# Section B

*Question 1.* Penny Pockerpitt is an expected utility maximizer who has a preference for more treats. In other words, her Bernoulli utility function $u(x)$ is strictly increasing in the amount of treats $x$ she gets. Suppose Penny faces the following situation: A fair coin (a penny) will be flipped. If the outcome is heads, then Penny will receive 2 treats. If the outcome is tails, then the coin will be flipped again. If the outcome of this second flip is heads, Penny will get 1 treat, and if it is tails, she will get 3 treats. Denote this lottery by $L$.

(i) Define formally the lottery $L$ and its set of outcomes $C$. Find the expectation and variance of $L$.

$C = \{1, 2, 3\}$, $L = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$.

$$E(L) = \frac{1}{4} \cdot 1 + \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 3 = 2$$

$$Var(L) = \frac{1}{4} \cdot (1-2)^2 + \frac{1}{2} \cdot (2-2)^2 + \frac{1}{4} \cdot (3-2)^2 = \frac{1}{2}$$

(ii) Suppose that Penny is **risk-neutral**. What are Penny's certainty equivalent and risk premium associated with the lottery $L$?

The certainty equivalent for the lottery $L$ is given as $c(L, u) = 2$. The risk premium is the difference between the expectation of $L$ and the certainty equivalent $c(L, u)$: $rp(L, u) = E(L) - c(L, u) = 2 - 2 = 0$.

(iii) Consider a lottery $L'$ under which Penny will receive 1 treat with probability $\frac{1}{4}$, 2 treats with probability $\frac{1}{4}$ or 3 treats with probability $\frac{1}{2}$. Does Penny prefer lottery $L$ or lottery $L'$? Would that preference between $L$ and $L'$ change if Penny were risk-averse instead of risk-neutral?

$L'$ is preferred to $L$ if

$$\frac{1}{4}u(1) + \frac{1}{2}u(2) + \frac{1}{4}u(3) \leq \frac{1}{4}u(1) + \frac{1}{4}u(2) + \frac{1}{2}u(3),$$

which simplifies to $\frac{1}{4}(u(2) - u(3) \leq 0$. This always holds as long as $u(x)$ is increasing, irrespective of risk preferences.

(iv) We say that lottery $L'$ weakly dominates lottery $L''$ if, for every amount of treats $x$, the probability of getting at least $x$ is (weakly) higher under $L'$ than under $L''$ (this is known as *first-order stochastic dominance*).

Consider a lottery $\hat{L} = (p, q, 1-p-q)$. According to this lottery, Penny receives 1 treat with probability $p$, 2 treats with probability $q$, or 3 treats with probability $1-p-q$. What is the condition on $p$ and $q$ so that lottery $\hat{L}$ weakly dominates lottery $L$?

The set of lotteries that weakly dominate $L$ is given by all lotteries for which $q+(1-p-q)\geq\frac{3}{4}$ and $(1-p-q)\geq\frac{1}{4}$, which is equivalent to $p\leq\frac{1}{4}$ and $p+q\leq\frac{3}{4}$.

(v) Provide an example of a lottery that neither weakly dominates $L$ nor is weakly dominated by $L$, and show why this is an appropriate example.

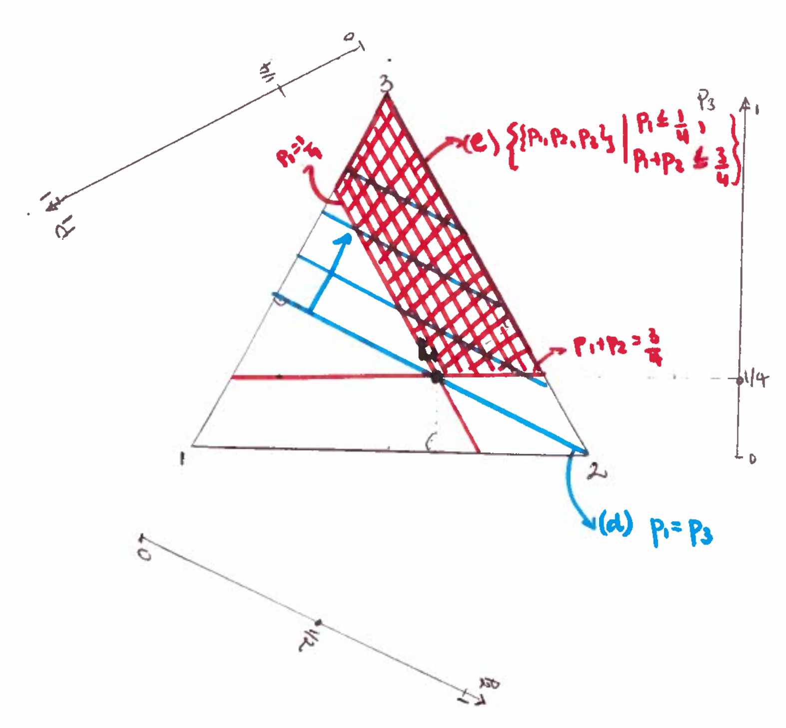
E.g., $\hat{L}=(p,q,1-p-q)=(1/2,0,1/2)$. Here $1-p-q=1/2>1/4$ but $q+(1-p-q)=1/2<1/2+1/4$. So $\hat{L}$ and $L$ are not comparable.

