Game Theory

Example

~~1/2~~ ~~1/2~~ Rain God

Player 1

	Sun	Rain
No Umbrella	5	0
Umbrella	1	3

Payoffs of player 1

beliefs

- structural uncertainty (Nature laws of physics)
- strategic — . — (endogenous, human-made)

Normal-form game

(strategic-form, simultaneous-move, matrix-form)

consists of 3 elements:

1) Set of players $\{1, 2, \ldots, N\}$

2) Set of actions A_i for each player i

$A = A_1 \times A_2 \times \ldots \times A_N$ ~ set of action profiles

$a = (a_1, a_2, \ldots, a_N) \in A$ ~ action profile (or a play)

3) Payoffs: $u_i : A \to \mathbb{R}$ for each $i = 1, 2, \ldots, N$

$u_i(a)$ = utility of player i when the play is $a = (a_1, \ldots, a_N)$

For today, $N = 2$

action = strategy

Dominance

Action a **strictly dominates** action a'
if $u(a,b) > u(a',b^*)$ for all b

Action a **weakly dominates** action a'
if 1) $u(a,b) \geq u(a',b)$ for all b
2) $u(a,b') > u(a',b')$ for some b'

Action a is **dominant** if it strictly dominates all other actions

Action a is **weakly dominant** if it weakly dominates all other actions.

Action a is **undominated** (admissible) if it is not weakly dominated by any other action.

Examples

		Player 2	
		C	D
Player 1	C	-1	-4
	D	0	-3

← payoffs of player 1

D strictly dominates C
D is undominated
D is dominant

		Pl.2	
		C	D
Pl.1	C	0	-4
	D	0	-3

D weakly dominates C
D is undominated
D is weakly dominant

Observations

A rational player:

→ will never play a strictly dominated action
→ will always play the dominant action (if there is one)
→ might play a weakly dominated action.

Belief Rationality and Best Response

Beliefs Player 1's beliefs about Player 2's actions a given by a probability distribution

$$p = [\, p(b_1), p(b_2), \dots, p(b_K) \,]$$

where $p(b_k)$ is the probability that player 2 chooses action b_k, $k = 1, 2, \dots, K$.

Expected payoff of player 1 from playing action a, given belief p is

$$u(a, p) = \sum_{k=1}^{K} p(b_k)\, u(a, b_k)$$

If player 2 plays action b_k with prob 1, so $p(b_k) = 1$, $p(b_j) = 0$ for all $j \neq k$,

then $u(a, p) = u(a, b_k)$.

Observation A rational player will always choose an action that maximizes $u(a, p)$ given belief p.

Def Action a is a best-response to belief p if $u(a,p) \geq u(a',p)$ for every a'.

Def The best-response correspondence $BR(p)$ is the set of best-response actions to belief p,

