to see that consumption is increasing in the average income of others and always higher than in the absence of social preferences. Equilibrium utility is

$$U(z) = \ln[1 + \alpha] + 2\ln\left[\frac{z}{2} - \frac{\alpha \mu}{2(1 + 2\alpha)}\right]. \quad (20)$$

It is clear that utility is decreasing in average income $\mu$, so that any individual will be worse off if the incomes of others increase.

## 2.2 Additively Separable Preferences

As noted, in the well-known model of Fehr and Schmidt (1999) utility is linear and separable in $D$ and $A$ and hence in $S$. In the current notation, one could write,

$$U(x, y, S) = v(x, y) + S(x, x_{-i}) \quad (21)$$

where $v(\cdot)$ is some standard utility function, so that overall utility $U$ is additively separable and linear in status. The first order condition will be similar to before and consumption will be monotone in income under similar conditions to those identified in Proposition 1 - that is, $\alpha$ and $\beta$ should not be too large.

Note that this specification does not satisfy assumption (vi) and further $U$ is not strictly concave in $S$. It can be shown that consequently Lemma 1 does not hold. The choice of conspicuous consumption by any individual is **not** affected by the consumption decisions of others. That is, the reaction function (9) will now be a function of income $z$ and rank $F(x)$ alone.

For concreteness, suppose that $v(x, y) = xy$, then one can solve for equilibrium consumption explicitly,

$$x(z) = \frac{z}{2} + \frac{\alpha(1 - G(z)) - \beta G(z)}{2} = \hat{x}(z) + \frac{S_x(x; z)}{2}. \tag{22}$$

That is, consumption is still distorted from the privately optimal level $\hat{x}$ by relative comparisons, in a qualitatively similar way to that illustrated in Figure 1. However, this distortion is now non-strategic and is no longer a function of others' consumption. Thus, the comparative statics results of Section 4 will not apply.

However, as observed by Dufwenberg et al. (2014), equilibria will still not be efficient because of the negative social externalities. One can easily check that the results of Section 3 on socially optimal consumption still apply to the additive case of this subsection.

## 2.3 Partial Pooling Equilibria

Up to now, the focus has been on strictly monotone equilibrium. However, in general there are other equilibria. These all involve some degree of pooling in which individuals with different incomes choose the same consumption but this may be combined with a strictly monotone consumption at other income levels. The intuition behind this is that under inequity aversion, there is a tendency to avoid consumption differences and choose similar levels of consumption. But working against this, individuals have different levels of income and, absent status concerns, would choose different consumption levels. Thus, pooling must be over small income intervals. Further, one can show that if $\beta$ is not too large, then all equilibria will be weakly monotone (so in the rivalrous case ($\beta < 0$), equilibria are necessarily weakly monotone), so all equilibria are broadly qualitatively similar.

More technically, pooling can be incentive compatible because it creates mass points in the distribution $F(x)$ of consumption (if a mass of agents choose the same consumption level $\tilde{x}$, then $F(x)$ is discontinuous at $\tilde{x}$, with $F_-(\tilde{x}) = \lim_{x \uparrow \tilde{x}} F(x) < F(\tilde{x})$). While payoffs are continuous in $x$ at such mass points, the status function is not differentiable there, with the right derivative lower than the left (except in the KUJ case). Thus marginal utility with respect to consumption can be strictly positive to the left and strictly negative to the right of the pooling level, so that there is no incentive to deviate

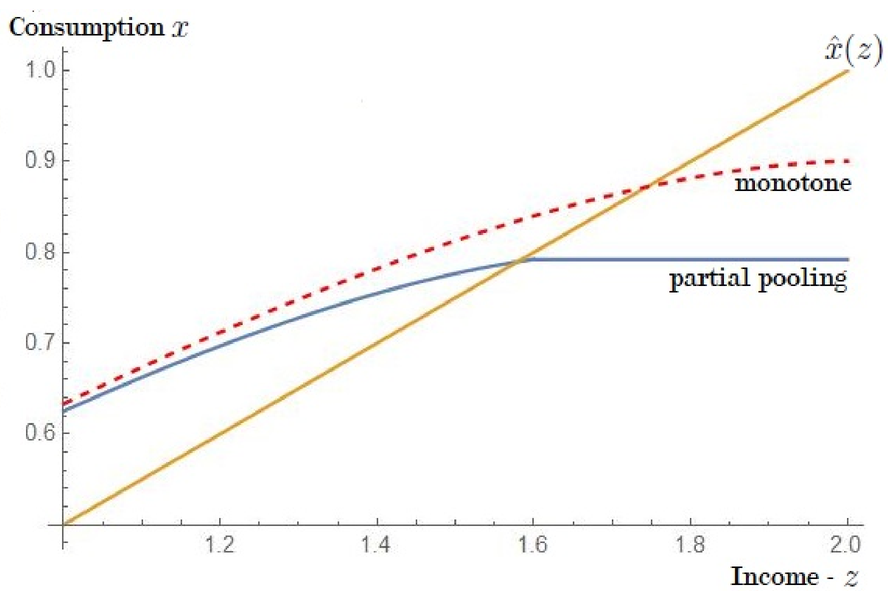


Figure 2: Strictly monotone (dashed; $\alpha = 0.5, \beta = 0.2$) and partial pooling (solid; $\alpha = 0.5, \beta = 0.4$) equilibria plotted against privately optimal consumption $\hat{x}(z)$.

either up or downwards.[8]

The following result characterizes equilibria that are not strictly monotone. They are qualitatively similar to strictly monotone equilibria in that the consumption function $x(z)$ is always continuous and at least weakly increasing. Further, it is similar in its relation to the privately optimal level of consumption. For example, the below result implies that in the rivalrous case ($\beta \leq 0$), equilibrium consumption always exceeds the privately optimal level.

**Proposition 2.** *For $\beta$ sufficiently small, all equilibrium functions $x(z)$ are weakly monotone and continuous. If $x < F_{-}^{-1}(\alpha/(\alpha+\beta))$, then $x(z) \geq \hat{x}(z)$, equilibrium consumption is above the privately optimal level. If $x > F^{-1}(\alpha/(\alpha+\beta))$, then $x(z) \leq \hat{x}(z)$, equilibrium consumption is below the privately optimal level.*

The amount of pooling supportable in such equilibria is increasing in $\alpha$ and in $\beta$. Simply put, a high $\alpha$ deters the lowest income types in a pooling group from deviating downwards because they then would suffer deprivation relative to the group of other agents. Similarly, a high $\beta$ deters upward deviations from high income types because they would suffer from the advantage relative to the group

More generally, a larger proportion of agents can pool where the income distribution is relatively compact so that there are not large differences in absolute income amongst the pooling types. But this shows the limitation of such equilibria. In real populations which have significant income differences between rich and poor, the values of $\alpha$ and $\beta$ would have to be implausibly large to compel all to choose the same level

[8]This observation that asymmetric relative concerns can induce non-differentiability and hence multiple equilibria was first made by Bhaskar (1990).

of consumption. And if instead pooling is limited and local and the equilibrium has strictly monotone components, then such equilibria are qualitatively similar to a strictly monotone equilibrium.

The existence of partial pooling equilibria also partially answers the question about what form do equilibria take when $\alpha$ and $\beta$ are too large for a strictly monotone equilibrium to exist. Instead, a weakly monotone equilibrium is possible, with pooling at high incomes.

For example, using the logarithmic preferences of Section 2.1, it is possible to construct some numerical examples. Suppose incomes are uniformly distributed on [1,2]. Then, first, assume $\alpha = 0.5$ and $\beta = 0.2$. There is a strictly monotone equilibrium, but one can see in Figure 2 that at high incomes the equilibrium consumption function $x(z)$ is almost flat as the rich, facing guilt as the parameter $\beta$ is strictly positive, moderate their consumption.

Second, assume now that $\alpha = 0.5$ and $\beta = 0.4$. For these values of the social preference parameters guilt is too strong for there to be a strictly monotone equilibrium. However, there is still a partial pooling equilibrium with pooling at the top - in which all agents with incomes on [1.6, 2] choose $x = 0.792$, while consumption is strictly monotone on [1, 1.6). This is also illustrated in Figure 2. The privately optimal consumption function $\hat{x}(z)$ is also illustrated for comparison. The strictly monotone and partial pooling equilibria are qualitatively similar.

## 2.4 Heterogeneity in Relative Concerns

One consistent finding of the experimental literature that attempts to measure social preferences is that there is diversity across subjects. For example, Iriberri and Rey-Biel (2013) find that the majority of subjects have some form of social preferences, self-interest (without apparent social preferences) is the biggest single group, with inequity aversion second. Thus, an important question (which has not previously been addressed in the status literature) is what happens when there are heterogeneous preferences?

Since the experimental literature finds that the plurality of subjects have no social preferences, in this section I analyze outcomes where there is a mix of status-conscious and status-neutral individuals. Let $\theta$ of the population have status concerns and $1-\theta$ have no status concerns so that they have neoclassical preferences (equivalently for them $\alpha = \beta = 0$). Let both types have the same income distribution $G(z)$. Let the status-concerned individuals use strategy $x(z;\theta)$ and the status-neutral individuals use strategy $\hat{x}(z)$, which as introduced earlier represents optimal consumption in the absence of status concerns. That is, for the status-concerned their own frequency $\theta$ will affect their consumption choice through relative consumption externalities. However, for the status-neutral choosing consumption is a single-person decision problem and so neither the population composition nor the consumption of others affects their choice.