(vi) Draw lottery $L$ in the 2-dimensional simplex. Label the corners of the triangle. Draw Penny's indifference curve that passes through lottery $L$, assuming that Penny is risk-neutral.

See the drawing below.

(vii) How is risk preference reflected in the shape of the indifference curve in the drawing of part (vi)? How would the indifference curve change if Penny was risk-averse? Provide a brief explanation to your answers.

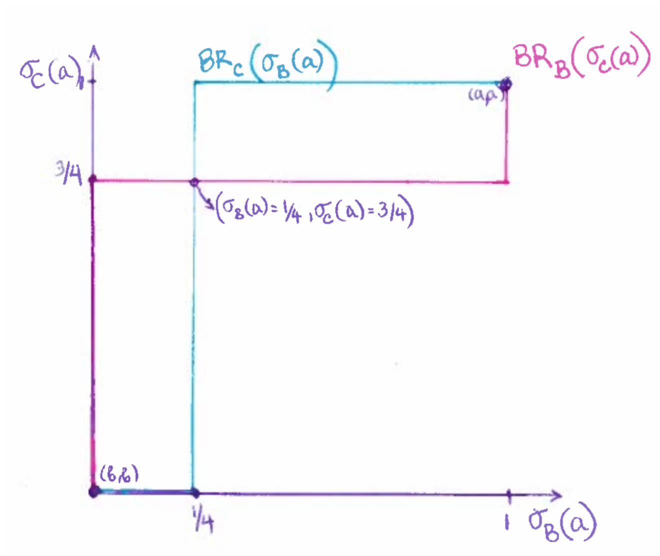
The indifference curves are straight lines for any risk preference. This is because the expected utilities are linear in the probabilities of different outcomes. For example, let $u(x)=\sqrt{x}$. Then the values $1,2,3$ will change into different values $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, but nothing else changes. However, risk preference may affect the slope of these indifference lines. For example, for $u(x)=x$ lottery $L$ is as good as the lottery $(0,1,0)$ that yields 2 with certainty, but for $u(x)=\sqrt{x}$ lottery $(0,1,0)$ would be strictly preferred to $L$.




*Question 2.* Consider the following normal form game played between Bonnie and Clyde:

| | | Clyde | |
|---|---|---|---|
| | | $a$ | $b$ |
| Bonnie | $a$ | 2,4 | 1,1 |
| | $b$ | 1,1 | 4,2 |

(i) What are the best response correspondences of the players? Draw the best response correspondences in the same graph. Put the probability of Bonnie playing $a$ on the horizontal axis and the probability of Clyde playing $a$ on the vertical axis.




(ii) Find *all* Nash equilibria of this game.
The Nash equilibria are $(a, a)$, $(b, b)$, and $(\sigma_B(a) = \frac{1}{4}, \sigma_C(a) = \frac{3}{4})$.

