Section B

2. [25 marks]

Two tech companies, TechCorp and Innovate Inc. (for short, denoted by T and I, respectively), are deciding whether to launch a new product in the same market. Both companies face high development costs of launch. If both companies launch their products simultaneously, the market will become saturated, leading to lower profits for both. However, if only one company launches, that company could dominate the market and earn higher profits. The companies must decide simultaneously whether to launch or not launch.

The payoffs of the companies are as follows:

- If both companies launch, they each make loss of £$2M$ due to high cost of launching and low revenues in the competitive market.
- If only one company launches, the launching company earns £$5M$ (monopoly profit), while the other company earns 0.
- If neither company launches, they both earn 0.

(a) Represent this game using a payoff matrix, where Row Player is TechCorp and Column Player is Innovate Inc.

Solution

	L	NL
L	$-2, -2$	$5, 0$
NL	$0, 5$	$0, 0$

(b) Find the Nash equilibrium or equilibria of the game.

Solution

Pure NE: (L,NL) and (NL,L)

Mixed NE: Solve $(-2)p + 5(1-p) = 0$, so $p = 5/7$. MSNE: $((\frac{5}{7}, \frac{2}{7}), (\frac{5}{7}, \frac{2}{7}))$.

(c) Is there a dominant strategy for either player? Explain.

Solution

No. When the other player chooses L, the best response is NL, and vice versa.

(d) Suppose the companies can communicate and exchange payments before making launch decisions. How might this affect the outcome of the game? Explain.

Solution

The firms would coordinate on one of the pure NE. The monopolist may make a payment to the other firm to compensate them for not entering the market, for example, by paying $t = 2.5$, which would lead to equal payoff of $(2.5, 2.5)$ in equilibrium.

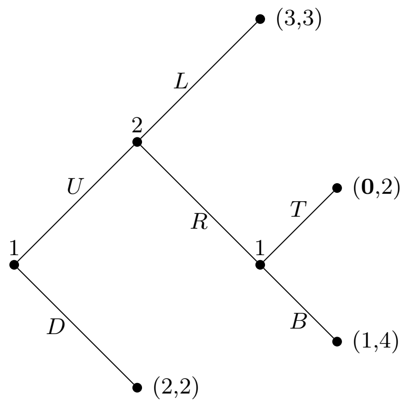
(e) How would the game change if the companies could launch the product at different times rather than simultaneously? What is an appropriate solution concept and why? What is the solution?

Solution

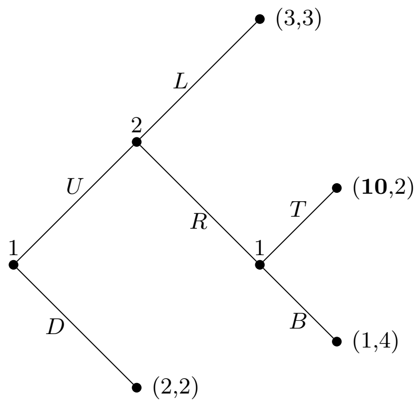
Suppose T launches first. Then I would stay out. Anticipating that, T should launch. This is the unique SPE by backward induction. As this is a sequential game of complete info, the appropriate concept is BI or SPE.

3. **[25 marks]**

Consider the following extensive form games (note that there is only one difference between these games, which is in the payoffs, and is highlighted in **bold**):



Game F



Game S

(a) Can backwards induction be used to solve for subgame perfect equilibria of each of these games? Explain briefly.

Solution

Yes, we can solve for the subgame perfect equilibria using backwards induction as these are games of perfect information.

Now, suppose that Game F is played with probability $1 - \varepsilon$ and Game S is played probability ε. This can be represented as Nature moving first and randomly choosing Game F with probability $1 - \varepsilon$ and Game S with probability ε. (Remember that Nature is a non-strategic player!) Assume that $0.1 < \varepsilon < 0.5$.

(b) Assume that neither Player 1 nor Player 2 observe Nature's choice. Find all subgame perfect equilibria of this new game.

Solution

Since neither player knows the game, they are symmetrically informed. So we can apply backward induction w.r.t. the players' expected payoffs. Calculating the expected payoff after path U, R, T, we obtain 10ε. By backward induction, we obtain the unique SPE $((U, T), L)$.

For the next two subquestions, assume that Player 1 observes the initial choice by Nature, but Player 2 does not.

(c) Argue that there does not exist a perfect Bayesian equilibrium in which Player 1 chooses U in both Game F and Game S.

Solution

Assuming Player 1 plays U both after observing F or S, Player 2's beliefs are given by the prior: $(1-\varepsilon, \varepsilon)$. Given these beliefs, and given that Player 1 will always choose B at in Game F and T at Game S, at any PBE, the expected payoffs of Player 2 from playing L or R at information are:
$EP_2(L) = 3$
$EP_2(R) = 4(1-\varepsilon) + 2\varepsilon = 4 - 2\varepsilon$
Since $\varepsilon < 0.5$, $EP_2(R) > EP_2(L)$ and so Player 2 will always choose to play R. But then, in Game F, Player 1 would rather choose D instead of U. Thus, choosing U in both games cannot occur in PBE.

(d) Find a perfect Bayesian equilibrium in which Player 1 mixes between U and D after observing that Nature has chosen Game F, and otherwise plays a pure strategy.

Solution

First note that in Game S, Player 1 will always play U (as it strictly dominates playing D) and then T.

Let p be the probability that Player 1 plays U in Game F.

Let q be the probability that Player 2 plays L.

To randomize between U and D in Game F, player 1 must be indifferent:
$EP_1(U|GameF) = 3q + 1(1-q)$
$EP_1(D|GameF) = 2$
Solving $3q + 1(1-q) = 2$, we obtain $q = 1/2$.

To randomize between L and R, player 2 must be indifferent. By Bayes rule, the beliefs of Player 2 are $(\mu, 1-\mu)$, where

$$\mu = \frac{(1-\varepsilon)p}{(1-\varepsilon)p + \varepsilon}.$$

The expected payoffs from playing L or R then are:
$EP_2(L) = 3$
$EP_2(R) = 4\frac{(1-\varepsilon)p}{(1-\varepsilon)p+\varepsilon} + 2\frac{\varepsilon}{(1-\varepsilon)p+\varepsilon} = 2 + 2\frac{(1-\varepsilon)p}{(1-\varepsilon)p+\varepsilon}$
Solving for p, we obtain $p = \frac{\varepsilon}{1-\varepsilon}$. [As a check, notice that $p = \frac{\varepsilon}{1-\varepsilon} \in (0, 1)$ since $\varepsilon < 0.5$, so p is a valid probability.]

Therefore, the following is a PBE:

- in Game F, Player 1 plays $((p, 1-p), B)$ (that is, U and D with probabilities p and $1-p$, respectively, and then B if that decision node is reached), where $p = \frac{\varepsilon}{1-\varepsilon}$

- in Game S, Player 1 plays (U, T)

- Player 2 plays $(1/2, 1/2)$.

- Player 2 beliefs that they are in Game F with probability μ and in Game S with probability $1-\mu$, where $\mu = \frac{(1-\varepsilon)p}{(1-\varepsilon)p+\varepsilon}$.