Then a status-concerned individual consuming $x$ will have rank,

$$F(x) = \theta G(x^{-1}(x)) + (1-\theta)G(\hat{x}^{-1}(x)) = \theta G(z) + (1-\theta)G(\hat{x}^{-1}(x)). \tag{23}$$

One can then derive the first order conditions for the status-conscious strategy in the same way as before. However, one now has,

$$S_x(x;z) = \alpha(\theta(1-G(z)) + (1-\theta)(1-G(\hat{x}^{-1}(x)))) - \beta(\theta G(z) + (1-\theta)G(\hat{x}^{-1}(x))). \quad (24)$$

This implies that $S_x$ is generally smaller as $G(\hat{x}^{-1}(x)) > G(z)$ because $x(z) > \hat{x}(z)$. Thus, conspicuous consumption is increasing in the proportion of the status-conscious $\theta$.[9] The first order condition implicitly defines $R(z; G(z;\theta); \alpha d(z;\theta) - \beta a(z;\theta))$, where

$$d(z;\theta) = \theta \int_{\bar{z}}^{z} x(t)g(t)\,dt + (1-\theta)\int_{\bar{z}}^{z} \hat{x}(t)g(t)\,dt. \quad (25)$$

The derivation of $a(z;\theta)$ is similar. One has

$$d'(z;\theta) = -\theta x(z)g(z) - (1-\theta)\hat{x}(z)g(z);\; a'(z;\theta) = -d'(z;\theta);\; d(\bar{z};\theta) = 0,\; a(\underline{z};\theta) = 0. \quad (26)$$

The main points are that, even under heterogeneous preferences, it is possible to calculate the equilibrium and that it is qualitatively similar to the earlier result when all are status-conscious. This is true even when the proportion of status-concerned individuals is small. Note that as $\theta \downarrow 0$, $x(z;\theta)$ approaches $R(z; G(z,0), \alpha\hat{d} - \beta\hat{a})$, where $\hat{d}(z) = \int_z^{\bar{z}} \hat{x}(t)g(t)dt$, and does not approach $\hat{x}(z)$. For example, for the log preferences with $\beta = 0$,

$$x(z,0) = \frac{z}{2} + \frac{\alpha\hat{d}(z)}{2(1+\alpha(1-G(z)))}, \quad (27)$$

where $\hat{d} = \int_z^{\bar{z}} \frac{t}{2}g(t)dt = d(z;0)$. So, status-conscious individuals will still spend more on consumption than is privately optimal, even when a vanishingly small proportion of the population.

**Proposition 3.** *The monotone equilibrium in the heterogenous case is a pair* $(x(z;\theta), \hat{x}(z))$ *where* $x(z;\theta)$ *solves the system (13), (24) and (26), and* $\hat{x}(z)$ *solves* $U_x(x, z-x, 0) - U_y(x, z-x, 0) = 0$. *In the rivalrous case* $\alpha > 0 \geq \beta$, $x(z;\theta)$ *is increasing in* $\theta$, *and* $x(z;0) > \hat{x}(z)$ *everywhere on* $[\underline{z}, \bar{z})$.

That is, the consumption by the status-conscious in the heterogeneous case is qualitatively similar to the case where all are status-conscious. However, clearly aggregate or average consumption is increasing in the proportion $\theta$ of status-conscious.

# 3 Welfare

The striking result of Frank (1985) is that, with ordinal relative concerns, the Nash equilibrium level of conspicuous consumption is Pareto inefficient. If all individuals

[9]This also suggests that this is another example where cardinal preferences are very different from ordinal. In the ordinal case, those who are concerned with status should have a greater incentive to consume when they are in a minority as it is easier to rise in rank when others, the status-neutral, spend less.

simultaneously reduced their consumption in a way that maintained their relative position, everyone would have the same status but higher utility because consumption decisions would be less distorted. The cardinal case is necessarily much more complicated as utility depends on the exact differences in consumption and not just relative position. Thus, while in the ordinal case, the optimal consumption schedule is simply what is privately optimal (the consumption chosen in the absence of relative concerns), here there is no such simple formula and privately optimal consumption $\hat{x}(z)$ is not socially optimal.

Nonetheless, one can show a simple result for rivalrous preferences, where others' consumption is always a negative. In this case, everyone can be made better off if everyone reduces consumption. However, the results are quite different under inequity aversion. I show that under the consumption schedule that maximizes utilitarian welfare either (more likely) the rich or (less likely) the poor are worse off than in Nash equilibrium. The case where $\alpha > \beta = 0$ is particularly clear. Since people only look upwards, the very rich have a negative externality on others, but suffer little from externalities themselves, so that their consumption choice is not much distorted. A planner would want them to reduce their consumption to reduce the externality on others, but this must make them worse off as they were already at their optimum.

The utility or payoff in a strictly monotone equilibrium will be

$$U(z) = U(x(z), z - x(z), S(z)), \tag{28}$$

where $x(z)$ is the equilibrium function that solves (13) and (14), and $S(z) = x(z)(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z)$, where in turn $d(z)$ and $a(z)$ are the solutions to the equation system (14). Utilitarian welfare is

$$W = \int_{\underline{z}}^{\bar{z}} U(z)\ dG(z). \tag{29}$$

I consider a limited social planner problem in that there is no redistribution.[10] Instead, the question is, keeping the distribution of incomes unchanged, what consumption schedule $x(z)$ would maximize welfare? I show in Lemma 2 below that the relevant first order condition is the following, using the abbreviation $U_x(z)$ for $U_x(x(z), z - x(z), S(z))$ and so on,

$$U_x(z) - U_y(z) + U_S(z)S_x(z) - \alpha k(z) + \beta m(z) = 0, \tag{30}$$

for every $z \in [\underline{z}, \bar{z}]$, where

$$k(z) \equiv \int_{\underline{z}}^{z} U_S(t)g(t)\ dt;\ m(z) \equiv \int_{z}^{\bar{z}} U_S(t)g(t)\ dt. \tag{31}$$

The first order condition (30) is like the Nash first order condition (8) plus two additional terms. The first, $-\alpha k(z)$, reduces consumption to take into account the negative externality of conspicuous consumption by an individual with income $z$ on those with

[10] If redistribution were allowed, one would obtain the well-known result that the utilitarian optimum is complete equality.

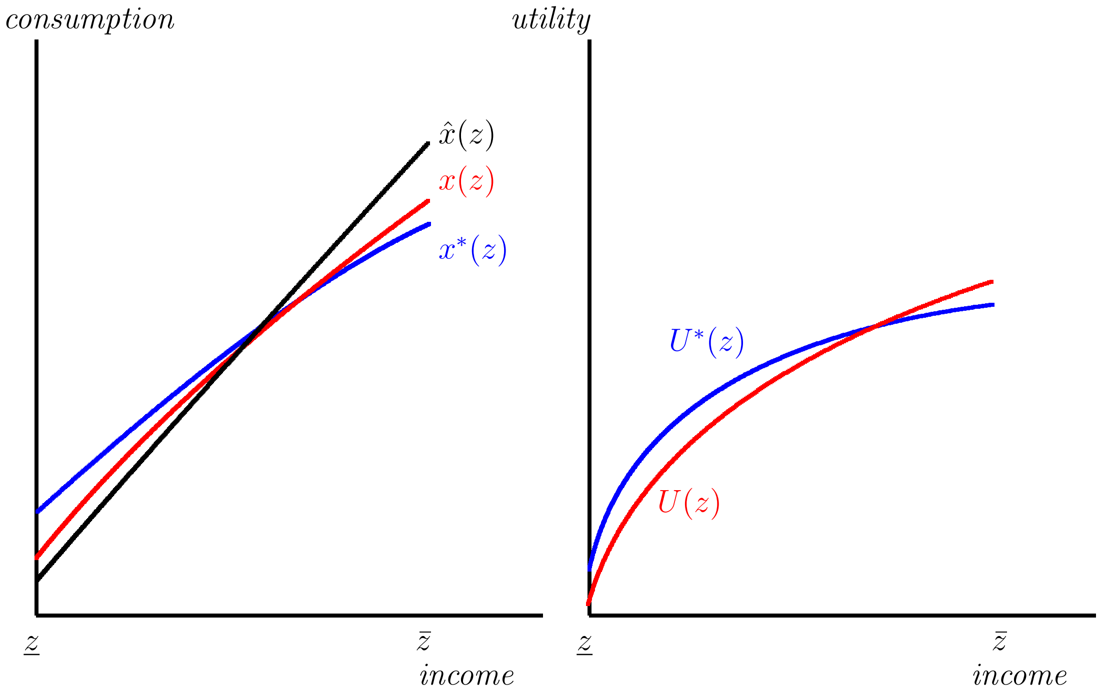


Figure 3: Illustration of Proposition 4: under inequity aversion $\alpha \geq \beta > 0$, the utilitarian optimal consumption schedule $x^*(z)$ is flatter than the equilibrium schedule $x(z)$ and differs further from the privately optimal consumption $\hat{x}(z)$. Optimal utility $U^*(z)$ is higher than the Nash equilibrium utility $U(z)$ for most but not the rich.

incomes less than $z$ who envy $z$. The second, $\beta m(z)$, if $\beta > 0$, increases consumption at income $z$ to reduce the guilt felt by those richer than $z$. If $\beta < 0$, the rivalrous case, then it also will decrease consumption to reduce the negative externality from conspicuous consumption.

Then the utilitarian solution solves the system that combines these differential equations

$$k'(z) = U_S(z)g(z), \;\; m'(z) = -U_S(z)g(z); \;\; k(\underline{z}) = 0, \;\; m(\bar{z}) = 0. \tag{32}$$

with those in (14) and where $x(z)$ now solves the condition (30) rather than (13). To be clear, in general one has to solve simultaneously a system of four differential equations plus the nonlinear first order condition. That is, solving for the utilitarian optimum in the cardinal case is vastly more difficult than in the ordinal case, where it is simply equal to the privately optimal consumption schedule. In any case, denote the solution to the above as $x^*(z)$ and the utility obtained under this allocation as $U^*(z)$.

**Lemma 2.** *The solution to the system (14), (30) and (32), denoted $x^*(z)$, maximizes utilitarian welfare (29).*

Starting with the case where $\alpha > \beta = 0$, note that this first order condition implies that the poorest agent should consume the same amount as in the Nash equilibrium but that all other agents should consume less than the Nash amount. Hence the richest agent consumes less than her optimal amount and thus must be worse off than in equilibrium. In other words, the socially efficient consumption schedule is not a Pareto improvement on the Nash equilibrium outcome.

