Section B

This section is worth half of the exam points and consists of two equally weighted questions (that is, each question is worth quarter of the points). Your answer will be assessed on three criteria: model formulation, knowledge of tools, and accuracy and rigour of analysis.

Question 1. Sonja is an expected utility maximizer whose Bernoulli utility function is given by $u(x) = x^2$. She has wealth $w > 0$, but with probability $\pi > 0$ she suffers a loss of $D > 0$ pounds. She can buy insurance. One unit of insurance pays off 1 pound in case of loss and costs p pounds ($p > 0$), so if she buys a units of insurance, she pays ap pounds. She cannot buy negative insurance, nor can she spend more than w on insurance. She can afford full insurance if she wanted to at any $p \in [0, 1]$ and she will still have some money left over, as she has had a very successful career.

(a) Determine the risk attitude of Sonja (i.e., is she strictly/weakly risk-averse, risk-loving or risk-neutral)?

(b) Set up Sonja's expected utility maximization problem. Write out the associated Lagrangian, and first-order (Kuhn-Tucker) conditions.

(c) Show that if $p = \pi$, then there is a solution to the first-order conditions where the amount of insurance that Sonja purchases is equal to D (that is, full insurance).

(d) Show, however, that if $p = \pi$, then full insurance does NOT maximize Sonja's expected utility. (HINT: Demonstrate that at $a = D$, the value of the Lagrangian increases when a increases.)

(e) Explain in words why Sonja prefers being over-insured ($a > D$) to having full insurance ($a = D$) when $p = \pi$.

Question 2. The country of Johnlandia is deciding how and whether to attack the country of Philipine. Johnlandia has three choices: it can 1) send a well-equipped invasion force (denoted by W), 2) send a poorly-equipped invasion force (denoted by P), or 3) stay at home (denoted by H). A well-equipped invasion force is more expensive than a poorly equipped one, but it is also more likely to win in battle. If Johnlandia chooses to invade, then Philipine has two choices: it can either fight (denoted by F) or surrender (denoted by S). Crucially, when making its decision, Philipine cannot tell whether Johnlandia sent a well-equipped or a poorly-equipped invasion force. That fact is common knowledge, as are the following payoffs:

> If Johnlandia stays home, its payoff is 900 and Philipine's is 600.
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> If Johnlandia sends a well-equipped force and Philipine fights, payoffs are 400 and –600 respectively.
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> If Johnlandia sends a poorly-equipped force and Philipine fights, payoffs are –600 and 600 respectively.
>
> If Johnlandia sends a well-equipped force and Philipine surrenders, payoffs are 1000 and 0 respectively.
>
> If Johnlandia sends a poorly-equipped force and Philipine surrenders, payoffs are 1200 and 0 respectively.

(a) Draw the extensive form of this game. What are the sets of pure strategies for each of the two countries? Can backwards induction be used to solve for the subgame perfect equilibria of this game? Explain briefly.

(d) Find all pure strategy subgame perfect equilibria. Explain briefly.

(e) Find all pure strategy perfect Bayesian equilibria.

(f) Find a perfect Bayesian equilibrium in which Johnlandia stays at home with probability one and Philipine randomizes non-trivially (i.e. puts strictly positive probabilities on more than one action).

(g) Find a perfect Bayesian equilibrium in which both Johnlandia and Philipine randomize with positive probability on all the actions in their respective action sets (that is, the supports of their mixed strategies are equal to their respective action sets).