Problem Set 3

Microeconomics I

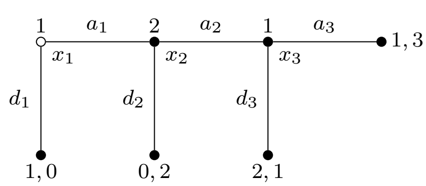
Exercise 1: Consider a first-price sealed-bid auction of an object with two bidders. Each bidder i's valuation of the object is v_i, which is known to both bidders. The auction rules are that each player submits a bid in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object and pays the auctioneer the amount of his bid. If the bidders submit the same bid, each gets the object with probability $\frac{1}{2}$. Bids must be in dollar multiples (assume that valuations are also).

(a) Are any strategies strictly dominated?

(b) Are any strategies weakly dominated?

(c) Is there a Nash equilibrium? If so, what is it? Is it unique?

Exercise 2: Consider the following extensive form game:



(a) Find all the subgame perfect Nash equilibria.

(b) Find all the Nash equilibria.

Exercise 3: The following normal form game is played twice (each player's final payoff is the sum of her payoffs in the two plays):

		Player 2	
		a	b
Player 1	A	10,10	5,20
	B	20,5	7,7

(a) Draw the extensive form game. How many pure strategies does each player have?

(b) Find all the subgame perfect Nash equilibria.

Exercise 4: A Stackelberg duopoly has two firms — firm 1 and firm 2 — with firm 1 choosing output first and firm 2 choosing output second, after observing the choice of firm 1. Suppose that the inverse demand function is $P(Q) = 6 - Q$, where $Q = q_1 + q_2$ is aggregate output. Each firm has constant marginal cost of £4 per unit, and a capacity constraint of 3 units.

(a) Define formally the strategy set of each firm. (*Hint:* Firm 2's strategy is a function.)

(b) Find a Nash equilibrium in which the Cournot outputs are produced.

(c) Find a Nash equilibrium in which firm 2 produces the monopoly output and firm 1 produces nothing.

(d) Find the subgame perfect Nash equilibria.

Exercise 5: Suppose n players use an ultimatum procedure to share an apple pie. First, player 1 proposes a division. Then the others simultaneously respond "yes" or "no." If they all say "yes" , the proposed division is implemented. Otherwise, the pie is fed to Penny the dog. Each player prefers more pie to less, and is indifferent about how much pie any other player or dog consumes.

(a) Define formally the strategy set of each player.

(b) Find the subgame perfect Nash equilibria when $n = 2$ and $n = 3$.

Exercise 6: Consider the following Pirate Game: There are R pirates who must decide how to divide 100 gold pieces among themselves. The gold pieces are indivisible, so a division is feasible only if each pirate gets a whole number of gold pieces. The mechanism they use is as follows: Pirate 1 proposes a division. Then Pirate 2 can accept or reject it. If he accepts, the proposed division is implemented and the game is over. If he rejects, Pirate 1 is thrown to the sharks, and then Pirate 2 proposes a division to Pirate 3, and so on. If pirate R rejects Pirate $R-1$'s proposal, then Pirate R gets all 100 gold pieces. Pirates prefer more gold to less and are indifferent about about how much gold any other pirate gets; being fed to the sharks is their least favorite thing. Watching another pirate being fed to the sharks gives a pirate positive utility, but a pirate always prefers an extra gold piece to watching sharks eat. Describe the subgame perfect Nash equilibria of this game. How does your answer depend on the value of R?