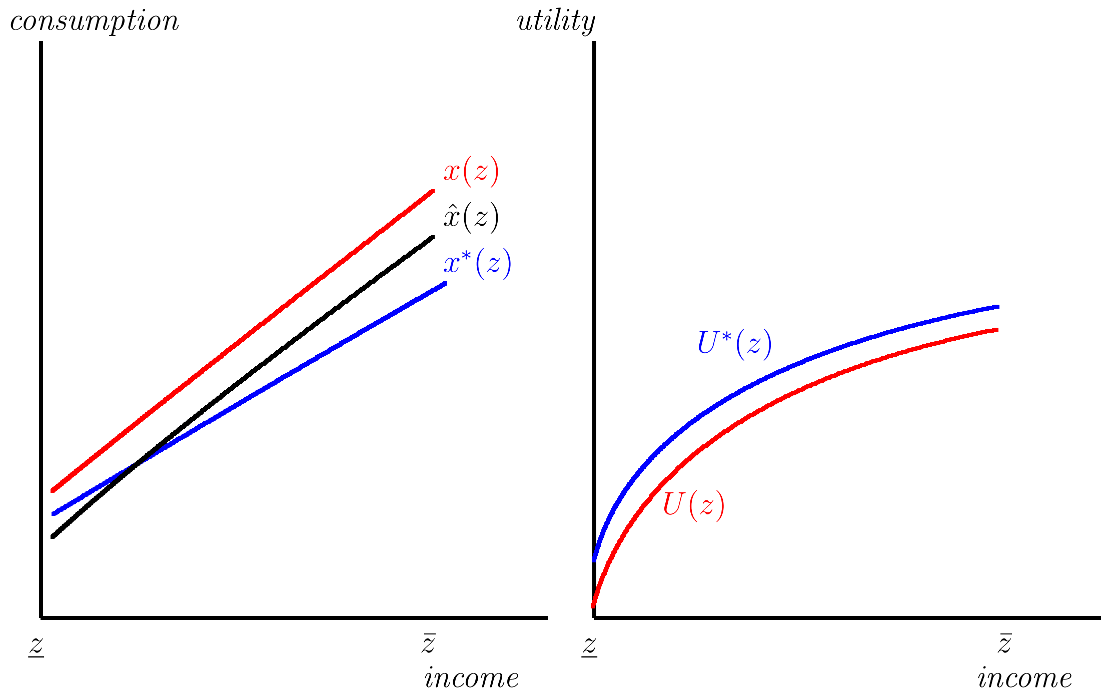


Figure 4: Illustration of Proposition 5: under rivalrous preferences $\alpha > 0 > \beta$, the utilitarian optimal consumption schedule $x^*(z)$ is everywhere lower than the equilibrium schedule $x(z)$ nonetheless it does not equal the privately optimal schedule $\hat{x}(z)$. Utilitarian utility $U^*(z)$ is higher than the Nash equilibrium utility $U(z)$ for all.

The same logic applies for $\alpha \geq \beta > 0$. The richest agent consumes less than the privately optimal amount in the Nash equilibrium but must consume still less to maximize welfare.

**Proposition 4.** *If $\alpha > \beta \geq 0$ then either $x^*(\bar{z}) < x(\bar{z})$ and consequently $U^*(\bar{z}) < U(\bar{z})$, the richest individual consumes less and is worse off under the utilitarian optimal consumption schedule than in the Nash equilibrium or $x^*(\underline{z}) > x(\underline{z})$ and $U^*(\underline{z}) < U(\underline{z})$, the poorest individual is worse off.*

Of course, given that average welfare is higher under the utilitarian scheme, there must be some tax and subsidy scheme that would make all better off. But if needed to balance its budget, it would need to subsidize the non-conspicuous consumption of the rich by taxing the non-conspicuous consumption of the poor.

Let us now turn to the case of rivalrous preferences $\alpha > 0 > \beta$. The analysis is much more like the ordinal case of Frank (1985) and Hopkins and Kornienko (2004). First, $x^*(z) < x(z)$, the socially optimal level of consumption is below the Nash equilibrium level at all income levels. Second, imposing the socially optimal consumption would result in a Pareto improvement. This is depicted in Figure 4. The explanation is that with rivalrous preferences everyone is engaged in an arms race in consumption and everyone would be better off if this consumption was lowered.

**Proposition 5.** *In the rivalrous case ($\alpha > 0 > \beta$), the utilitarian optimal level of conspicuous consumption is below the NE level at all income levels so that $x^*(z) < x(z)$ everywhere on $[\underline{z}, \bar{z}]$. Further, all agents are better off. That is, $U^*(z) > U(z)$ everywhere on $[\underline{z}, \bar{z}]$.*

However, what is different from the ordinal case is that the optimal consumption schedule is not simply privately optimal consumption. Rather it is the solution to the system of differential equations derived earlier in this section. Further, as depicted in Figure 4, the optimal schedule tends to be flatter than the privately optimal. To see this note that the utilitarian solution solves (30) while the privately optimal solves $U_x - U_y = 0$, so the two solutions are equal only when $U_S(z)S_x(z) = \alpha k(z) - \beta m(z)$. This will not be true in general. Further, given that $U_{SS} \leq 0$ and $\partial S_x/\partial z = -(\alpha+\beta)g(z) < 0$, the utilitarian schedule will be lower than the privately optimal at high incomes.

How could the utilitarian outcome be implemented? The usual suggestion is a Pigouvian tax on conspicuous consumption. This can be integrated into a Mirlees optimum tax framework. See Ireland (2001), Kanbur and Tuomala (2013) for some results in this direction but in signalling and non-strategic settings respectively. A full analysis of optimal tax in the presence of cardinal relative concerns is left for later research, but note the following. Looking at Proposition 4, in the inequity averse case the effect of the optimal tax would have to be redistributive, raising the consumption of the poor and reducing it for the rich. However, note the important difference from the standard optimal tax framework. There, optimality of redistribution follows from an exogenous social welfare function that places greater weight on the welfare of the poor. Here, reducing inequality in consumption follows purely from concern for efficiency.

# 4 The Effects of Greater Relative Deprivation and Greater Inequality

Suppose the distribution of income changes? How does this change behavior and welfare? Here, with relative concerns, equilibrium behavior depends on the incomes of others and so changes to others' resources can have direct effects. For reasons of tractability, in this section, I specialize to the log preferences introduced in Section 2.1.

In general, comparative static results are both more difficult and qualitatively different than under ordinal preferences or in signalling models. For example, Glazer and Konrad (1996) analyse the effects of greater inequality in a signalling model, results which were then applied to conspicuous consumption by Charles et al. (2009). In these signalling models, the equilibrium strategy does not depend on the distribution of income, but total expenditure can be affected by changes in the distribution due to changes in composition - total expenditure goes up if high spenders increase in relative frequency. The crucial difference in both the ordinal and cardinal games of status is that changes in inequality can have direct effects on behavior - the equilibrium strategy can itself change - as well as there being compositional effects. Further, one can observe, that as in the ordinal status model, signalling equilibria build from the bottom, so changes at the top of the income distribution have no effect on those below. Second, when the marginal propensity to consume is declining (which is the usual assumption), consumption is concave in income, the composition effect necessarily implies that greater inequality will decrease consumption expenditure.[11] Here, I find that both that changes in the income of the rich can affect behavior of the poor and that higher inequality can increase conspicuous consumption.

## 4.1 Greater Deprivation

In this section, I use the approach of Hopkins and Kornienko (2009) and analyze the effect of greater relative deprivation, comparing at constant rank rather than at constant income. For example, one can compare the choices and outcomes of the median individual before and after a change in the distribution of income. This is useful as it makes clearer who benefits from changes in the income distribution. In particular, I want to investigate what happens when the rich become richer. It is simply not possible to investigate the effect of increased income by making comparisons at a constant income.

First, let us rewrite some of the earlier results in terms of rank. Let $r = G(z)$, an agent's rank in the distribution of income. Let $Z(r) = G^{-1}(r)$ be the inverse distribution of income. Next, if $x(z)$ is the strictly monotone equilibrium strategy in terms of income derived in Section 2, then $x(r) = x(G^{-1}(r))$. Given this monotone relationship between $x(z)$ and $x(r)$, there will exist a monotone equilibrium in terms of rank if and only if there exists one in terms of income. Thus, as pointed out in Hopkins and Kornienko (2009), this is not a new model, it is just a different way of presenting the existing approach.

For what follows, it will be helpful to add that, given $r = G(z)$ one has $g(z)dz = dr$, so that

$$d(r) = \int_r^1 x(t)\ dt;\ a(r) = \int_0^r x(t)\ dt. \tag{33}$$

Status becomes $S(x; r) = x(\alpha(1-r) - \beta r) - \alpha d(r) + \beta a(r)$. Rewriting the reaction function for log preferences (16) in terms of rank, one obtains,

$$x(r) = \frac{Z(r)}{2} + \frac{\alpha d(r) - \beta a(r)}{2(1 + \alpha(1-r) - \beta r)}. \tag{34}$$

Now let us consider a change in the income distribution in which only the rich get richer. Under normal assumptions this of course would represent a Pareto improvement. Here, under the assumption of rvialrous preferences, while the very rich will become better off from the increase in income, even some of the gainers in income will lose in utility. Those whose incomes do not rise are all worse off. The reasons are twofold. First, the increase in income leads the very rich to increase their expenditure, leading to increased relative deprivation for others. Second, this leads everyone to increase conspicuous consumption. It is easy to see this from the reaction function (34) where

[11] Heffetz (2011) estimates Engel curves (that is, how consumption changes with income) for a number of goods. There is no particular indication that one should reject (weakly) decreasing marginal propensity to consume even for visible or luxury goods.

