

## Section B

This section is worth half of the exam points and consists of two equally weighted questions (that is, each question is worth quarter of the points). Your answer will be assessed on two criteria: knowledge of tools, and accuracy and rigour of analysis.

*Question 2.* Penny Pockerpitt is an expected utility maximizer who has a preference for more treats. In other words, her Bernoulli utility function  $u(x)$  is strictly increasing in the amount of treats  $x$  she gets. Suppose Penny faces the following situation: A fair coin (a penny) will be flipped. If the outcome is heads, then Penny will receive 2 treats. If the outcome is tails, then the coin will be flipped again. If the outcome of this second flip is heads, Penny will get 1 treat, and if it is tails, she will get 3 treats. Denote this lottery by  $L$ .

- (i) Define formally the lottery  $L$  and its set of outcomes  $C$ . Find the expectation and variance of  $L$ .
- (ii) Suppose that Penny is **risk-neutral**. What are Penny's certainty equivalent and risk premium associated with the lottery  $L$ ?
- (iii) Consider a lottery  $L'$  under which Penny will receive 1 treat with probability  $\frac{1}{4}$ , 2 treats with probability  $\frac{1}{4}$  or 3 treats with probability  $\frac{1}{2}$ . Does Penny prefer lottery  $L$  or lottery  $L'$ ? Would that preference between  $L$  and  $L'$  change if Penny were risk-averse instead of risk-neutral?
- (iv) We say that lottery  $L'$  weakly dominates lottery  $L''$  if, for every amount of treats  $x$ , the probability of getting at least  $x$  is (weakly) higher under  $L'$  than under  $L''$  (this is known as *first-order stochastic dominance*).  
Consider a lottery  $\hat{L} = (p, q, 1 - p - q)$ . According to this lottery, Penny receives 1 treat with probability  $p$ , 2 treats with probability  $q$ , or 3 treats with probability  $1 - p - q$ . What is the condition on  $p$  and  $q$  so that lottery  $\hat{L}$  weakly dominates lottery  $L$ ?
- (v) Provide an example of a lottery that neither weakly dominates  $L$  nor is weakly dominated by  $L$ , and show why this is an appropriate example.
- (vi) Draw lottery  $L$  in the 2-dimensional simplex. Label the corners of the triangle. Draw Penny's indifference curve that passes through lottery  $L$ , assuming that Penny is risk-neutral.
- (vii) How is risk preference reflected in the shape of the indifference curve in the drawing of part (vi)? How would the indifference curve change if Penny was risk-averse? Provide a brief explanation to your answers.

*Question 3.* Consider the following normal form game played between Bonnie and Clyde:

		Clyde	
		<i>a</i>	<i>b</i>
Bonnie	<i>a</i>	2,4	1,1
	<i>b</i>	1,1	4,2

- (i) What are the best response correspondences of the players? Draw the best response correspondences in the same graph. Put the probability of Bonnie playing *a* on the horizontal axis and the probability of Clyde playing *a* on the vertical axis.
- (ii) Find **all** Nash equilibria of this game.

Suppose a third action is added, giving the following game *G*:

		Clyde		
		<i>a</i>	<i>b</i>	<i>c</i>
Bonnie	<i>a</i>	2,4	1,1	0,0
	<i>b</i>	1,1	4,2	0,0
	<i>c</i>	0,6	0,0	5,5

Consider the repeated game  $G(\delta)$ , where the game *G* is played infinitely many times, and both players have the same discount factor  $\delta \in (0, 1)$ .

- (iii) Find the minmax payoffs and minmax strategies for both players in the stage game *G*, restricting attention to pure strategies. Draw the set of all possible payoffs for the players in the stage game. Identify which payoffs are feasible SPE payoffs of  $G(\delta)$  for high enough  $\delta$  (Fudenberg and Maskin's Folk Theorem).
- (iv) Construct a SPE of the repeated game  $G(\delta)$  such that  $(c, c)$  is played in every period, provided  $\delta$  is high enough. [NOTE: You need to construct a strategy profile which specifies the play after each possible history and check that there are no profitable deviations both on and off the equilibrium path.]

For what values of  $\delta$  the constructed strategy profile is indeed a SPE of  $G(\delta)$ ?

END OF PAPER