$$BR(p) = \operatorname*{argmax}_{a \in A} u(a,p).$$

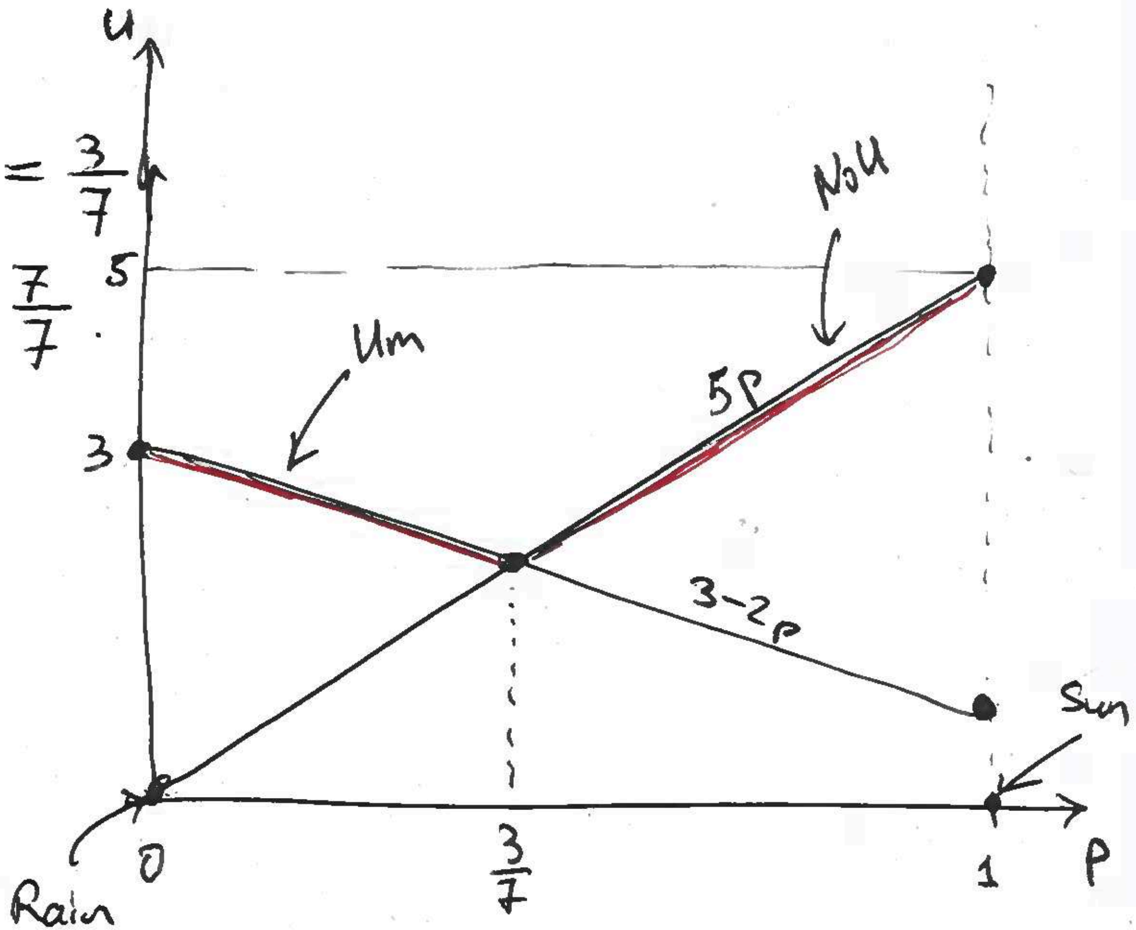
Rain-Sun Game

$$u(NoU,\ p) = 5\cdot p + 0\cdot(1-p) = 5p$$
$$u(Um,\ p) = 1\cdot p + 3(1-p) = 3-2p$$

Pl. 1

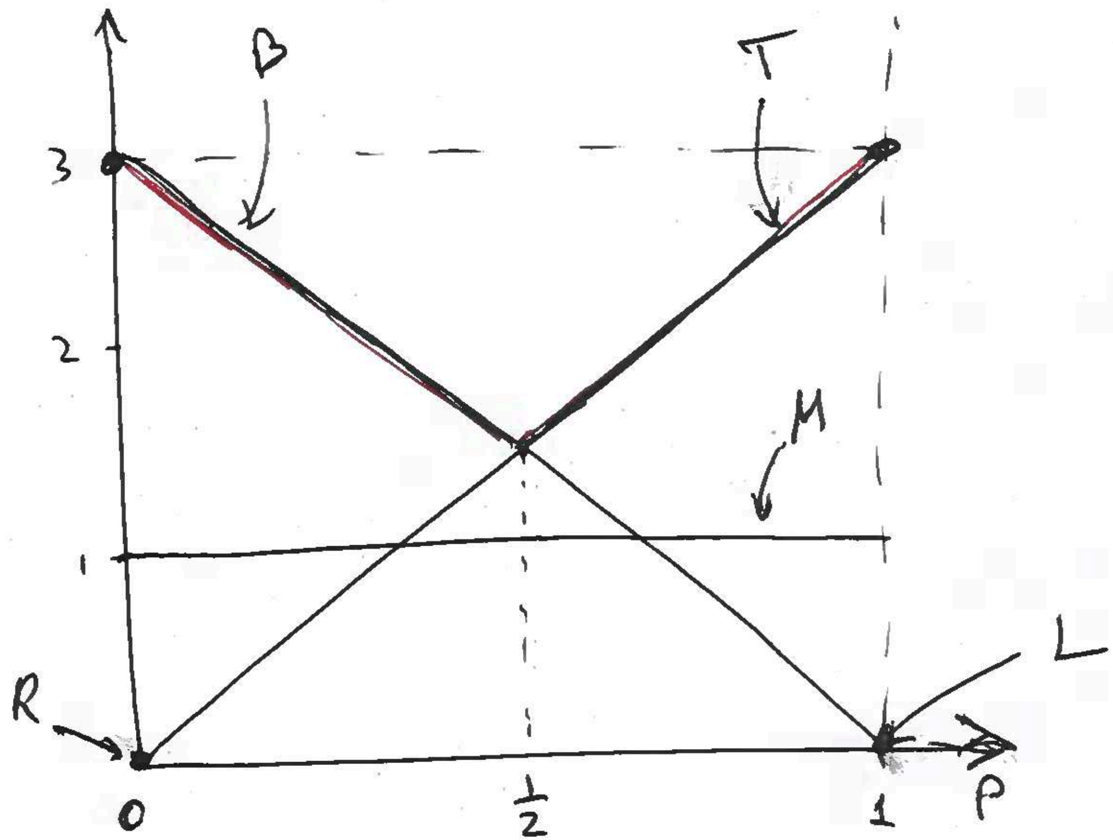
		Rain God	
		Sun	Rain
Pl. 1	No umb.	5	0
	Umb.	1	3
		p	$1-p$

$$BR(p) = \begin{cases} Um, & \text{if } p < \frac{3}{7} \\ \{Um, NoU\}, & \text{if } p = \frac{3}{7} \\ NoU, & \text{if } p > \frac{7}{7}. \end{cases}$$



