Problem Set 2

Microeconomics I

Exercise 1: The game matrix below gives Player 1's payoffs:

Player 2

		L	R
Player 1	U	x	0
	D	0	y

where $x > y > 0$. Let p be the probability with which Player 1 believes that Player 2 will play L. Derive the best response correspondence $BR(p)$.

Exercise 2: The game matrix below gives Player 1's payoffs:

Player 2

		S	D
Player 1	U	15	90
	M	B	75
	D	55	40

Let q be the probability with which Player 1 believes that Player 2 will play S.

(a) Suppose that $B = 35$. Find the three ranges of values of q for which U, M and D are optimal, respectively (and draw a picture of expected utility versus q). Is any action strictly dominated, and if so, by what mixed action? (Draw another picture, utility when Player 2 plays S versus utility when Player 2 plays D.)

(b) Repeat (a), assuming now that $B = 20$.

(c) For what range of values of B is action M strictly dominated?

Exercise 3: Solve the following game by iteratively deleting strictly dominated strategies:

Player 2

		a	b	c	d
	A	3,1	0,0	1,0	0,0
Player 1	B	1,1	1,0	1,1	1,2
	C	1,2	0,4	6,2	1,1
	D	0,4	1,0	1,1	2,3

Exercise 4: Consider the following game:

		Player 2		
		L	C	R
Player 1	U	50,0	5,5	1,-1000
	D	50,50	5,0	0,-1000

Show that the set of strategies that survive the iterated deletion of *weakly* dominated strategies depends on the order of deletion.

Exercise 5: Consider the following symmetric, two-player, simultaneous move game: each player i chooses an action from the set $A_1 = A_2 = \{100, 200, 300\}$. The payoffs are as follows:

$$u_i(a_i, a_{-i}) = \begin{cases} a_i + 200 & \text{if } a_i < a_{-i} \\ a_i & \text{if } a_i = a_{-i} \\ a_i - 200 & \text{if } a_i > a_{-i} \end{cases}$$

(a) Write down the normal form payoff matrix for this game.

(b) Which actions are strictly dominated? Which actions are weakly dominated?

(c) Find all of the pure-strategy Nash equilibria.

(d) Find all of the Nash equilibria, including those in mixed strategies.

Exercise 6: Consider the following auction, known as a *second-price*, or *Vickrey*, auction. An object is auctioned off to N bidders. Bidder i's valuation of the object in monetary terms is v_i. The auction rules are that each player submit a bid (a non-negative number) in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object but pays the auctioneer the amount of the second-highest bid. If more than one bidder submits the highest bid, each gets the object with equal probability. Show that submitting a bid of v_i with certainty is a weakly dominant strategy for bidder i. Also argue that this is bidder i's unique weakly dominant strategy.

Exercise 7: Consumers are uniformly distributed along a boardwalk that is 1 mile long. Ice cream prices are regulated so consumers go to the nearest vendor because they dislike walking. Assume that at the regulated price all consumers will purchase an ice cream even if they have to walk a full mile. If more than one vendor is at the same location, they split the business evenly.

(a) Consider a game in which two ice cream vendors pick their locations simultaneously. Show that there exists a *unique* pure strategy Nash equilibrium and that it involves both vendors locating at the midpoint of the boardwalk.

(b) Show that with three vendors, no pure strategy Nash equilibrium exists.