THE UNIVERSITY *of* EDINBURGH

Microeconomics 1
ECNM11023

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Number of questions: 3
Total number of marks: 100

IMPORTANT PLEASE READ CAREFULLY

Before the examination

1. Put your university card face up on the desk.
2. **Complete PART A and PART B above**. By completing PART B you are accepting the University Regulations on student conduct in an examination (see back cover).
3. Complete the attendance slip and leave it on the desk.
4. This is a closed-book examination. No notes, printed matter or books are allowed.
5. A calculator is permitted in this examination. It must not be a programmable or graphic calculator. It must not be able to communicate with any other device.

During the examination

1. Write clearly, in ink, in the space provided after each question. If you need more space then please ask an invigilator for additional paper.
2. **This paper has 2 sections. Section A (worth 50%) has 1 question. Section B (worth 50%) has 2 equally weighted questions. Answer all questions**.
3. If you have rough work to do, simply include it within your overall answer – put brackets at the start and end of it if you want to highlight that it is rough work.

At the end of the examination

1. This examination script must not be removed from the examination venue.
2. Additional paper and graph paper, if used, should be attached to the back of this examination script. Write your examination number on the top of each additional sheet.

Examiners: Prof Jonathan Thomas (Chair), Prof Edmund Cannon (External)

Section A

1. [50 marks]

Suppose each household consists of a mother and a daughter, who have different preferences from each other. Each household chooses how much land to buy for their home, and how much each member works and eats. Households are endowed with land, and a single farm makes food from labour and land.

(a) Write down a competitive model of the land, labour and food markets.

(b) Reformulate the household's problem using a Bellman equation, and a value function for each household member.

(c) Prove that if the price of land increases, then the farm's demand for land decreases.

(d) Suppose that at some non-equilibrium prices, there is unemployment and vacant land. Prove that this implies there is a famine (food shortage).

(e) Is every competitive equilibirum efficient, when considering welfare from the point of view of each person (not household)?

Solution

Full sample solutions for Section A are provided separately

Section B

2. [25 marks]

Taiwei is a decision maker whose preferences are represented by a von Neumann-Morgenstern utility function. Taiwei considers investing in a startup company. There are three possible scenarios. (i) The company goes bankrupt, in which case Taiwei's investment is worthless. (ii) The company shows mediocre performance, in which case the investment is worth \$10M. (iii) The company shows stellar performance, in which case the investment is worth \$36M. Suppose that the probabilities of these scenarios are 1/3, 1/2, and 1/6, respectively.

Taiwei's friend Aera advises Taiwei, instead of investing in the startup, to deposit money in the bank, in which case the worth of the investment will be \$10M with certainty.

Denote by L_S the choice to invest in the startup and by L_B the choice to deposit in bank.

(a) Find and compare the expected values of L_S and L_B.

Solution

$E[L_S] = 0 \cdot \frac{1}{3} + 10 \cdot \frac{1}{2} + 36 \cdot \frac{1}{6} = 11$ and $E[L_B] = 10$.

(b) Using notation $u_T(x)$ for Taiwei's Bernoulli utility function, write down Taiwei's problem of choice between L_S and L_B according to the expected utility theory.

Solution

$$E[U(L_S)] = u_T(0) \cdot \frac{1}{3} + u_T(10) \cdot \frac{1}{2} + u_T(36) \cdot \frac{1}{6}$$
$$E[U(L_B)] = u_T(10).$$

Choose the lottery L in $\{L_B, L_S\}$ that maximizes $E[U(L)]$.

(c) Assume that Taiwei is risk-loving (i.e., $u_T(x)$ is convex). Can you tell whether Taiwei will follow Aera's advice? Would Taiwei prefer L_S or L_B, or is there not enough information to decide? Explain briefly.

Solution

Taiwei would prefer L_S. By choosing L_S over L_B, he gets more of both: the expected value and risk (which he likes).