Example

		L	R
Player 1	T	3	0
	M	1	1
	B	0	3
		p	$1-p$



$$BR(p) = \begin{cases} B, & \text{if } p < \frac{1}{2} \\ \{B, T\}, & \text{if } p = \frac{1}{2} \\ T, & \text{if } p > \frac{1}{2} \end{cases}$$

M is never a best response

__Def__ Action a is __never a best response__ (NBR) if it is not a best response to any belief $p \in \Delta(A_2)$, that is

$a \notin BR(p)$ for all $p \in \Delta(A_2)$

→ A rational player will never choose a NBR action.

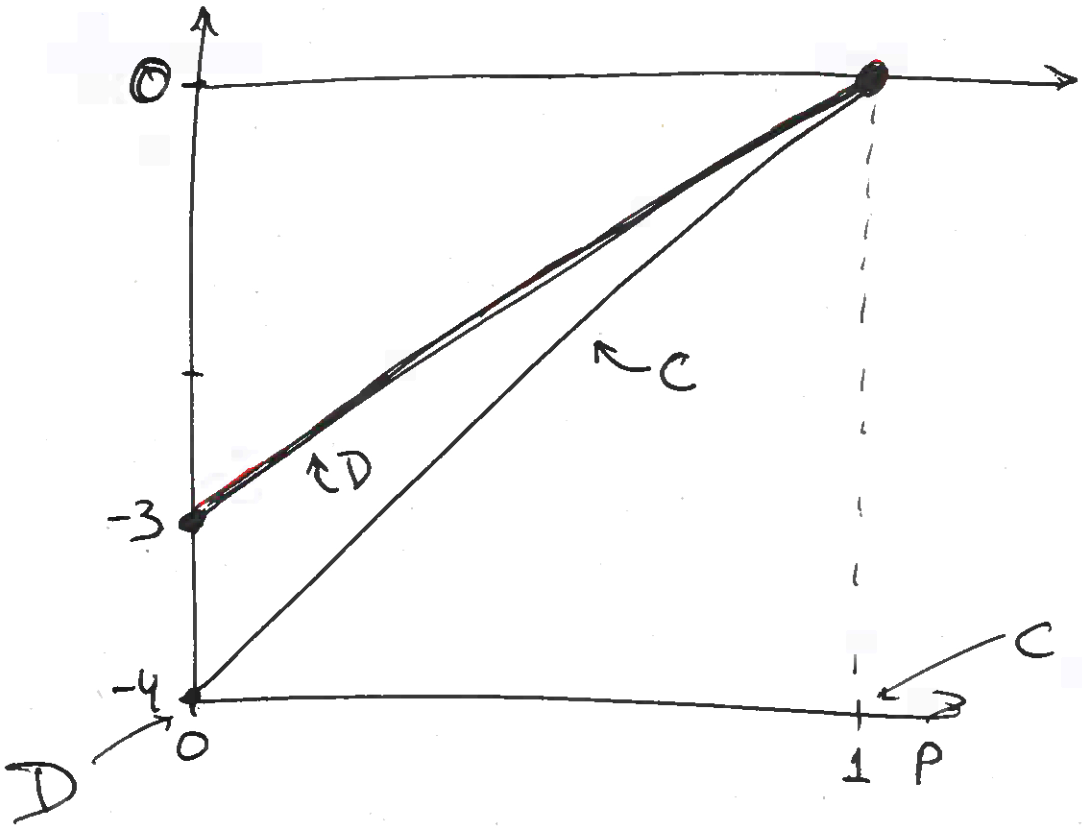
→ A strictly dominated action is NBR

→ A NBR action may or may not be dominated

→ A weakly dominated action may or may not be a NBR

		Pl. 2	
		C	D
Pl. 1	C	0	-4
	D	0	-3
		p	1-p

$$BR(p) = \begin{cases} D, & \text{if } p<1 \\ \{C, D\}, & p=1 \end{cases}$$



C is weakly dominated (by D) but C is a best response when $p=1$.