Section B

Question 1. Brendan is an expected utility maximizer with the following Bernoulli utility function:

$$u(x) = \sqrt{x}.$$

Brendan holds a portfolio of stocks of local companies. The local economy can go either up or down. If the economy goes up, then the portfolio will be worth 2000. If the economy goes down, these stocks will be worthless, so the portfolio will be worth zero. The probability that the local economy will go bad is 0.2. Brendan also has some safe assets which are worth W no matter what happens.

(a) Calculate the certainty equivalent of Brendan's current holdings as a function of W. How would this certainty equivalent change if Brendan's Bernoulli utility function were linear in his wealth (instead of $\sqrt{x}$)?

Suppose now that Brendan can buy insurance. One unit of insurance costs p pounds, and pays 1 pound in the event that the stocks become worthless. Brendan can buy any nonnegative number of units of insurance, but he cannot spend more than his wealth W on insurance. Use letter q to denote the number of insurance units bought by Brendan, and assume $W > 1000$.

(b) Set up Brendan's expected utility maximization problem and write out the Kuhn-Tucker conditions that characterize the optimal number q^* of insurance units he should buy.

Consider for the rest of the problem the case of actuarially fair insurance.

(c) How is the price p of a unit of insurance defined for the case of actuarially fair insurance? What are the profits of the insurance company in this case?

(d) How many units of insurance will Brendan buy (that is, what is the value of q^*)? *Hint: Use the Kuhn-Tucker conditions to show that* $q^* \in (0, \frac{W}{p})$, *i.e. the solution is interior, and then to calculate* q^*.

Question 2. Consider the following extensive form game between two co-workers named Ken and Barbie.

Stage 1. Barbie (Player 1) decides whether to start a project or not. If not, then the game ends with both players getting 0. Otherwise, move to stage 2.

Stage 2. Ken (Player 2) decides to contribute to the project or to free ride. If Ken contributes, then the game ends with both players getting 1. Otherwise, move to stage 3.

Stage 3. Barbie decides whether to file a complaint to the line manager about Ken's lack of effort. If she does not complain, then the game ends with the payoffs -1 for Barbie and 2 for Ken. If she does complain, then there is a small probability $\varepsilon = 0.01$ that the line manager is prejudged against Barbie. If the line manager is fair, then Ken will be punished for his lack of effort, and the payoffs will be 2 for Barbie and -2 for Ken. However, if the line manager is prejudged, then Ken will be rewarded and Barbie punished, and the payoffs will be -2 for Barbie and 2 for Ken.

Consider two versions of this game.

In **version A**, both players have the same uncertainty about whether or not the line manager is prejudged.

In **version B**, only Barbie is uncertain about the line manager's attitude, whereas Ken knows exactly whether or not the line manager is prejudged.

(a) Depict versions A and B of this extensive-form game graphically, using tree diagrams. In what way are these diagrams different?

(b) Can backwards induction can be used to solve version A? Explain briefly. Apply an appropriate equilibrium concept and find an equilibrium in pure strategies for version A. Please clearly describe the strategies (and beliefs, if needed) of the players, and argue why this is an equilibrium.

(c) Repeat exercise (b) for version B of the game.

(d) Draw economic conclusions about how uncertainty and asymmetric information of coworkers may influence incentives for teamwork.