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# Price formation in a matching market with targeted offers ☆

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ABSTRACT

We model a market where the surpluses from seller–buyer matches are heterogeneous but common knowledge. Price setting is synchronous with search: buyers simultaneously make one personalized offer each to the seller of their choice. With impatient players efficient coordination is not possible, and both temporary and permanent mismatches occur. Nonetheless, for patient players efficient matching (with monopsony wages) is an equilibrium. The setting is inspired by a labor market for highly skilled workers, such as the academic job market, but it can be easily adapted to, for example, the housing market or Internet advertising auctions.



## 1. Introduction

We consider a labor market for starting professionals whose "quality" is public information. They could be doctors, lawyers, MBAs, PhDs, fund managers, athletes, musicians, chefs etc. There is commonly a one-shot, though dynamic, market for them, with no new entrants: the market is active until there is no co-existence of unfilled vacancies and suitable applicants. A common characteristic of these markets is that the firms are vertically differentiated as well, and hence the productivity of a worker varies with who has hired her. Fitting this scenario, the distinguishing feature of our model is that – as often in real life[1] – wages are set by the firms, who – in every period and for each of their vacancies – make a single personalized wage offer to the worker of their choice. Of course, in the presence of frictions, the firms need to balance their wishes against their realistic chances to hire a worker who is higher in the pecking order than they are.

While motivated by the labor market, our model can be interpreted as depicting any two-sided market with transferable utility and unit supply and demand, where each "buyer" chooses a "seller" to make an offer to.

Search and matching theory has been the standard – and rather successful – method for the analysis of labor markets, both theoretically and empirically.[2] Our contribution belongs to the family of complete information models within this literature. This sub-field can be split into two camps. One of them uses *ex post* wage setting: first firms and workers meet (according to some well-specified procedure, described via a *matching function*) and once they are matched they negotiate the wages. These models typically exhibit a hold-up-like feature, often referred to as the Diamond (1971)

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[1] Even if actually workers apply first, they typically use "blanket" application strategies, effectively giving the relevant choice over to the firms.

[2] See Rogerson et al. (2005) and the Scientific Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010, for surveys.

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paradox[3]: despite the existence of either unemployed workers or unfilled vacancies, the terms of trade – wages – are determined as if the negotiation among the matched parties were taking place in isolation, with no outside opportunities, no matter how inexpensive it is to switch partners. The alternative family of models has *ex ante* wage setting (directed search), where the firms commit to wage offers before the matching occurs (see Butters, 1977; Montgomery, 1991; Peters, 1991 and their followers[4]). Here hold up is no longer a problem and the matching process is also more interesting, as now the workers can condition their search strategy on the posted wages, which then feeds back into the competition among firms. Our model does not fit into either camp neatly. In principle, the targeted nature of the offers could be interpreted as an extreme form of directed search, where a worker can only search within the pool of firms that have made her an offer.[5] Nonetheless, our model also has some *ex post* flavor, as wages can be "re-negotiated": a worker can reject all her offers, if she expects better ones to materialize in the future.

Naturally, we are interested in the efficiency and distributional properties of this market. Note that it is liable to suffer from two types of inefficiency, caused by market imperfections:[6] the coexistence of unfilled vacancies and qualified job seekers (*frictional* unemployment); and *mismatch*, where matched workers could be reassigned to different jobs in a way to increase aggregate production.[7]

We derive the unique sub-game perfect equilibrium for the case of two firms – and at least two workers – first. When the workers are sufficiently impatient, the equilibrium involves "double mixing": firms use mixed strategies both to select the worker to target and the wage offered to the best worker. Consequently, with positive probability, the outcome exhibits both (temporary) frictional unemployment and (permanent) mismatch. Wages are below the firm-optimal competitive wage and with positive probability they are as low as the monopsony wages.

As the workers' discount factor rises, the upper end of the support of the wage distribution for the best worker stays constant, at her lowest competitive wage, as it is the weaker firm's option of hiring the second best worker what limits how much it is willing to bid for the best worker. Note that this implies that the firms' expected payoffs are unchanged. The increased patience of the best worker manifests itself in an increase of the *lower* bound of the common support of the mixed wages offered to her (which, in fact, is her continuation value when she receives two offers). As the firms' – and the rest of the workers' – payoffs stay constant, any resulting increase in her payoff would come from capturing efficiency gains. The latter are not guaranteed though as – in order to keep the weaker firm indifferent between offering to the two best workers – the best firm must increase the probability of offering a wage equal to her outside option to the best worker, countervailing the effect of the (stochastically) higher mixed offers.

