Section B

1. **[25 marks]**

Consider the following stage game G:

		Player 2 C	D
Player 1	C	3,3	0,4
	D	5,0	0,0

(a) Find all Nash equilibria of game G, including those where one or both players play mixed strategies (if any).

Solution

Marking note: This question tests the understanding of the concept of Nash equilibrium. Students are expected to explain how NE are found, and to identify all of them. Partial credit ('concepts') is given for employing the notions of (weakly) dominated strategies and best response, and ('analysis') for an attempt to solve for a MSNE.

Let p and q be the probabilities of C played by Players 1 and 2, respectively. Player 1's best response to q is

$$p^*(q) = \begin{cases} \{0\}, & \text{if } q > 0 \\ [0,1] & \text{if } q = 0. \end{cases}$$

Similarly, Player 2's best response to p is

$$q^*(p) = \begin{cases} \{0\}, & \text{if } p > 0 \\ [0,1] & \text{if } p = 0. \end{cases}$$

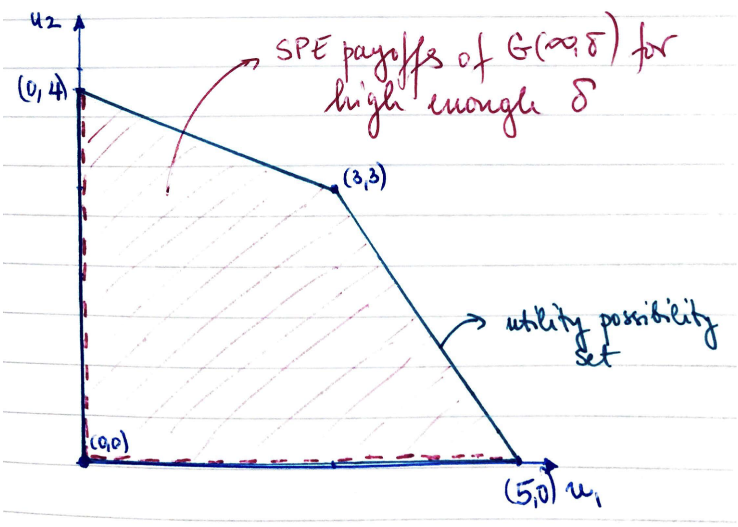
NE are mutual best responses. Pure NE: (D, D). Other NE: (D, q) for any $q \in (0, 1)$; (p, D) for any $p \in (0, 1)$.

Consider now the infinitely repeated game $G(\infty, \delta)$ with a discount factor $\delta \in (0, 1)$.

(b) Find the minmax payoffs of Players 1 and 2. Explain briefly. Draw the set of all possible payoffs. Indicate the area that contains all the subgame perfect equilibrium payoffs of the repeated game $G(\infty, \delta)$ for high enough δ, according to Fudenberg and Maskin's Folk Theorem.

Solution

The best-response payoff to Player 2 is $w_2(p) = \max\{3p, 4p\} = 4p$. This is minimized at $p = 0$, i.e. $\underline{p} = 0$. That is, by playing D Player 1 minimizes the value of Player 2's best response payoff $w_2(p)$, and the corresponding minmax value is $\underline{v}_2 = \min_{p\in[0,1]} w_2(p) = 0$. Similarly, $w_1(q) = \max\{3q, 5q\} = 3p$, and the minmax value for Player 1 is $\underline{v}_1 = \min_{q\in[0,1]} w_1(q) = 0$.



(c) What is the smallest value of δ such that cooperation can be achieved in a subgame perfect equilibrium (SPE) of the repeated game $G(\infty, \delta)$? Provide an example of such an SPE.

Solution

Consider a 'grim trigger' strategy: Play C, then in each period play C if the opponent always played C in the past, and play D otherwise.

By the one-shot deviation principle, we only need to consider a deviation from C to D in a single round.

Player 1 cooperates: $3 + 3\delta + 3\delta^2 + ... = 3/(1-\delta)$. Player 1 deviates once: 5 and then zero forever. $3/(1-\delta) \geq 5$ if and only if $\delta \geq 0.4$.

Player 2 cooperates: $3 + 3\delta + 3\delta^2 + ... = 3/(1-\delta)$. Player 2 deviates once: 4 and then zero forever. $3/(1-\delta) \geq 4$ if and only if $\delta \geq 0.25$.

Thus, cooperation can be sustained if and only if $\delta \geq \max\{0.4, 0.25\} = 0.4$. As the grim trigger strategy imposes the maximum punishment of deviations, cooperation is not sustainable for any smaller δ.