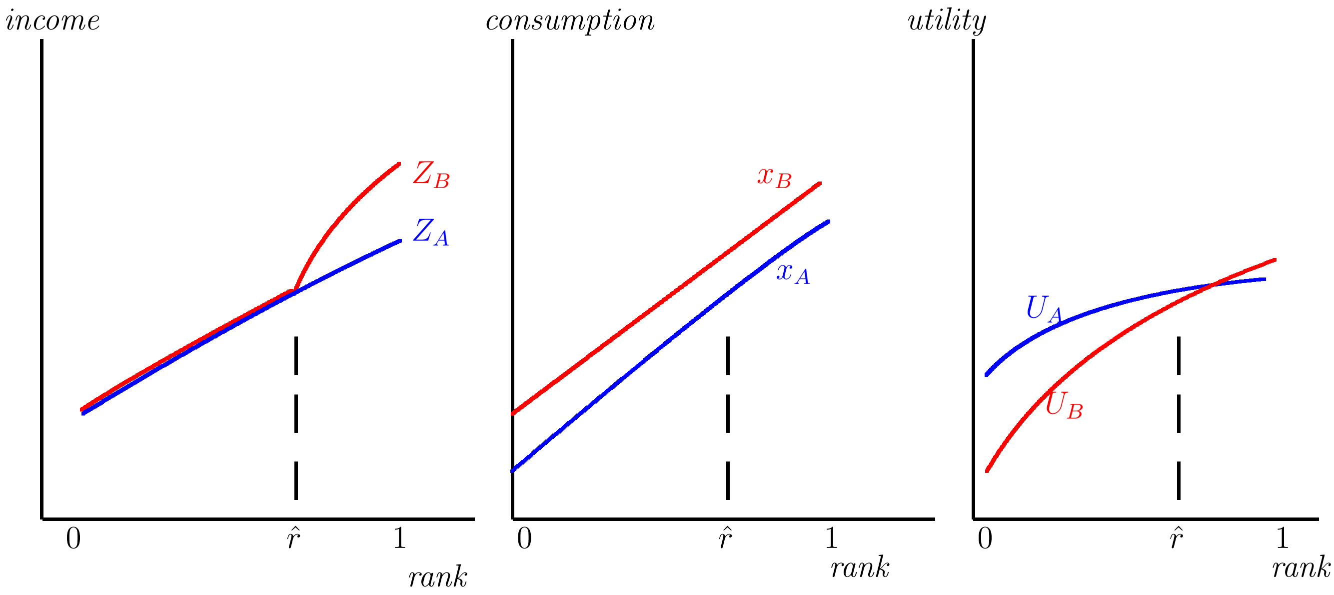


Figure 5: Illustration of Proposition 6: an increase in income only for the rich in inverse income distribution $Z_B$ (first panel), leads to higher conspicuous consumption at all income levels (second panel) and lower utility for most (third panel) evaluated at a constant rank in the income distribution.

the increase in consumption by those at the top increases relative deprivation and hence consumption for lower ranked individuals through the $d(r)$ term. The result is illustrated in Figure 5.

**Proposition 6.** *Suppose $\alpha > 0 \geq \beta \geq -\alpha$ and the rich become richer so that $Z_B(r) > Z_A(r)$ on $(\hat{r}, 1]$ for some $\hat{r} \in (0, 1)$ but $Z_A(r) = Z_B(r)$ on $[0, \hat{r}]$, with $Z'_B(r) > Z'_A(r)$ on $(\hat{r}, 1)$. Let $x_A(z)$ and $x_B(z)$ be the resulting monotone equilibrium strategies. Then consumption is higher, $x_B(r) > x_A(r)$, everywhere on $[0, 1]$ and utility is lower for all except the very rich, that is $U_A(r) > U_B(r)$ on $[0, \tilde{r}_+)$ for some $\tilde{r}_+ \in (\hat{r}, 1)$ but $U_A(r) < U_B(r)$ on $(\tilde{r}_+, 1]$.*

Note that the above result is impossible under the ordinal concerns model (Hopkins and Kornienko, 2004, 2009) or the signalling models following Ireland (2001). In these other models, the equilibrium strategy at a given income only depends on the distribution of income below that level. Thus changes at the top end of the income distribution will have no effect on utility or behavior of those below.

## 4.2 Inequality

In this section, I derive some further comparative statics on the effect of greater inequality. I again use the log utility introduced in Section 2.1. Assume income distributions $G_A(z)$ and $G_B(z)$ that have the same support $[\underline{z}, \bar{z}]$. Next one needs a formal definition of being more unequal. As introduced in Hopkins and Kornienko (2004), I use the Unimodal Likelihood Ratio order, defined below, which is a refinement of second order stochastic dominance. That is, if $G_A$ dominates $G_B$ in the ULR order, it is more equal than $G_B$ or stochastically higher than $G_B$. See for example the first panel of Figure 6.

**Definition (ULR):** Two distributions $G_A$, $G_B$ satisfy the Unimodal Likelihood Ratio (ULR) order and write $G_A \succ_{ULR} G_B$ if the ratio of their distribution functions $L(z) = g_A(z)/g_B(z)$ is unimodal and $\mu_A \geq \mu_B$. That is, $L$ is strictly increasing for $z < \hat{z}$ and it is strictly decreasing for $z > \hat{z}$ for some $\hat{z} \in (\underline{z}, \bar{z}]$.

The main result of this section finds that greater inequality lowers equilibrium utility under rivalrous preferences. I make this comparison at constant levels of income so that these changes are entirely driven by changes in the negative externalities driven by others' consumption. In particular, the increase of conspicuous consumption at the top end of society has a negative effect on others through an increase in their relative deprivation, and so equilibrium utility falls at most and possibly all income levels. The effect on conspicuous consumption is less clear. While consumption rises for the rich, the poor do not necessarily follow. See Figure 6 for an illustration. The welfare results are therefore not driven by wasteful emulation, rather by the direct psychological effect of higher relative deprivation.

**Proposition 7.** *Suppose $\alpha > \beta = 0$ and society $B$ is more unequal than $A$, $G_A \succ_{ULR} G_B$, but $A$ and $B$ have the same mean income $\mu_A = \mu_B$. Let $x_A(z)$ and $x_B(z)$ be the resulting monotone equilibrium strategies. Further, let $\hat{z}_-$ and $\hat{z}_+$ satisfy $\underline{z} \leq \hat{z}_- < \hat{z} < \hat{z}_+ < \bar{z}$.*
*(a) In the more unequal society $B$ the rich spend more on consumption than in the more equal society $A$, but the effect on the poor is ambiguous. That is, first $x_A(z) < x_B(z)$ on $(\hat{z}_+, \bar{z})$; second there can be a further crossing at $\tilde{z}$ in $(\hat{z}_-, \hat{z}_+)$ and if so also a final crossing in $(\underline{z}, \hat{z}_-)$.*
*(b) Utility is lower for most income levels in the more unequal society, so that for $\tilde{z} < \hat{z}$, it holds that $U_A(z) > U_B(z)$ on $[\tilde{z}, \bar{z})$ and possibly for all $z$ in $[\underline{z}, \bar{z})$.*

To give some further intuition for these results, from the first order condition (13), the marginal return to conspicuous consumption depends on $S_x$ which in turn depends on the income distribution. In particular, $S_{xz} = -(\alpha + \beta)g(z)$ so that the marginal return to conspicuous consumption will increase with greater inequality which lowers the density $g(z)$. Thus, greater inequality will tend to raise consumption, which then has spillover effects on others through relative deprivation. However, greater inequality also raises the density at low income levels, which is why consumption does not rise everywhere.

These comparative static results on the effect of greater inequality are almost the complete opposite of those obtained in Hopkins and Kornienko (2004). There, greater inequality reduced competition and increased utility. It is possible to generate similar results here, but under different assumptions on the form of cardinal preferences. In particular, if $\alpha = 0$ but $\beta < 0$, then individuals only make downward negative comparisons. Perhaps strangely, then the comparative static results are now almost identical to those obtained under ordinal status concerns.

**Proposition 8.** *Suppose $\alpha = 0 > \beta$ and society $B$ is more unequal than $A$, $G_A \succ_{ULR} G_B$, but $A$ and $B$ have the same mean income $\mu_A = \mu_B$. Assume $\underline{z} \leq \hat{z}_- < \hat{z} < \hat{z}_+ < \bar{z}$.*
*(a) In the more unequal society $B$ the poor spend more on consumption than in the*

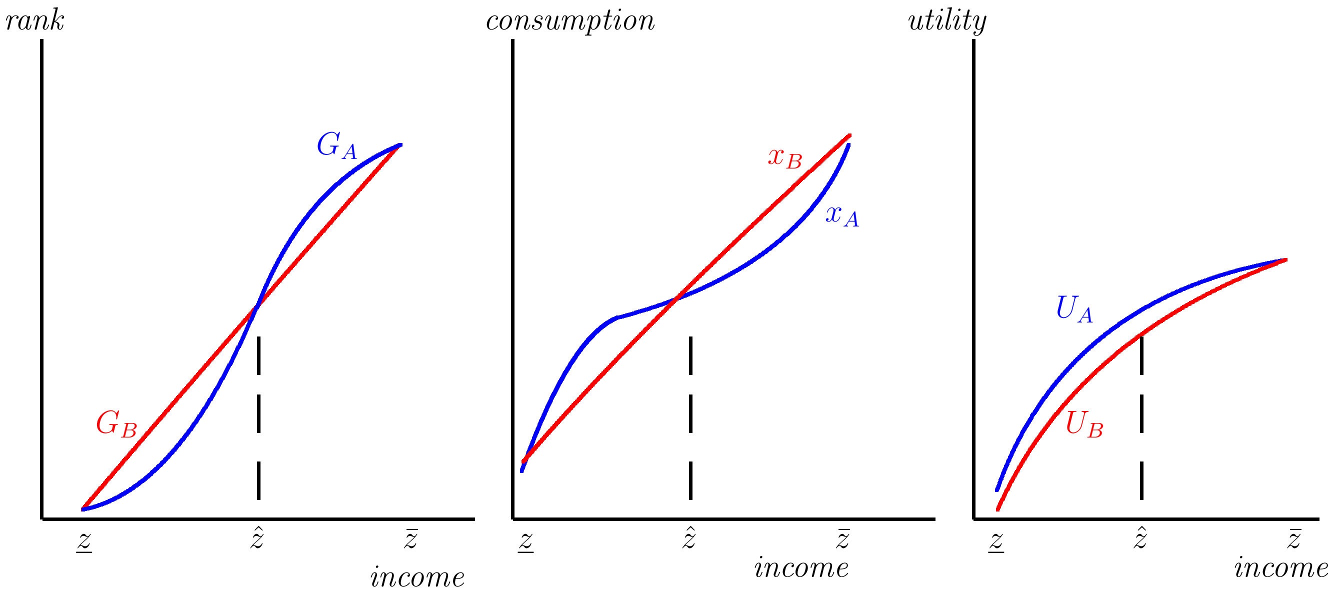


Figure 6: Illustration of Proposition 7: Greater inequality under income distribution $G_B$ (first panel), leads to higher conspicuous consumption for the rich and possibly all (second panel) and lower utility at almost all income levels (third panel).