When both discount factors are sufficiently high, the lower bound of the mixing support hits the upper bound and the equilibrium undergoes a metamorphosis: the weaker firm gives up on trying to compete for the best worker, and in equilibrium each firm targets its efficient match. In the resulting absence of competition the wages are the monopsony ones. While efficient matching when frictions are still present is remarkable, even more striking is that the equilibrium has a distinct Diamond paradox flavor: we have a nearly frictionless decentralized market leading to the monopsony wages. As we explain below, the underlying logic is entirely different though, it has nothing to do with the hold-up scenario.

Take the efficient strategy profile, where each firm makes an exclusive offer to its corresponding worker and hence wages are the monopsony ones (call them zero). At first glance, one might think that this cannot be part of an equilibrium. If both firms offer zero then there seems to exist a profitable deviation where the weaker firm offers $\varepsilon > 0$ to "poach" the better worker. However, outbidding one's competitor is not sufficient to obtain the services of a worker. It is also necessary that the worker be willing to accept this higher wage. As it happens, the fact that the worker was willing to accept zero in the putative equilibrium does not imply that she would also accept a deviant offer of $\varepsilon$ by a competing firm. The difference is that, in equilibrium, rejecting the offer would only delay the inevitable, as no other firm would be around to put an upward pressure on the wage. Yet, rejecting *both* offers following the deviation would lead to a subgame where there are still two firms left. As described above, the continuation value of the best worker following such a double rejection approaches the (lowest) competitive wage as the discount factors tend to 1. Since she would reject any lower offer, by the very definition of this competitive wage, the incentive to poach disappears exactly at the limit, where the higher wage cancels out the higher productivity.

Our model of the labor market also includes a (small) vetting cost, which comes into play here. This cost is incurred by the firms when they make the first binding offer to a worker (subsequent offers to the same worker are free). As a result, if – following a deviation by the weaker firm – the best worker receives two offers, her continuation value is that of a game with these two firms, where the vetting cost of (only) this worker has already been incurred by both firms. Such a game is biased in favor of the best worker, as firms now need to pay another vetting cost to make an offer to a weaker worker but not if they continue to bid for her. Consequently, the upper bound of the wage distribution for the best worker shifts up

[3] Several alternative versions of the paradox circulate, but this is our preferred one.

[4] There is also a smaller literature, started by McAfee (1993), on competing mechanism designers, where instead of wages, entire mechanisms (for wage determination) are posted by the firms.

[5] It is important to observe that the most important feature is not that the offers are personalized, rather that they are restricted to a single recipient. If firms could make a personalized offer to each worker simultaneously, the outcome would be a competitive equilibrium.

[6] Of course, there are many other inefficiencies associated with the labor market, like structural unemployment, discrimination, distortions caused by labor laws etc. However, these are not caused by the market institution itself and hence are not subjects of this study.

[7] Note that this is a different definition of mismatch from Shimer's (2007), which is closer to structural unemployment (in a multimarket context).

by the value of the vetting cost. That is, in the continuation game the highest possible wage offers are strictly higher than in the first period. As the mixing interval collapses on its upper bound, for high enough discount factors the continuation value of the best worker is strictly higher than her lowest competitive wage, which is the highest wage the weaker firm is willing to pay her in the first period. Hence, neither poaching nor mixing can happen and we end up with the "Diamond" equilibrium as the unique outcome even for discount factors strictly below 1.

Note that it is exactly the improvement in the workers' bargaining position that leads to the equilibrium with the lowest possible wages. Because the workers are so powerful when there is competition for them, the firms shy away from competition.[8] Workers would actually benefit from being able to commit to accepting below competitive wages!

The characterization of equilibria becomes exceedingly difficult as the number of firms grows. Nevertheless, we show that the Diamond outcome continues to be an equilibrium for an arbitrary number of firms, if the discount factors are sufficiently high. We can do that because in the continuation following a unilateral deviation by a firm from the efficient equilibrium there are always only two firms left – since all the others will have traded according to the equilibrium strategies – which is exactly the situation we have already characterized.

We also show that the above result is robust: it does not matter how many vacancies firms have; whether there are more workers than firms; whether workers can be vetted in batches; or whether the workers can hold on to an offer or not.

On the other hand, when firms can commit not to make a second offer to the same worker, the Diamond equilibrium is no longer possible: as the combination of commitment and lack of direct competition eliminates the high continuation value for a worker who receives two offers. When there are only two firms, the equilibrium is like the mixed strategy one above, with the only difference that now workers have a zero continuation value, so the support for the wage distribution starts at the better worker's outside option, leading to a lower expected wage for her.