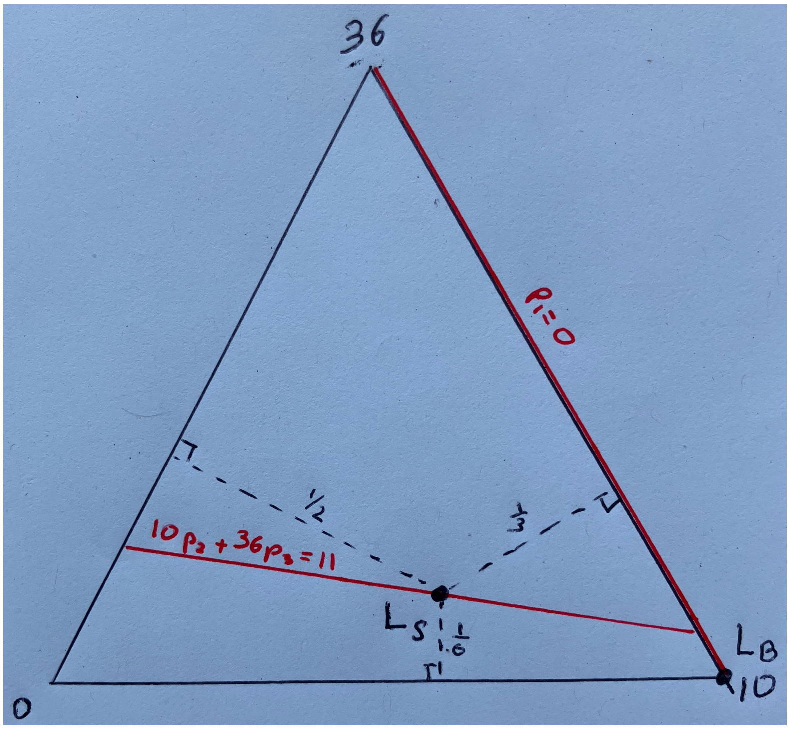
(d) Continue assuming that Taiwei is risk-loving. What is Taiwei's certainty equivalent of L_S? Is it greater or less than the expectation of L_S, and why?

Solution

$CE(L_S)$ is defined as the solution x of the equation $u_T(x) = E[U(L_S)]$. It is greater than $E[L_S] = 11$, because risk-loving individuals have a positive value of risk. (A complete answer should invoke either the definition of risk loving or the convexity of u_T).

(e) Draw L_S and L_B in the 2-dimensional simplex. Carefully label the corners of the triangle and the probabilities.

Solution



See the drawing.

(f) Derive the set of all lotteries that have the same expectation as L_S, and depict that set in the 2-dimensional simplex you drew for part (e).

Solution

(p_1, p_2, p_3) such that $p_1 + p_2 + p_3 = 1$ and $10p_2 + 36p_3 = 11$. See the tilted red line in the above drawing.

(g) Lottery L' dominates a lottery L'' if, for every amount of money x, the probability of getting at least x is higher under L' than under L''. Derive the set of all lotteries that dominate L_B and depict that set in the 2-dimensional simplex.

Solution

(p_1, p_2, p_3) such that $p_1 + p_2 + p_3 = 1$, $p_2 + p_3 \geq 1$. So, $p_1 = 0$. See the red line coinciding with the left edge of the triangle in the above drawing. (To be accurate, the dot indicating L_B should be depicted as an unfilled circle, to indicate that L_B does not dominate itself.)

3. [25 marks]

A seller has a good of quality Q, which only she knows. To the buyer, Q takes values 1 and 2 with probabilities $1 - \pi$ and π respectively. A good of quality Q breaks down with probability $1 - Q/2$. A buyer receives a utility of Q if the good does not break down and 0 if it breaks down. Thus, if the buyer purchases the good for price $p \geq 0$, his utility is $Q - p$ if it does not break down, and $-p$ if it does, while the seller's utility is p in both cases. If no trade occurs, then the payoffs of both traders are 0.

Suppose that the seller can sell the good with a warranty. The warranty is characterized in terms of a donation $D \geq 0$. If the good breaks down, the warranty commits the seller to donating D to penguins' flood relief in Antarctica. This donation does not directly affect the buyer's utility, but it does affect the seller's. The seller makes a take-it-or-leave-it offer (p, D). The buyer then accepts or rejects the offer, after which the corresponding payoffs are realized.