*more equal society A. That is, $x_A(z) < x_B(z)$ on $(\underline{z}, \hat{z}_-)$.*
*(b) In the more unequal society B the poor have higher utility, $U_A(z) < U_B(z) \in (\underline{z}, \hat{z}_-)$.*

Let us turn to the KUJ formulation as given in (19). There inequality has no effect on behavior because in general the distribution of income has no effect, only its mean. I give no proof for the result below as it follows from simple observation of (19) and (20).

**Proposition 9.** *Suppose $-\alpha = \beta < 0$ (KUJ case) and society B is more unequal than A, $G_A \succ_{ULR} G_B$, but A and B have the same mean income $\mu_A = \mu_B$. Then, there is no difference between A and B in consumption, $x_A(z) = x_B(z)$, or utility, $U_A(z) = U_B(z)$.*

In contrast, suppose rather than a change in inequality, one considers an example of unequal growth in which the incomes of the rich increase at greater rate than the rest of society. Then conspicuous consumption rises and utility falls. Again, given that average income rises, the result follows from inspection of (19) and (20).

**Proposition 10.** *Suppose $-\alpha = \beta < 0$ (KUJ case) and the rich become richer so that $G_B(z) < G_A(z)$ on $(\hat{z}, \bar{z})$ for some $\hat{z} \in (\underline{z}, \bar{z})$ but $G_A(z) = G_B(z)$ on $[\underline{z}, \hat{z}]$, then average income is higher $\mu_B > \mu_A$ which implies that consumption is higher $x_B(z) > x_A(z)$ and utility is lower $U_B(z) < U_A(z)$ everywhere on $[\underline{z}, \bar{z}]$.*

These distributional comparative statics need to be interpreted with care. For example, this final result and Proposition 7 earlier do not in general imply that society $A$ Pareto dominates society $B$ or that its distribution of utility stochastically dominates that in $B$. The issue is that by changing the distribution some individuals have been made richer and therefore may be better off, even if utility falls at a constant level of income. Simply put, those who gain in income can be better off, but those who see no income gains are made worse off.[12] One can see these issues more clearly by comparison at a constant rank rather at a constant income, which is why I presented the rank-based result, Proposition 6, first.

# 5 Conclusions

This is the first detailed analysis of the interaction between conspicuous consumption and inequality under the assumption of cardinal preferences. We find that the results here depend heavily on which exact form of cardinal preferences are assumed, inequity averse or rivalrous. But in either case, the results are different from those obtained under ordinal preferences by Frank (1985), Hopkins and Kornienko (2004, 2009) or using signalling models such as those in Ireland (2001) or Charles et al. (2009).

One of the main findings is that here there is a negative effect from increased inequality with the possibility of utility falling at every income level. This partially works through increased conspicuous consumption by the rich leading to an increase in consumption by those with lower income. However, the negative effect also works through the consequent increase in relative deprivation, people are worse off because of increased negative relative comparisons.

I hope these results, by contrasting the results under cardinal and ordinal preferences, will be useful in empirical investigations of relative concerns in actual consumption behavior. However, there is one further point about empirical identification of such effects that is not considered sufficiently frequently. Samuelson (2002) points out that relative consumption effects can also be produced by social learning. Seeing others consume could induce consumption through a learning or demonstration effect. However, the big difference in predictions between Samuelson's (2002) learning model and the ordinal status model of Hopkins and Kornienko (2004) was on the effect of inequality. It is welfare-decreasing in the former, but tends to reduce wasteful consumption in the latter. However, given the results here on cardinal status preferences, it is less clear how to test between status and social learning. Equally, if social learning and cardinal status concerns are qualitatively similar in their effects, it may be more important to separate the cardinal and ordinal models.

Finally, welfare analysis is more complicated than in the ordinal case because under cardinal preferences welfare depends on the exact level of others' consumption. It is also more complex in its implications for policy. Here, with inequity averse preferences, the rich are worse off with utilitarian optimal consumption than in the non-cooperative equilibrium. With rivalrous preferences, it is possible to construct a Pareto-improving policy, but it requires much more extensive calculations than under ordinal status concerns. Further, the effect of inequality is also complex. For example, rank-based comparisons show that the rich gain from greater inequality even if others are worse off.

[12] A simple analysis of equilibrium utility (20) in the KUJ case finds that one is better off if one's own increase in income is greater than the increase in average income multiplied by $\alpha/(1+2\alpha) < 1$.

These results may also interact with further behavioural issues. For example, Dal Bó et al. (2017) conduct a laboratory experiment which has a social dilemma similar to that considered here. Similarly, there exists a policy that reduces the externality and hence results in a Pareto improvement, but many subjects vote against it. Thus, policy design and political economy in situations where negative externalities are important, including, as analyzed in recent work by Gitmez et al. (2020), the current coronavirus pandemic, remain a complex and important topic.

## Appendix: Proofs

**Proof of Lemma 1:** This follows from the application of the implicit function theorem. Write the left-hand side of (13) as $\psi(x, z, S)$. Then $\partial R/\partial d = -(\partial\psi/\partial d)/(\partial\psi/\partial x)$. One has $\partial\psi/\partial d = \alpha(U_{yS} - U_{xS} - U_{SS}S_x)$. Now, given $S_x = \alpha - (\alpha+\beta)F(x)$, one has $S_x > 0$ everywhere when $\beta < 0$ and, for $\beta > 0$ for $F(x) < \alpha/(\alpha+\beta)$. When $S_x > 0$, by inspection of (8), one can see that $x > \gamma(z, S)$ and thus by condition (vi) on the utility function, $U_{xS} - U_{yS} < 0$. One also has $\alpha > 0$ and $U_{SS} \leq 0$ by assumption and thus $\partial\psi/\partial d > 0$. Similarly, it is easy to verify that, first if $F(x)$ is differentiable then $\partial\psi/\partial x$ exists, and, second, that $\partial\psi/\partial x < 0$ given the assumptions on $U(\cdot)$. Thus, $\partial R/\partial d > 0$ and the result follows. The derivation of $\partial x/\partial a$ is similar for $z < z^*$. But when $z > z^*$, one has $S_x < 0$ and the general case is ambiguous. $\square$

**Proof of Proposition 1:** Suppose that

$$\alpha\frac{g(\bar{z})U_S}{U_{yS} + g(\bar{z})U_S} + \beta < \frac{U_{xy} - U_{yy}}{U_{yS} + g(\bar{z})U_S} > 0 \Rightarrow \beta < \frac{U_{xy} - U_{yy}}{U_{yS} + g(\bar{z})U_S}. \tag{35}$$

for the individual with the highest income $\bar{z}$ and highest rank $G(z) = 1$ so that $d(z) = 0$ and $a(z) = \mu_X$. Thus, the arguments of $U_{xy}$ etc. should be evaluated at $(x, \bar{z}-x, -\beta(x-\mu_X))$, where $\mu_X$ is the average expenditure on $x$ across the population.[13] I will show that this inequality is sufficient for monotonicity.[14]

First, I show that an equilibrium $x(z)$ is necessarily weakly increasing because best responses are increasing in income $z$. Differentiating the first order condition (8) with respect to income $z$ one obtains,

$$\frac{\partial^2 U(x, z-x, S)}{\partial x \partial z} = U_{xy} - U_{yy} + U_{yS}(\alpha(1 - F(x)) - \beta F(x)). \tag{36}$$

[13] This implies the righthand side of the inequality is also a function of $\beta$ and also of $\alpha$ (through $x$ and $\mu_X$). But given the assumption (v) that $U_{xy} - U_{yy} > 0$, and assuming all of the derivatives of $U$ are bounded for finite positive $x$, it is still clearly the case that there are $\alpha$, $\beta$ small enough that the inequality holds.

[14] To give an indication of how restrictive this is, for the log preferences in Section 2.1, when income is distributed uniformly on [1,2], then the constraint is approximately $\alpha + 2.2\beta < 1$. So, $\alpha = 0.5$ and $\beta = 0.2$ or $\alpha = 1$ and $\beta < 0$ would satisfy the constraint, but not $\alpha = \beta = 0.5$. Empirical estimates of similar parameters (Drechsel-Grau and Schmid, 2014; Alvarez-Cuadrado et al., 2016) are lower than this.

This is greater than zero for $F(x) = 0$ and is still greater than zero at $F(x) = 1$ for $\beta < (U_{xy} - U_{yy})/U_{ys} > 0$ which holds if (35) holds - that is, if $\beta$ is sufficiently small. This implies that the best response for each agent is (weakly) increasing in own income $z$.

By definition, in a strictly monotone equilibrium $F(x)$ is continuous and strictly increasing which implies that its inverse $x(F)$ is continuous and strictly increasing. This in turn implies that, given $G(z)$ is continuous and strictly increasing, $x(z)$ is continuous and strictly increasing and that, in equilibrium, $F(x) = G(z)$. Thus, (13) is continuous and differentiable.

