

TRACTABLE VALUATIONS UNDER UNCERTAINTY*

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November 4, 2014

Abstract

I put forward a concise and intuitive formula for the calculation of the valuation for a good in the presence of the expectation that further, related, goods will soon become available. This valuation is tractable in the sense that it does not require the explicit resolution of the consumer's life-time problem.

JEL classification: D01, D03, D11, D91.

Keywords: distributed choice, quasi-linear utility, value for money.

1 Introduction

The literature on price determination generally treats an agent's valuation of an indivisible object as an exogenous parameter. But where does this value

*I thank Dan Friedman and Kohei Kawamura for their constructive comments.

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come from? Let us make the brave assumption that the agent knows by how much his utility would increase relative to any given *status quo* in case he obtained the object. The question remains: How would a boundedly rational agent incorporate into his valuation the – positive or negative – synergies with other goods, which he might obtain later but consumes together with the object? We resolve that question in this note.

Consider the following scenario: our agent is about to participate in a trading mechanism where he can possibly obtain good A . The difficulty he faces is that there exists a good¹ B , which will become available later² and is not want independent of A : the utility derived from owning A depends on whether or not the agent will own B as well. Of course, the ideal way to resolve this problem would be to make a joint decision on the purchase of A and B . However, it is often the case that this is not possible: at the time of the opportunity to buy A it may well be that the price (or even the availability) of B is not yet known to the agent, and as a result he cannot predict with certainty whether he will end up owning B .

Of course, an – impractical – alternative would be to solve the agent's entire stochastic life-time problem. Barring that, surprisingly, a tractable solution to this basic problem is not known within the standard context of consumer choice. The reason for that is the straight jacket imposed on us by the universally accepted view of the consumer problem (due to Hicks and Allen, 1934), which frames it as utility maximization subject to a budget constraint: even if – and this is a big if – our agent knew his budget for buying A and B , he would only be able to satisfy his budget constraint in

¹To simplify the equations, we consider a single alternative good. The generalization to many different potential baskets is straightforward.

²For simplicity, we do not model uncertainty over the time when B becomes available, and also assume no discounting.

expectation, which would generically lead to an *ex post* suboptimal decision.³

Recently, Friedman and Sákovics (2014) developed an alternative model of *tractable* consumer choice.⁴ It is based on the Marshallian concept of the *marginal utility of money*, λ , defined as the slope of the life-time indirect utility function evaluated at the current wealth (and forgoing the current shopping opportunity). According to them, the pecuniary connection between current and future choices is a trade-off rather than a (budget) constraint, and λ – that can be learned and/or approximated – is the measure of it. Crucially, λ need not be updated in between a series of small purchases. Thus, instead of framing the consumer’s problem as $\max u(\mathbf{x})$ subject to $\mathbf{p} \cdot \mathbf{x} \leq m$, they advocate $\max u(\mathbf{x}) - \lambda \mathbf{p} \cdot \mathbf{x}$. A nice thing about the resulting quasi-linear utility is that it is well suited to handle a probabilistic problem as above.

Before turning to the model with multiple goods, it is worthwhile to ponder the implications of the quasi-linear foundation of valuation for a single item. Recall that the valuation v of an object A is typically defined implicitly by $\bar{u}(A, m - v) = \bar{u}(0, m)$, where m is the money holding and we ignore the additional arguments in the (not fully specified) utility function \bar{u} . By contrast, from the quasi-linear approach it is immediate that the explicit value is $v = (u(A) - u(0))/\lambda$. Also the utility function becomes clearly understood as the utility derived from the basket of goods considered together with A , which are want independent from any other goods: defining a subproblem for which λ can be considered constant.

The above decomposition of v into a utility factor, $u(A) - u(0)$, and a value of money factor, λ , has useful consequences. For example, we can endogenize v without requiring the consumer to change her taste ($u(\cdot)$). In

³See Sákovics (2011) for a model where the budget constraint is “satisfied” for a misperceived, and therefore incorrect, (but fixed) price, also leading to an *ex post* under- or overspend.

⁴For the corresponding theory of revealed preference see Sákovics (2013).

other words, we can make willingness to pay dependent on the circumstances – such as reference prices – maintaining the object’s utility value, and hence the agent’s welfare, unchanged. Previously, in order to introduce welfare neutral distortions, researchers needed to resort to an “as if” approach, where it was counterfactually assumed that it was the perception of prices that was distorted.⁵ Using the demand function developed by Friedman and Sákovics (2014), all we need is a discrete change in λ to achieve the same behavior without interfering with welfare.

2 The main result and its derivation

Let $u(A, B)$ denote the (incremental) utility of obtaining both A and B , $u(A, \bar{B})$ the utility of obtaining A but not B , etc. Let the price of good X be denoted by p_X . Also let $F_B(\cdot)$ denote the cumulative distribution function of the agent’s belief about the price of B . Finally, assume that the marginal utility of money in the continuation following the decisions over A and B , λ , is approximately constant within the range of prices considered.⁶ In sum, $u(\cdot, \cdot)$, p_A , $F_B(\cdot)$ and λ are the only exogenous parameters of the agent’s decision problem.