Suppose a third action is added, giving the following game $G$:

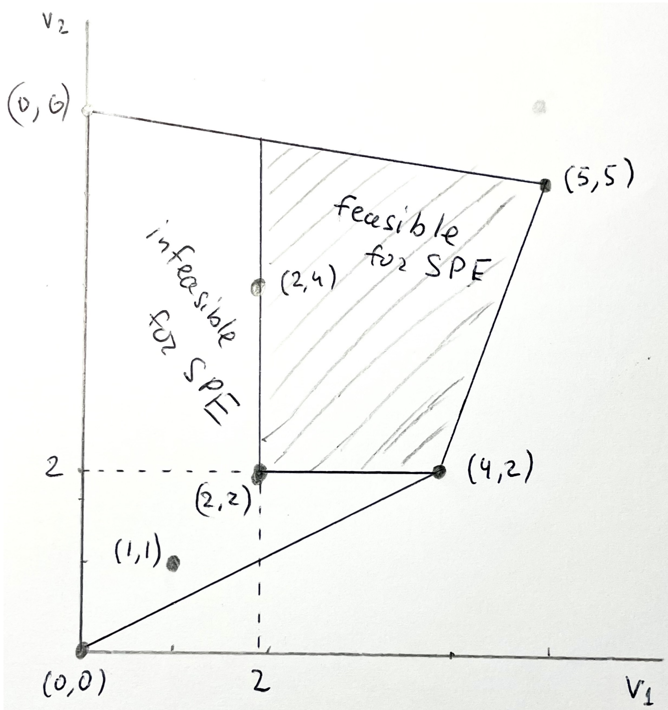
| | | Clyde $a$ | $b$ | $c$ |
|---|---|---|---|---|
| Bonnie | $a$ | 2,4 | 1,1 | 0,0 |
| | $b$ | 1,1 | 4,2 | 0,0 |
| | $c$ | 0,6 | 0,0 | 5,5 |

Consider the repeated game $G(\delta)$, where the game $G$ is played infinitely many times, and both players have the same discount factor $\delta \in (0, 1)$.

(iii) Find the minmax payoffs and minmax strategies for both players in the stage game $G$, restricting attention to pure strategies. Draw the set of all possible payoffs for the players in the stage game. Identify which payoffs are feasible SPE payoffs of $G(\delta)$ for high enough $\delta$ (Fudenberg and Maskin's Folk Theorem).

The minmax values and strategies for both players: $m_1 = a$, $\underline{v}_1 = 2$; $m_2 = b$, $\underline{v}_2 = 2$

See attached drawing.




(iv) Construct a SPE of the repeated game $G(\delta)$ such that $(c, c)$ is played in every period, provided $\delta$ is high enough. [NOTE: You need to construct a strategy profile which specifies the play after each possible history and check that there are no profitable deviations both on and off the equilibrium path.]

For what values of $\delta$ the constructed strategy profile is indeed a SPE of $G(\delta)$?

Consider the following strategy for each player $i \in \{B, C\}$ :
1) in period 1 Player $i$ plays $c$;
2) in every period after period 1, Player $i$ plays $c$ if $(c, c)$ has been played in all previous periods; and
3) play $b$ forever otherwise *[could also be a; or a for Bonnie and b for Clyde]*.

If this strategy profile constitutes an equilibrium, then $(c, c)$ will be played in each period, which is what we want. In this case, each player gets a payoff of 5 each period, i.e., $\frac{5}{1-\delta}$. After a deviation, both players will play $(b, b)$ forever, which gives Clyde his minmax payoff.

The conditions for no profitable deviations on the equilibrium path are:
—for Player B, on the equilibrium path, following the above strategy gives a payoff of 5 in every period, i.e. $\frac{5}{1-\delta}$. A one-shot deviation and then playing the strategy subsequently gives a payoff of at most $0 + \delta\frac{4}{1-\delta}$. Deviating is

not profitable for Player B if

$$\frac{5}{1-\delta} \geq 0 + \delta\frac{4}{1-\delta}$$

which holds for any $\delta \in (0, 1)$.

—for Player C, on the equilibrium path, following the above strategy gives a payoff of 5 in every period, i.e. $\frac{5}{1-\delta}$. The most profitable one-shot deviation is to $a$ and then playing the strategy subsequently gives a payoff of $6+\delta\frac{2}{1-\delta}$. Deviating is not profitable for Player C if

$$\frac{5}{1-\delta} \geq 6 + \delta\frac{2}{1-\delta} \Leftrightarrow \delta \geq \frac{1}{4}.$$

Since we need to ensure this is a SPE, we also need to look at profitable deviations off the equilibrium path. In any history after any deviation we have the play $(b, b)$, which is a NE of the stage game, so there are no profitable deviations.