I show that the solution to the first-order conditions (13), $R(z; G(z); \alpha d(z) - \beta a(z))$ is an optimum. A sufficient condition is pseudoconcavity. That is, $U$ is increasing in $x$ for $x < R(\cdot)$ and decreasing for $x > R(\cdot)$. Now, take $\tilde{x} < R(\cdot)$ and let $\tilde{z}$ be such that $\tilde{x} = R(\tilde{z})$. Then, $\tilde{z} < z$. Conditional on $dU/dx = 0$ and keeping $x_{-i}$ fixed, one has

$$\frac{\partial^2 U(x, z - x, S)}{\partial x \partial z} = U_{xy} - U_{yy} + U_{yS}(\alpha(1 - G(z)) - \beta G(z)). \tag{37}$$

This is greater than zero for $G(z) = 0$ and is still greater than zero at $G(z) = 1$ for $\beta < (U_{xy} - U_{yy})/U_{ys} > 0$ which holds if (35) holds - that is, if $\beta$ is sufficiently small. Hence, for some $\tilde{x} = x(\tilde{z}) < x(z)$, $dU(\tilde{x}, z)/dx \geq dU(\tilde{x}, \tilde{z})/dx = 0$. Thus, $U$ is increasing in $x$ for $x < R(\cdot)$ and we have pseudoconcavity.

Now let us consider monotonicity and uniqueness. Applying the implicit function theorem to (13), one has $x'(z) > 0$ if $U_{xy} - U_{yy} + U_{yS}S_x - U_S(\alpha + \beta)g(z) > 0$. Now, $S_x$ is at it lowest at $\bar{z}$ where $S_x = -\beta$. Thus, if the inequality (35) is satisfied, the equilibrium is monotone. Further, given that $x(z)$ is differentiable, by the fundamental theory of differential equations, the differential equations (14) have a unique solution for the given boundary conditions, so that there is exactly one strictly monotone equilibrium.

Further, the monotone equilibrium is the unique equilibrium when $\beta = -\alpha$. I have shown above that strategies must be increasing, but consider a candidate equilibrium that is not strictly monotone so that a mass of agents choose some $\tilde{x}$. The left derivative of $S$ with respect to $x$ then is $\alpha(1 - F_-(\tilde{x})) - \beta F_-(\tilde{x})$, while the right derivative is $\alpha(1 - F(\tilde{x})) - \beta F(\tilde{x})$ with the difference being $(\alpha + \beta)(F(\tilde{x}) - F_-(\tilde{x}))$, where $F_-(\tilde{x}) = \lim_{x \uparrow \tilde{x}} F(x)$. Thus, the difference is zero when $\beta = -\alpha$ and thus (13) is continuous and differentiable. Hence, it could not be the best response for this mass all to choose $\tilde{x}$. So, any equilibrium must be strictly monotone and, by the previous argument, unique.

Equilibrium status $S(x(z); z)$ has total derivative

$$\frac{dS(x(z); z)}{dz} = x'(z)(\alpha(1 - G(z) - \beta G(z)) - (\alpha + \beta)x(z)g(z) - \alpha d'(z) + \beta a'(z).$$

But given (14), this simplifies to $x'(z)(\alpha(1 - G(z) - \beta G(z))$ which is strictly positive for $z < z^*$ and strictly negative for $z > z^*$.

Finally, let us turn to the comparisons with the privately optimal consumption

$\hat{x}(z) = \gamma(z, 0)$. For $\beta < 0$, $S_x = \alpha(1 - G(z)) - \beta G(z) > 0$ for all $[\underline{z}, \bar{z}]$; for $\beta = 0$, $S_x > 0$ everywhere except $\bar{z}$ where it is zero. Inspecting the first order condition (8), we have $x(z) \geq \hat{x}(z)$ with equality only where $S_x = S = 0$ which is only the case for $\beta = 0$ and at $\bar{z}$.

For case (i), I show that $x(z)$ and $\hat{x}(z)$ cross only once and at a point $\tilde{z} > z^* = G^{-1}(\alpha/(\alpha + \beta))$. For $\beta > 0$, $S_x > 0$ for $z < z^*$ and so by the above argument, $x(z) > \hat{x}(z)$. Given $x(\bar{z}) = \hat{x}(\bar{z})$ when $\beta = 0$, $x(\bar{z}) < \hat{x}(\bar{z})$ for $\beta > 0$. So there must be a crossing point in $(z^*, \bar{z})$. At any point of crossing on $(z^*, \bar{z})$, one has, again by the implicit function theorem, $x'(z) = (U_{xy} - U_{yy} + U_{ys} S_x - (\alpha+\beta) g(z) U_S)/(b+c) < (U_{xy} - U_{yy})/b = \hat{x}'(z)$, where $b = -U_{xx} - U_{yy} + 2U_{xy} > 0$ and $c = -2S_x(U_{xS} - U_{yS}) - (S_x)^2 U_{SS} > 0$. Thus the crossing is unique. □

**Proof of Proposition 2:** First, an equilibrium strategy $x(z)$ is necessarily weakly increasing as shown in the proof of Proposition 1.

Second, $x(z)$ is continuous. Given the previous result, only upward jumps are potentially possible. The first order condition is continuous and differentiable in $x$ where $F(x)$ is continuous and differentiable.[15] Thus, it implicitly defines a continuous $x(z)$. If there were an upward jump in $x$, say from $x_1$ to $x_2$, then $F(x)$ would be constant on $[x_1, x_2]$ and thus continuous and differentiable. Thus, $x(z)$ would be continuous, a contradiction.

Finally, $x(z)$ will have the same qualitative properties as a strictly monotone equilibrium wherever $F(x)$ is continuous and the first order conditions (13) and (8) are identical. But if $F(x)$ is discontinuous at some point $\tilde{x}$, from (8), $S_x$ is positive for sure if $F_{-}(x) < \alpha/(\alpha + \beta)$ and negative for sure if $F(x) > \alpha/(\alpha + \beta)$. □

**Proof of Proposition 3:** Given the new expression for $S_x$ given in (24), one can see that, if $\alpha > 0 \geq \beta$, then clearly $S_x > 0$ and $S_x$ is increasing in $\theta$. The result then follows directly by applying the methods in the proof of Proposition 1. □

**Proof of Lemma 2:** One needs to choose $x(z)$ to maximize welfare as given in (29). I use the maximum principle approach to maximization, with $x$ as the control variable and $d$ and $a$ as the state variables. The equations of motion are those given in (14). This leads to the Hamiltonian

$$H = g(z)U(x, z - x, x(\alpha(1 - G(z)) - \beta G(z)) - \alpha d(z) + \beta a(z)) - \lambda x g(z) + \xi x g(z),$$

where $\lambda$ and $\xi$ are the costate variables for $d$ and $a$ respectively. Thus the first order condition here is

$$\frac{\partial H}{\partial x} = g(z)(U_x - U_y + U_S S_x) - \lambda(z) g(z) + \xi(z) g(z) = 0. \tag{38}$$

[15] At points where $F(x)$ is discontinuous, $F(x)$ jumps upwards because a mass of agents on an interval, say $[\tilde{z}_{-}, \tilde{z}_{+}]$, choose the same $\tilde{x}$. That is, $F(x)$ is discontinuous but $x(z)$ is constant and thus continuous on $[\tilde{z}_{-}, \tilde{z}_{+}]$.

One has further

$$\lambda'(z) = -\frac{\partial H}{\partial d} = \alpha U_S(z)g(z);\ \xi'(z) = -\frac{\partial H}{\partial a} = -\beta U_S(z)g(z).$$

The boundary condition is $\lambda(\underline{z}) = 0$, because the lowest income agent has no negative externality on others through her influence on $d$. Thus, one has

$$\lambda(z) = \alpha \int_{\underline{z}}^{z} U_S(t)g(t)\ dt = \alpha k(z).$$

Applying a similar method to solve for $\xi(z)$, then substituting into (38), I obtain the first order condition (30) and (31) given in the text. $\square$

**Proof of Proposition 4:** Start with $\beta = 0$ and obtain

$$U(x;\bar{z}) = U(x, \bar{z} - x, 0)$$

because $d(\bar{z}) = 0$ and $G(\bar{z}) = 1$ so that $S(\bar{z}) = 0$ irrespective of choice of own consumption $x$. So, clearly this utility is maximized by $x = \hat{x}(\bar{z})$ where $U_x - U_y = 0$. So the Nash choice of $x$ is optimal for the richest agent, irrespective of the choices of others. However, evaluating (30) at $\bar{z}$, one has

$$U_x - U_y - \alpha k(\bar{z}) = 0.$$

This condition only differs from the equilibrium first order condition by the extra negative term $-\alpha k(z)$, so clearly $x^*(\bar{z}) < x(\bar{z}) = \hat{x}(\bar{z})$.

In the general case with $\beta > 0$, there are two possibilities. Either (more likely) $a^*(\bar{z}) = \mu_X^* \leq \mu_X = a(\bar{z})$, optimal total consumption is lower than in equilibrium to reduce the negative externality, or possibly $\mu_X^* > \mu_X$ if guilt dominates. In the first case, let $\tilde{x}(\bar{z})$ maximize the richest agent's utility $U(\bar{z}; x, \mu_X^*)$, that is taking the socially optimal total consumption $\mu_X^*$ as given. Then, because the agent's utility is increasing in $a(\bar{z}) = \mu_X$, we have $U(\bar{z}) \geq U(\bar{z}; \tilde{x}(\bar{z}), \mu_X^*)$. But also clearly, we have $U(\bar{z}; \tilde{x}(\bar{z}), \mu_X^*) \geq U^*(\bar{z})$ and the result follows. For the other case, where $\mu_X^* > \mu_X$, one can apply a similar argument to show that, because the utility of the individual with income $\underline{z}$ is decreasing in $\mu_X$, she is worse off because $U(\underline{z}) \geq U(\bar{z}; \tilde{x}(\underline{z}), \mu_X^*) \geq U^*(\underline{z})$. $\square$

**Proof of Proposition 5:** First, given $\beta < 0$, the first order condition for the socially optimal consumption (30) has two additional negative terms compared to the Nash equilibrium first order conditions (13). Further, reductions in expenditure by others will reduce $p \equiv \alpha d(z) - \beta a(z)$ and thus the other terms in the first order conditions, $U_x - U_y + U_S S_x$, will also be lower. Thus, $x^*(z) < x(z)$ everywhere.