Denote by $\Pi(B|A)$ the probability the agent assigns to buying B if he also purchases A and by $\Pi(B|\bar{A})$ if he does not. Also, write the expected price of B , conditional on buying both it and A , as $E(p_B|B, A)$. Then – using the quasi-linear set-up of Friedman and Sákovics (2014) described above –

⁵See Sákovics (2011).

⁶In other words, the decisions are over small/inexpensive items. To model decisions over a wider range of prices we could not maintain the quasi-linear approximation and would need to work with a range of λ s.

we can write the agent's expected utility if he buys A for p_A as

$$\Pi(B|A) (u(A, B) - \lambda(p_A + E(p_B|B, A))) + \Pi(\bar{B}|A) (u(A, \bar{B}) - \lambda p_A).$$

Similarly, his expected utility in case he renounces to buy A – normalizing $u(\bar{A}, \bar{B})$ to zero – is

$$\Pi(B|\bar{A}) (u(\bar{A}, B) - \lambda E(p_B|B, \bar{A})).$$

By definition, the agent's valuation for A , v_A , is the price of A at which he is indifferent between the two expected values:

$$v_A = \frac{\Pi(B|A)u(A, B) + \Pi(\bar{B}|A)u(A, \bar{B}) - \Pi(B|\bar{A})u(\bar{A}, B) + \Pi(B|\bar{A})E(p_B|B, \bar{A}) - \Pi(B|A)E(p_B|B, A)}{\lambda}.$$

Next, note that the agent will eventually purchase B if his direct gain in utility – evaluated at that point in time – exceeds the shadow utility value of the monetary cost: $u(\cdot, B) - u(\cdot, \bar{B}) \geq \lambda p_B$. We can thus write the conditional valuations for B as $\frac{u(A, B) - u(A, \bar{B})}{\lambda} = v_B^A$ and $\frac{u(\bar{A}, B)}{\lambda} = v_B^{\bar{A}}$.⁷

We are now ready express the conditional purchasing probabilities as functions of the exogenous parameters: $\Pi(B|A) = F_B(v_B^A)$, $\Pi(\bar{B}|A) = 1 - F_B(v_B^A)$ and $\Pi(B|\bar{A}) = F_B(v_B^{\bar{A}})$. Finally, $E(p_B|B, A) = \frac{\int_0^{v_B^A} z dF_B(z)}{F_B(v_B^A)}$ and $E(p_B|B, \bar{A}) = \frac{\int_0^{v_B^{\bar{A}}} z dF_B(z)}{F_B(v_B^{\bar{A}})}$. Pulling everything together, we have

$$v_A = \frac{u(A, \bar{B})}{\lambda} + F_B(v_B^A)v_B^A - F_B(v_B^{\bar{A}})v_B^{\bar{A}} + \int_{v_B^{\bar{A}}}^{v_B^A} z dF_B(z). \quad (1)$$

Note that the last three terms of (1) can be interpreted as the result of integration by parts. “Reverse integrating” them we obtain our main result:

⁷Note that if A and B were want independent then v_B^A would equal $v_B^{\bar{A}}$.

Proposition 1. *The valuation for good A before the agent learns the price of good B is given by*

$$v_A = \frac{u(A, \bar{B})}{\lambda} + \int_{v_B^{\bar{A}}}^{v_B^A} F_B(z) dz. \quad (2)$$

The first term is the straightforward valuation that the agent would have if he knew that good B was not available (or if A and B were want independent). The second term captures the interdependence between A and B . For example, if A and B are complements, then $v_B^A > v_B^{\bar{A}}$ (the utility gained by obtaining B is greater if the agent also owns A) and the second term is positive – unless B is priced out of the agent’s reach: $F_B(v_B^A) = F_B(v_B^{\bar{A}}) = 0$. That is, if it is possible that a complementary good becomes available in the (near) future, the valuation of the currently considered good increases. Naturally, we have the opposite result for substitutes.

The size of the additional effect depends on the distribution of p_B . The more likely it is that p_B is low – say, in terms of first-order stochastic dominance of the distribution functions – the more likely it is that B will be bought and the more it affects the valuation for A . In the extreme case, when the agent is certain that he will buy good B , $F_B(v_B^{\bar{A}}) = F_B(v_B^A) = 1$ and the second term becomes $v_B^A - v_B^{\bar{A}} = \frac{u(A,B) - u(A,\bar{B}) - u(\bar{A},B)}{\lambda}$. Substituting back into (2), we obtain $v_A = \frac{u(A,B) - u(\bar{A},B)}{\lambda}$, as we should.

There are two possible interpretations of the Proposition. One is that we have identified the error term, in case the agent – or the modeler(!) – mistakenly treats the purchase of A as a separable – in terms of want independence – subproblem. In such a case the second term in (2) is missed out. The other side of the same coin is to see that when the purchase of A is not a separable subproblem, a “virtual income effect” comes into play even with quasi-linear utilities.

3 Conclusion

In this note we have derived the valuation for a good at a time when the conditions under which another, related, good will be available are not yet known. The sufficient statistic for the rest-of-life problem was the marginal utility for money, which can be learned and/or estimated. This result can be interpreted as a tractable micro-foundation for using a valuation as a parameter in these circumstances.

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