Problem Set 4

Microeconomics I

Exercise 1: Consider the infinitely repeated game $G(\infty, \delta)$ based on the stage game below. Use the principle of optimality to find the set of discount factors for which the following strategy profile is a subgame perfect equilibrium:
1) in period 1 Player i plays a_i;
2) in every period after period 1, Player i plays b_i if (b_1, b_2) or (c_1, c_2) was played in the previous period; and
3) in every period after period 1, Player i plays c_i if (b_1, b_2) or (c_1, c_2) was not played in the previous period.

Player 2

Player 1	a_2	b_2	c_2
a_1	4,4	3,2	1,1
b_1	2,3	2,2	1,1
c_1	1,1	1,1	-1,-1

Exercise 2: (Repeated Prisoner's Dilemma) Consider the following stage game G:

Player 2

Player 1	C	D
C	-1,-1	-4,0
D	0,-4	-3,-3

Show that for high enough δ there is a subgame perfect equilibrium (SPE) σ of the infinitely repeated game $G(\infty, \delta)$ for which $u_1(\sigma) = u_2(\sigma) = -1$.

Exercise 3: Consider the following stage game G (G is a game of "Chicken"):

Player 2

Player 1	S	KG
S	4,4	1,6
KG	6,1	0,0

where S stands for "Swerve" and KG stands for "Keep Going".

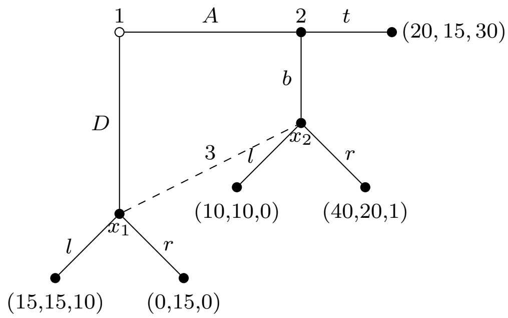
(a) Find every Nash equilibrium of G.

(b) Find the strategy m_1 of Player 2 that minmaxes Player 1, and the corresponding minmax value $\underline{v}_1$ of Player 1. (G is symmetric, so that is also Player 2's minmax value.)

(c) Carefully draw the utility possibility set, and the set of payoffs identified in the Fudenberg and Maskin Folk Theorem as SPE payoffs of $G(\infty, \delta)$ for high enough δ.

(d) For what values of δ is $((S, S), (S, S), (S, S), \dots)$ a Nash equilibrium outcome of $G(\infty, \delta)$? NOTE: The equilibrium need not be subgame perfect.

Exercise 4: Consider the following extensive form game:



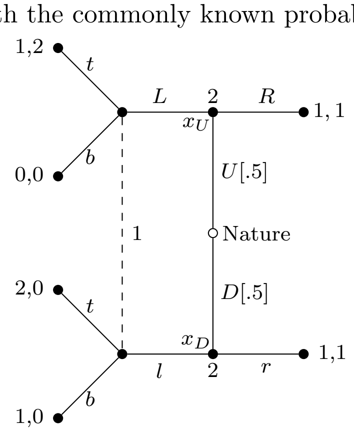
(a) Write the corresponding normal form.

(b) Show that no player has any weakly dominated strategies.

(c) Let p be the probability that Player 3 chooses action l. For what values of p is (A, t, p) a Nash equilibrium.

(d) Show that whenever (A, t, p) is a Nash equilibrium, then it is also part of a perfect Bayesian equilibrium.

Exercise 5: Consider the following extensive form game in which Nature moves first with the commonly known probabilities given in brackets:



The first payoff is Player 1's, and the second payoff is Player 2's.

(a) Find all the perfect Bayesian equilibria (PBE).

(b) Find a Nash equilibrium that is not a part of a PBE.