Second, the first order condition for a social optimum (30) implicitly defines a reaction function $x^*(z;p)$, with $x^*(z;p^*) = x^*(z)$. It is easy to show that $dx^*(z;p)/dp \leq dx(z;p)/dp$ where $x(z;p)$ is non-cooperative equilibrium consumption. Because $x^*(z) < x(z)$ everywhere, one can find a sequence of consumption functions that start at the Nash equilibrium solution $x(z)$ and converges uniformly to $x^*(z)$ with $p$ decreasing monotonically along the sequence. Let $\langle \hat{p} \rangle$ be the induced sequence of $p$. Consider the utility $U(z, x^*; \hat{p})$ of an individual who chooses $x^*(z; \hat{p})$. First, note that the limit of the sequence $\langle U(z; x^*; \hat{p}) \rangle$ is $U^*(z)$, the utility at the social optimum. Second, utility increases at every point in the sequence because

$$\frac{dU}{dp} = \frac{\partial U}{\partial p} + \frac{\partial U}{\partial x}\frac{dx^*(z;p)}{dp} \leq \frac{\partial U}{\partial p} + \frac{\partial U}{\partial x}\frac{dx(z;p)}{dp} < 0,$$

with the final inequality following from application of the envelope theorem. Thus, $U^*(z) > U(z)$ for arbitrary $z$. □

**Proof of Proposition 6:** Note that from (34), $x(1) = Z(1)/2 - \beta\mu_X/(2(1-\beta))$, because $a(1) = \mu_X$, average expenditure on consumption. Further, given $Z_B'(r) > Z_A'(r)$ on $(\hat{r}, 1)$, the maximum income difference is at $r = 1$, so that $Z_B(1) - Z_A(1) > \mu_{XB} - \mu_{XA}$. Thus, one has $x_B(1) > x_A(1)$. Define $p(r) = \alpha d(r) - \beta a(r)$ so that $p'(r) = -(\alpha + \beta)x(r)$ and $p(1) = -\beta\mu_X > 0$. Given $\beta \leq 0$, one has $p(r) > 0$. We have $\mu_B > \mu_A$, average income is higher in $B$, and this implies through (34), that $\mu_{XB} \geq \mu_{XA}$, average conspicuous consumption is higher. Specifically, integrating (34), one has $\mu_X = \mu/2 + q$ where

$$q = \int_0^1 \frac{\alpha d(r) - \beta a(r)}{2(1 + \alpha(1-r) - \beta r)}\, dr.$$

Thus $\mu_{XA} - \mu_{XB} > 0$ only if $q_A - q_B > \mu_B - \mu_A$. However, note that $q_A - q_B < \alpha\mu_{XA}/(1+\alpha) + \beta\mu_{XB}/(1-\beta) < \mu_{XA} - \mu_{XB} < \mu_B - \mu_A$, given that $\alpha > 0 \geq \beta \geq -\alpha$. Thus, $p_B(1) = \beta a_B(1) = \beta\mu_{XB} \geq p_A(1) = \beta\mu_{XA}$. Then, $x_A(1) < x_B(1)$ implies $p_A(r) < p_B(r)$ for some interval $(\tilde{r}, 1)$ where $\tilde{r}$ is the largest $r \in (0,1)$ such that $x_A = x_B$ - at any other potential crossing point of $p_A$ and $p_B$ in $(\tilde{r}, 1)$ we have $p_B' = -(\alpha+\beta)x_B < -(\alpha+\beta)x_A = p_A'$. Thus, the greatest crossing point of $p_A$ and $p_B$ in $[0,1)$ is to the left of the greatest crossing point of $x_A$ and $x_B$. This in turn implies that $x_A(r) < x_B(r)$ everywhere on $[0,1]$ from (34) because $p_B(r) > p_A(r)$ and $Z_A(r) \leq Z_B(r)$.

Turning to utility, one has, after some manipulation,

$$U(r) = \ln[1 + \alpha(1-r) - \beta r] + 2\ln[y(r)],$$

where non-conspicuous consumption $y(r) = Z(r) - x(r) = Z(r)/2 - (\alpha d(r) - \beta a(r))/(2(1 + \alpha(1-r) - \beta r))$. Thus, $U_A$ and $U_B$ cross only when $y_A$ crosses $y_B$. Further, $y'(r) = Z'(r)/2 + (\alpha+\beta)y(r)/(2(1+\alpha(1-r)-\beta r))$ so that at a point of crossing of $y_A$ and $y_B$, the relative slopes are determined by comparing $Z_A'$ and $Z_B'$. We have $Z_A'(r) < Z_B'(r)$ on $(\hat{r}, 1)$ so that there is at most one point of crossing of $y_A$ and $y_B$ and thus at most one crossing of $U_A$ and $U_B$. Clearly, $U_A(1) < U_B(1)$ because again $Z_B(1) - Z_A(1) > \mu_{XB} - \mu_{XA}$ so that $y_B(1) > y_A(1)$. But, one can see that, because income at $\hat{r}$ is unchanged but $p_A(\hat{r}) < p_B(\hat{r})$ as shown above, then we have $U_A(\hat{r}) > U_B(\hat{r})$. So the only crossing of $U_A$ and $U_B$ is on $(\hat{r}, 1)$ and not on $(0, \hat{r})$. □

**Proof of Proposition 7:** It will be useful to differentiate the equilibrium consumption function (16) with respect to $z$, using the derivatives given in (14), to obtain,

$$x'(z) = \frac{1}{2} - \frac{(\alpha+\beta)g(z)(z-x)}{2(1+\alpha(1-G(z))-\beta G(z))} = \frac{1}{2} + \frac{(z-x)}{2}\phi(z), \tag{39}$$

where $\phi(z) = -(\alpha+\beta)g(z)/(1+\alpha-(\alpha+\beta)G(z)) < 0$. Define

$$Q(z) = \frac{G_A(z) - \frac{1+\alpha}{\alpha+\beta}}{G_B(z) - \frac{1+\alpha}{\alpha+\beta}} \tag{40}$$

Note that $(1+\alpha)/(\alpha+\beta)$ is greater than one and thus $Q(z) > 0$ for all $[\underline{z}, \bar{z}]$. Given $L(z) = g_A(z)/g_B(z)$, one has

$$\frac{\phi_A(z)}{\alpha+\beta} = \frac{g_A(z)}{G_A(z) - \frac{1+\alpha}{\alpha+\beta}} < \frac{g_B(z)}{G_B(z) - \frac{1+\alpha}{\alpha+\beta}} = \frac{\phi_B(z)}{\alpha+\beta} \Leftrightarrow L(z) > Q(z). \tag{41}$$

The following result is effectively the reverse of Lemma A2 in Hopkins and Kornienko (2004).

**Lemma 3.** *If $G_A(z) \succ_{ULR} G_B(z)$ then for all $\alpha \geq 0$, $Q(z)$ has two extremes, a maximum at $\hat{z}_-$ and a minimum at $\hat{z}_+$, such that $\underline{z} \leq \hat{z}_- < \hat{z} < \hat{z}_+ < \bar{z}$. Further $L(z)$ and $Q(z)$ cross only at $\hat{z}_-$ and $\hat{z}_+$. Thus, $\phi_A(z) > \phi_B(z)$ on both $[\underline{z}, \hat{z}_-)$ and $(\hat{z}_+, \bar{z}]$ but $\phi_A(z) < \phi_B(z)$ on $(\hat{z}_-, \hat{z}_+)$.*

*Proof.* Note that $Q(\underline{z}) = Q(\hat{z}) = Q(\bar{z}) = 1$. Therefore, there is at least one extreme point for $Q(z)$ on each of the two intervals $(\underline{z}, \hat{z})$ and $(\hat{z}, \bar{z})$. Note that $dQ(z)/dz = 0$ if and only if $Q(z) = L(z)$. That is, $Q(z)$ and $L(z)$ cross at the turning points of $Q(z)$. Now since $L(z)$ is increasing on $(\underline{z}, \hat{z})$, at any crossing point $L(z)$ must cross $Q(z)$ from below and so there can only be one crossing point on $(\underline{z}, \hat{z})$. Equally there can be only one extreme for $Q(z)$ on $(\hat{z}, \underline{z})$. Lastly, $Q(z)$ is increasing iff $L(z) < Q(z)$ and we can see that the maximum is at $\hat{z}_-$ and the minimum at $\hat{z}_+$. The final result then follows from (41). □

(a) Because, by definition $d_A(\bar{z}) = d_B(\bar{z}) = 0$, we have from (16) that $x_A(\bar{z}) = x_B(\bar{z})$. Second, because the ULR order implies that $g_A(\bar{z}) < g_B(\bar{z})$ (and $G_A(\bar{z}) = G_B(\bar{z}) = 1$), by (39) we have $x'_A(\bar{z}) > x'_B(\bar{z})$. Thus, $x_A(z)$ crosses $x_B(z)$ from below at $\bar{z}$ so that $x_A(z) < x_B(z)$ on $(\bar{z} - \varepsilon, \bar{z})$ for some $\varepsilon > 0$. Third, there can be no crossing point in $(\hat{z}_+, \bar{z})$. Suppose there was such a point then simultaneously one would have $x_A = x_B$ and $x'_A < x'_B$. But $\phi_A(z) > \phi_B(z) \in (\hat{z}_+, \bar{z})$ by Lemma 3, so that $x'_A > x'_B$ wherever $x_A = x_B$ on this interval. Fourth, there can be a crossing point on $(\hat{z}_-, \hat{z}_+)$ because there $\phi_A(z) < \phi_B(z)$. If there is a crossing on $(\hat{z}_-, \hat{z}_+)$, then there can be a crossing on $(\underline{z}, \hat{z}_-)$, with $x'_A > x'_B$ because $\phi_A(z) > \phi_B(z)$ on this interval.
(b) Again because $d_A(\bar{z}) = d_B(\bar{z}) = 0$, and $x_A(\bar{z}) = x_B(\bar{z})$, it follows that $U_A(\bar{z}) = U_B(\bar{z})$. We have by the envelope theorem