(a) What is the expected utility of a seller who offers a good of quality Q for price p and warranty D if the buyer accepts?

Solution

The seller's expected utility is

$$p - \left(1 - \frac{Q}{2}\right) D.$$

(b) What is an appropriate equilibrium concept and why? Describe informally (without calculations) the conditions that an equilibrium of this game satisfies. (Use the notation (p_Q, D_Q) for the contract offered by the seller with good's quality $Q \in \{1, 2\}$.)

Solution

Because this is a game of incomplete information, Perfect Bayesian equilibrium is an appropriate concept. Equilibrium conditions:

(i) The seller of each type $Q = 1, 2$ chooses a contract (p_Q, D_Q) that maximises her expected utility given the buyer's strategy;

(ii) The buyer accepts or rejects an offered contract to maximise his expected utility given his posterior beliefs about the type Q of the seller who offers this contract;

(iii) The buyer's beliefs about the seller's strategy are consistent and satisfy Bayes' rule where possible.

(c) Find a "pooling" equilibrium where the seller offers the same contract, irrespective of the good's quality (i.e., $(p_1, D_1) = (p_2, D_2)$).

Solution

There are multiple pooling equilibria (as many buyer's beliefs would support the same behavior). For example consider the following pooling equilibrium. The buyer's posterior belief is equal to the prior. The buyer accepts a contract (p, D) if

$$\frac{1}{2}(1 - \pi) + 2\pi - p \geq 0.$$

The maximum acceptable price is thus $p^* = \frac{1}{2}(1 - \pi) + 2\pi$. The seller with $Q = 2$ maximizes p; the seller with $Q = 1$ maximizes $p - D/2$. Since $D \geq 0$, the seller's payoffs are maximized by $(p, D) = (\frac{1}{2}(1 - \pi) + 2\pi, 0)$.

PBE: The seller offers $(p^*, D^*) = (\frac{1}{2}(1 - \pi) + 2\pi, 0)$. The buyer believes that $Pr(Q = 2) = \pi$ (irrespective of the seller's contract) and accepts contract (p, D) if and only if $p \leq \frac{1}{2}(1 - \pi) + 2\pi$.

(d) Is there a "separating" equilibrium, where the seller with different good's quality offers different contracts (i.e., $(p_1, D_1) \neq (p_2, D_2)$)? Find such an equilibrium or show that it does not exist.

Solution

In a separating equilibrium, different seller types offer different contracts. There are multiple separating equilibria (as many buyer's beliefs would support the same behavior). For example, consider the following separating equilibrium.

Let (p_2, D_2) be a contract offered by type $Q = 2$. The buyer's posterior belief is 1. The buyer accepts a contract (p, D) if

$$2 - p_2 \geq 0.$$

The maximum acceptable price is thus $p_2 = 2$.

Let the buyer believe that any contract $(p, D) \neq (p_2, D_2)$ is offered by type $Q = 1$. The buyer thus accepts this contract if and only if

$$1/2 - p \geq 0.$$

Type $Q = 1$ maximizes $p_1 - D_1/2$ subject to $p_1 \leq 1/2$ and $D_1 \leq 0$. So

$$(p_1, D_1) = (1/2, 0).$$

Type $Q = 2$ maximizes p_2 subject to $p_2 \leq 2$, $D_2 \geq 0$, and that type $Q = 1$ does not want to use contract (p_2, D_2):

$$p_2 - \frac{1}{2}D_2 \leq p_1 - \frac{1}{2}D_1 = \frac{1}{2}.$$

Thus, $(p_2, D_2) = (2, 3)$.

PBE: The seller type $Q = 1$ offers $(p_1, D_1) = (1/2, 0)$, and type $Q = 2$ offers $(p_2, D_2) = (2, 3)$. When offered $(p_2, D_2) = (2, 3)$, the buyer believes that $Pr(Q = 2) = 1$ and accept the contract. When offered any other contract $(p, D) \neq (p_2, D_2)$, the buyer believes and $Pr(Q = 2) = 0$, and accepts the contract if and only if $p \leq 1/2$.