### *1.1. A brief review of the closely related literature*

The most relevant direct precursor to this contribution is De Fraja and Sákovics (2001).[9] They allow for many-to-one matching (together with *ex post* price determination) and show that this potentially creates local market conditions that reverse the aggregate ones. However, their matching function is exogenously given. In this paper we endogenize who matches with whom, while maintaining the possibility of market power reversal. In the literature with *ex ante* wage setting mentioned above, not only is there no reversal, but one side of the market sets the conditions of trade and the other chooses who to attempt to trade with. In the current model the same side of the market takes both decisions, thereby changing the nature of competition.

Shi (2001) also presents a model with two-sided heterogeneity, where firms set wages and they can specify the type of worker they would like to hire. The equilibrium is constrained efficient and involves no competition for workers. His model differs from ours in two major respects: First, there is a large number of workers of each skill level. Consequently, targeting a skill level does not imply targeting an individual. Second, there is free entry of firms, leading to zero profits in equilibrium. This makes it easy to discourage poaching. Shi (2002) has a similar market but the mechanism is directed search with priority, where firms post wages for each type of worker and they also state which type of worker they prefer in case they receive both types of applicant. The resulting equilibrium is similar to what we obtain for low patience, but with the roles of firms and workers reversed: high type workers only search for high type jobs, while low type workers mix between high and low type firms. In his model, this equilibrium is (constrained) socially optimal – as all jobs get filled and a low type worker never takes the job of a high type one – but it does involve unemployment as workers have no second chance if their applications fails, unlike in the current model.

We use the same set-up as Bulow and Levin (2006), except that they consider universal wages: a firm must hire the best worker that shows up for the wage it has advertised. While this is the opposite of targeting, their model provides an interesting benchmark to compare our results to. Their unique (mixed strategy) equilibrium exhibits some mismatch but no frictional unemployment. Wages are not only infra-competitive but they are compressed: the better the worker the farther below competitive her (expected) wage is. Importantly, due to the relatively high efficiency of the matching, the firms benefit from the losses of the workers: they earn supra-competitive profits.

The closest paper to ours is Konishi and Sapozhnikov (2008). While they do not have the same motivation, they also present a model with targeted offers – in the context of an abstract assignment problem. The dynamic variant of their model is cleverly set-up in a way that avoids simultaneous competition in equilibrium. By assuming that offers to a worker are made once and for all and that there is no cost of delay, they are able to construct (pure strategy) equilibria where

[8] While reminiscent of it, this effect is distinct from that of "potential competition". In the industrial organization literature (cf. Dasgupta and Stiglitz, 1988) it has been observed that the disciplining effect of potential competition diminishes with the intensity of the ex post competition (e.g. no firm is willing to pay a fix cost to engage in a symmetric Bertrand competition). In our case the intensity of competition is unchanged; it is the "demand" what decreases with patience.

[9] Julien et al. (2000) analyze a model with *homogeneous* firms and workers, where it is the workers who start by announcing a reserve wage. This is followed by the firms simultaneously approaching one worker each. Workers finally auction their labor to the firms who have approached them (the latter having observed the number of their competitors). Note that bidding and search are not synchronous, even if there is some "targeting". The model is geared towards the effects of coordination frictions on wage patterns.

only a single firm makes an offer in each period. Note that their assumptions amount to giving the last word to the firm moving later, implying that wage competition for a worker cannot occur, as whoever attempts to overbid a follower will be matched by it anyway and hence will not be able to hire that worker. The main point of our model is to draw attention to the intrinsic interest of (endogenous) instantaneous local competition in the dynamic context, which was finessed by Konishi and Sapozhnikov (2008).

While their set-up is different – they look at a market in its steady state with endogenous entry, matching is random and goods are perishable – Ponsatí and Sákovics (2008) also describe a decentralized market where increasing a friction (in their case the per period cost of waiting) can improve efficiency. As in the current paper, the increased cost helps to make agents internalize the negative externality they impose on the rest of the participants.

Finally, we should mention that there exist models of *centralized* labor matching markets which involve firms targeting workers and endogenous wages.[10] The seminal work in this area is Crawford and Knoer (1981). Their model requires that a firm – myopically – always offer to its most preferred worker at the "going" wage vector, thereby enforcing competition and ensuring a competitive outcome.

## 2. The model

There are $M$ firms, each with a single vacancy, and $N \geq M$ workers, each looking for a job.[11] It is common knowledge that the joint output of Firm $I$ and Worker $j$ would be $p_{Ij} > 0$. We assume that the output matrix is *decreasing* in both indices and it is (reverse) supermodular: firms with a *lower* index appreciate more a switch to a worker with a *lower* index. Supermodularity arises naturally when the "innate" qualities of firms and workers are complements, while at the same time it simplifies the analysis and provides a straightforward benchmark for efficient matching.