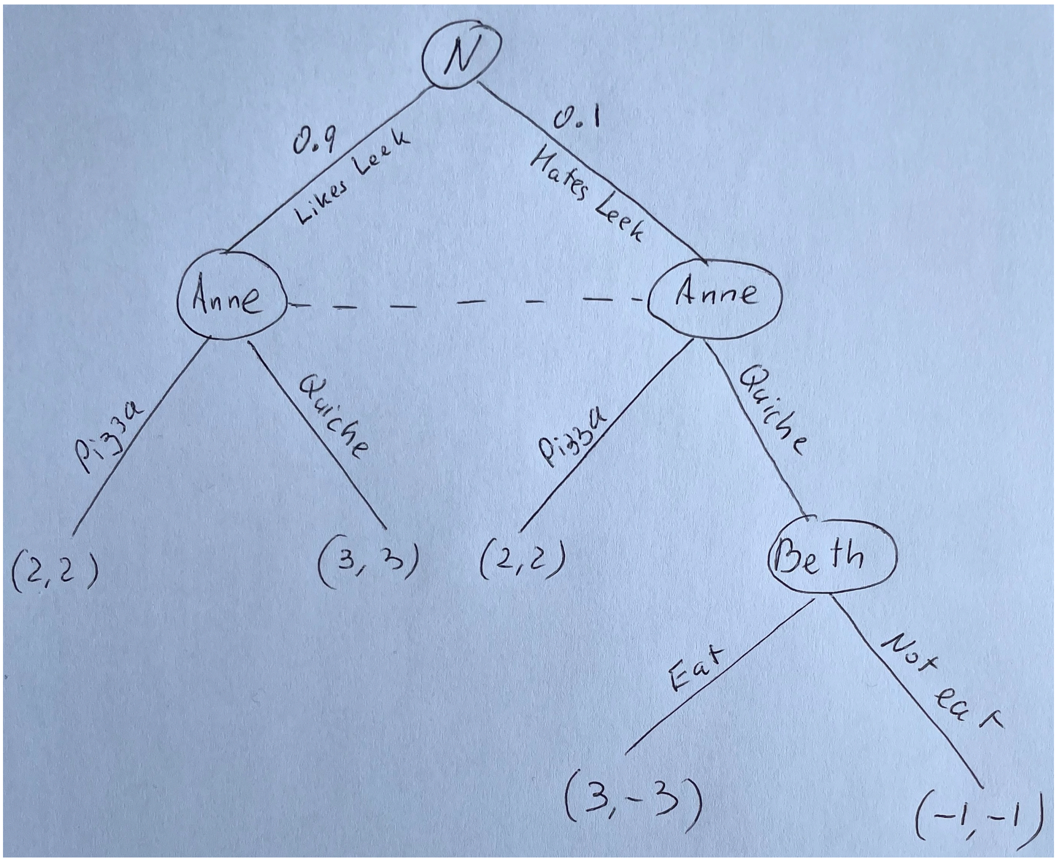
2. [25 marks]

Anne is ordering a takeaway dinner for herself and her roommate Beth (who is not home yet). Anne can either order a pepperoni pizza or a parmesan-and-leek quiche. Anne is pretty sure (with probability 0.9) that Beth likes leek, but there is a chance of 0.1 that Beth hates leek and would be very disappointed the order of quiche. If Beth hates leek, then she can either pretend that she likes it (in order not to upset Anne) or angrily throw away the dinner.

If Anne orders the pizza, the payoffs will be $(2, 2)$ for Anne and Beth. If Anne orders the quiche and Beth likes it, the payoffs will be $(3, 3)$; if Beth does not like the quiche but pretends she does, the payoffs will be $(3, -3)$; and if Beth throws away the dinner, the payoffs will be $(-1, -1)$.

(a) Draw this game in the tree form.

Solution



(b) What type of game is this? What would be an appropriate solution concept to solve this game? Motivate your choice by briefly comparing and contrasting different solution concepts (iterated deletion of dominated strategies, Nash equilibrium, subgame perfect equilibrium, perfect Bayesian equilibrium). Solve the game using your chosen solution concept.

Solution

Marking note: This question is designed to test the understanding of different solution concepts, the ability to judge the appropriateness of a method and to apply the selected method.

Formally, this is a game of incomplete (or asymmetric) information, as Beth has private information that Anne is uncertain about. A shallow answer is that, because this is a game of incomplete information, the concepts of IDDS, NE, and SPE are not appropriate here, and only PBE is appropriate. The complete answer demonstrates understanding of other solution concepts and arguments why these concepts can be applied in this specific game: Either using the argument that Nature can be treated as the third player whose move is a fixed mixed strategy $(0.9, 0.1)$, or working with the game structure directly.

At least one of the solutions must be provided:

PBE (solved by backward induction): Consider Beth's move. Beth prefers 'Not Eat', as $-1 > -3$. Anticipating Beth's choice, Anne gets in expectation $0.9 \cdot 3 + 0.1 \cdot (-1) = 2.6$ if she orders quiche, which is greater than the payoff of 2 if she orders pizza. Thus, PBE strategies are (Quiche, Not

Eat), and Anne's PBE beliefs are equal to the prior probabilities, (0.9,0.1).

SPE: Consider Nature as the third player whose move is a fixed mixed strategy $(0.9, 0.1)$. The game has a clear subgame where Beth moves. Beth strictly prefers 'Not Eat' to 'Eat'. Solving this subgame yields that Beth chooses 'Not Eat', with payoffs $(-1, -1)$. The remaining game is as if it is a simlutaneous-move game between Nature and Anne. Given Nature's mixed strategy $(0.9, 0.1)$, Anne's best response is 'Quiche'. Thus, the only SPE is (Quiche, Not Eat).

NE: Write down the normal-form representation of the game. The payoff matrix for Anne and for the type of Beth who hates leek (the other type of Beth does not make a move, and thus can be ignored), where Anne calculates her expected payoff anticipating Beth to like or hate leek with probabilities $(0.9, 0.1)$:

	Beth	
Anne	Eat	Not Eat
Pizza	$2, 2$	$2, 2$
Quiche	$3, -3$	$2.6, -1$

After calculating the best responses, derive that the only NE is (Quiche, Not Eat).

IDDS: When Beth makes her move, she has a stictly dominated strategy, as 'Not Eat' strictly dominates 'Eat'. After eliminating 'Eat', the in the remaining game only Anne has a move, and her expected payoffs are well defined: 'Pizza' yields 2, and 'Quiche' yields $0.9 \cdot 3 + 0.1 \cdot (-1) = 2.6$. Thus, Anne has a stictly dominated strategy, as 'Quiche' strictly dominates 'Pizza'. After deleting 'Pizza', the only remaining choices are (Quiche, Not Eat).

(Alternatively, one can apply IDDS to the above payoff matrix, the deletion order may be different but the result will be the same.)