$$U'(z) = \frac{1}{z-x};\ U''(z) = \frac{x'(z)-1}{(z-x)^2}. \tag{42}$$

So, $U'_A(\bar{z}) = U'_B(\bar{z})$, but as shown above we have $x'_A(\bar{z}) > x'_B(\bar{z})$ so that $U''_A(\bar{z}) > U''_B(\bar{z})$ so that $U_A(z) > U_B(z)$ for $z$ immediately lower than $\bar{z}$. First, take the case where $x_A(z) < x_B(z)$ everywhere on $(\underline{z}, \bar{z})$. Then at any potential crossing point of $U_A$ and $U_B$ we would have $U'_A(z) < U'_B(z)$ so there could only be one such crossing point and $U_B$ would cross $U_A$ from below. But such a crossing point would contradict that $U_A(z) > U_B(z)$ in the neighbourhood of $\bar{z}$. Second, suppose $x_A$ crosses $x_B$ at some point $\tilde{z} < \hat{z}$. By the above argument we have $U_A(z) > U_B(z)$ on $(\tilde{z}, \bar{z})$. In fact, at $\tilde{z}$ the function $U_A(z) - U_B(z)$ has its maximum. Since $U_A(\tilde{z}) - U_B(\tilde{z}) > 0$, and because $x_A(\tilde{z}) = x_B(\tilde{z})$ so that $\alpha d_A(z)/(1 + \alpha(1 - G_A(z)) = \alpha d_B(z)/(1 + \alpha(1 - G_B(z))$, it must be from (17) that $G_A(\tilde{z}) < G_B(\tilde{z})$ so that it follows that $\tilde{z} < \hat{z}$. Further, if there is further crossing point of $x_A$ and $x_B$ in $(\underline{z}, \hat{z}_-)$ then this is a minimum for $U_A - U_B$ and it also holds that $U_A - U_B > 0$ there, so that $U_A(z) > U_B(z)$ for all of $[\underline{z}, \bar{z})$. $\square$

**Proof of Proposition 8:** (a) In this case $x_A(\underline{z}) = x_B(\underline{z}) = \underline{z}/2$. We have $g_B(z) > g_A(z)$ on $[\underline{z}, \hat{z}_-)$ so that, from (39) and given $\beta < 0$, it follows that $x'_B(\underline{z}) > x'_A(\underline{z})$, implying $x_B(z) > x_A(z)$ in the neighbourhood of $\underline{z}$, and second, one would have $x'_B > x'_A$ at any potential crossing point in the interval $[\underline{z}, \hat{z}_-)$. So there can be no further crossing until $z > \hat{z}_-$.
(b) Since $x_A(\underline{z}) = x_B(\underline{z}) = \underline{z}/2$, one has $U_A(\underline{z}) = U_B(\underline{z})$ and $U'_A(\underline{z}) = U'_B(\underline{z})$ from (42). But because $x'_B(\underline{z}) > x'_A(\underline{z})$, one has $U''_A(\underline{z}) < U''_B(\underline{z})$ so that $U_B > U_A$ in the neighbourhood of $\underline{z}$. Further, since $x_B > x_A$ on $(\underline{z}, \hat{z}_-)$, there can be no further crossing. $\square$

# References


Alvarez-Cuadrado, F., Casado, J. M., Labeaga, J. M. (2016) “Envy and habits: Panel data estimates of interdependent preferences”, *Oxford Bulletin of Economics and Statistics*, 78 (4), 443-469.

Azmat, G., and N. Iriberri (2010) “The importance of relative performance feedback information: Evidence from a natural experiment using high school students”, *Journal of Public Economics*, 94(7), 435452.

Barnett, Richard C., Joydeep Bhattacharya, and Helle Bunzel (2019) “The fight-or-flight response to the Joneses and inequality”, *Journal of Economic Dynamics and Control*, 101, 187-210.

Battigalli, Pierpaolo and Martin Dufwenberg (2020) “Belief-Dependent Motivations and Psychological Game Theory”, *Journal of Economic Literature*, forthcoming.

Becker, Gary S., Kevin M. Murphy and Ivan Werning (2005) “The Equilibrium Distribution of Income and the Market for Status”, *Journal of Political Economy*, 113 (2), 282-310.

Bellet, Clément, and Eve Colson-Sihra (2018) “The Conspicuous Consumption of the Poor: Forgoing Calories for Aspirational Goods”, Working Paper.

Bertrand, Marianne, and Adair Morse (2016) "Trickle-down consumption", *Review of Economics and Statistics*, 98(5), 863-879.

Bhaskar, V. (1990) "Wage Relativities and the Natural Range of Unemployment", *The Economic Journal*, 100(400), 60-66.

Bilancini, Ennio, and Leonardo Boncinelli (2012) "Redistribution and the notion of social status", *Journal of Public Economics* 96.9: 651-657.

Bolton, Gary E., and Axel Ockenfels (2000) "ERC: A theory of equity, reciprocity, and competition", *American Economic Review*, 90(1), 166-193.

Brown, Gordon DA, Jonathan Gardner, Andrew J. Oswald, and Jing Qian (2008) "Does wage rank affect employees' well-being?", *Industrial Relations: A Journal of Economy and Society*, 47(3), 355-389.

Charles, Kerwin Kofi, Erik Hurst, and Nikolai Roussanov (2009) "Conspicuous consumption and race", *The Quarterly Journal of Economics*, 124(2), 425-467.

Clark, Andrew E., Paul Frijters, and Michael A. Shields (2008) "Relative income, happiness, and utility: An explanation for the Easterlin paradox and other puzzles", *Journal of Economic Literature* 46(1), 95-144.

Clark, Andrew E., and Andrew J. Oswald (1998) "Comparison-concave utility and following behaviour in social and economic settings", *Journal of Public Economics*, 70(1), 133-155.

Cooper, David J., and John H. Kagel (2016) "Other-regarding preferences", *The Handbook of Experimental Economics*, Vol 2, J. Kagel and A. Roth (eds.), Princeton: Princeton University Press.

Dal Bó, Ernesto, Pedro Dal Bó, and Erik Eyster (2017) "The demand for bad policy when voters underappreciate equilibrium effects", *The Review of Economic Studies*, 85(2), 964-998.

Deaton, Angus (2003) "Health, inequality and economic development", *Journal of Economic Literature*, 41(1), 113-158.

Drechsel-Grau, M., Schmid, K. (2014) "Consumption savings decisions under upward looking comparisons", *Journal of Economic Behavior and Organization*, 106, 254-268.

Dufwenberg, Martin, Paul Heidhues, Georg Kirchsteiger, Frank Riedel, Joel Sobel (2011) "Other-Regarding Preferences in General Equilibrium", *Review of Economic Studies*, 78(2), 613-639.

Engelmann, Dirk, and Martin Strobel (2004) "Inequality aversion, efficiency, and maximin preferences in simple distribution experiments", *American Economic Review*, 94.4, 857-869.

Fehr, Ernst and Klaus Schmidt (1999), "A theory of fairness, competition, and cooperation", *Quarterly Journal of Economics*, 114, 817-868.

Frank, Robert H. (1985) "The demand for unobservable and other positional goods", *American Economic Review*, 75 (1), 101-116.

Frank, Robert H., Adam Seth Levine, and Oege Dijk (2014) "Expenditure cascades", *Review of Behavioral Economics*, 1 (1-2), 55-73.

Friedman, Daniel, and Daniel N. Ostrov (2008) "Conspicuous consumption dynamics", *Games and Economic Behavior*, 64(1): 121-145.

Galí, Jordi (1994) "Keeping up with the Joneses: Consumption externalities, portfolio choice, and asset prices", *Journal of Money, Credit and Banking*, 26(1), 1-8.

Gitmez, Arda, Konstantin Sonin and Austin L. Wright (2020) "Political Economy of Crisis Response", working paper.

Glazer, Amihai, and Kai A. Konrad (1996) "A signaling explanation for charity", *American Economic Review*, 86(4), 1019-1028.

Heffetz, O. (2011). A test of conspicuous consumption: Visibility and income elasticities. *Review of Economics and Statistics*, 93(4), 1101-1117.

Hopkins, Ed (2008) "Inequality, Happiness and Relative Concerns: What Actually is their Relationship?", *Journal of Economic Inequality*, 6, 351-372.

Hopkins, Ed and Tatiana Kornienko (2004) "Running to Keep in the Same Place: Consumer Choice as a Game of Status", *American Economic Review*, September 2004, 94 (4), 1085-1107.

Hopkins, Ed and Tatiana Kornienko (2009) "Status, Affluence, and Inequality: Rank-Based Comparisons in Games of Status", *Games and Economic Behavior*, 67, 552-568.

Hopkins, Ed and Tatiana Kornienko (2010) "Which Inequality? The Inequality of Endowments Versus the Inequality of Rewards", *American Economic Journal: Microeconomics*, August 2010, 2(3), 106-137.

Ireland, Norman J. (2001) "Optimal income tax in the presence of status effects", *Journal of Public Economics*, 81.2, 193-212.

Iriberri, Nagore, and Pedro Rey-Biel (2013) "Elicited beliefs and social information in modified dictator games: What do dictators believe other dictators do?", *Quantitative Economics*, 4.3, 515-547.

Jinkins, David (2016) "Conspicuous consumption in the United States and China", *Journal of Economic Behavior & Organization*, 127: 115-132.

Kanbur, Ravi, and Matti Tuomala (2013) "Relativity, inequality, and optimal nonlinear income taxation", *International Economic Review*, 54.4, 1199-1217.

Moav, Omer, and Zvika Neeman (2012) “Saving rates and poverty: The role of conspicuous consumption and human capital”, *The Economic Journal*, 122(563), 933-956.

Samuelson, Larry (2004) “Information-Based Relative Consumption Effects”, *Econometrica*, 72(1), 93-118.

Shaked, Moshe and J. George Shanthikumar (2006), *Stochastic Orders*, Springer.

Tincani, Michela M. (2018) “Heterogeneous Peer Effects in the Classroom”, working paper.

Yitzhaki, Shlomo (1979) “Relative deprivation and the Gini coefficient”, *Quarterly Journal of Economics*, 93 (2), 321-324.