If Firm $I$ hires Worker $j$ at wage $w_{Ij}$ then the firm's payoff is $p_{Ij} - w_{Ij}$, while the worker obtains $w_{Ij}$. For convenience, the no-trade payoffs of both firms and workers are normalized to zero.

### 2.1. Competitive equilibria

We start by establishing the competitive benchmark: the hypothetical outcome in a centralized, frictionless market. The defining characteristic of such an equilibrium is that – taking the wages paid in equilibrium as given – no firm would want to hire a worker different from the one it hires in equilibrium. Recall that due to the complementarity of worker and firm types, the efficient matching is positively assortative (cf. Becker, 1973).

**Proposition 1.** *All competitive equilibria are efficient: Firm $I$ hires Worker $i$. Moreover, in the firms'-best competitive equilibrium, wages are $w^c_M = 0$ and $w^c_j = \sum_{I=i=j+1}^{M}(p_{I,i-1} - p_{I,i})$ for $j \in \{1, ..., M-1\}$.*

The proof is in Appendix A.

It is useful to observe that this wage vector would be the outcome of a Vickrey–Clarke–Groves auction, where firms bid their valuation for each worker (truthfully, in equilibrium), the assignment is efficient, and the wages paid by a firm reflect the externality it imposes on the rest of the firms: the presence of Firm $j$ forces each firm with index above $j$ to hire a worker of one lesser rank than they would otherwise.

## 3. Targeted wage setting

We assume that the decentralized market operates as follows. In period 1, simultaneously and independently, each firm makes an offer to a single worker of their choice. For each firm it costs $c > 0$ to approach a worker for the first time. Any subsequent offers to the same worker are free.[12] We assume that $c < \min p_{Ij}$, so that it does not discourage any match. The workers who receive (one or more) offers either accept the highest of those (in case of a tie they go with the better firm)[13] – in which case the firm whose offer has been accepted and the worker collect their payoffs and exit the market – or reject all offers (we discuss the case where workers can hold on to offers in Section 4.4). In the subsequent periods there is no

[10] We consider the large body of models with non-transferable utility too far removed to discuss them in this short overview.

[11] We analyze the case of $N < M$ in Section 4.1 and we extend our results to multiple vacancies in Section 4.2.

[12] One can interpret $c$ as the administrative cost of vetting a worker. Say, the work permit must be checked. Or, $c$ could be the (search) cost of sifting through applications. Alternatively, it could be that only the ordinal ranking of workers is common knowledge and the vetting is needed to find out the exact productivity.

We could also extend the model to endogenize the decision to vet. Say, there is a small probability that the candidate is not suitable. For a small vetting cost, the optimal policy would be to vet candidates with a probability high enough so that an unsuitable candidate would be indifferent to chance getting caught. In that case $c$ would be the expected vetting cost and $p_{Ij}$ the expected productivity (as with positive probability unsuitable workers would be hired).

We could also have a vetting cost that is proportional to the risk incurred (that is, to wages). As long as this were always positive – in other words taking into account that the zero minimum wage is a normalization – our results would be unaffected.

[13] That is we assume that workers have lexicographic preferences, with wages dominating the preference for working at a better firm.

further entry: the firms with unfilled vacancies keep making offers to the available workers until all vacancies get filled. Firms and workers discount the future by discount factors $\delta$ and $\beta$, respectively.

We start the analysis with the "simple" case of only two firms:

### *3.1. Duopsony*

Let us denote the more productive (lower index) firm by $H$ and the other one by $L$. Also let the efficient (lowest index) partner of $H$ be denoted by $h$, and the efficient partner of $L$ by $l$. The rest of the workers will not play any role. We will refer to the firms as *it*s, to Worker $h$ as *she* and Worker $l$ as *he*. We can characterize the equilibrium of our decentralized market as follows:

**Proposition 2.** *When $c < p_{Hh} + p_{Ll} - p_{Hl} - p_{Lh}$,*[14] *the two-firm game has a generically unique subgame-perfect equilibrium (SPE). There exists a well-defined value,*[15] $\underline{w} \in [0, w_h^c + c]$*, such that*

(i) *if $\underline{w} \le w_h^c$: $L$ with probability $\Pi_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - w_h^c}{p_{Hh} - \delta(p_{Hl} - c)}$, offers a zero wage to $l$, while with the remaining probability it makes an offer to $h$, mixing with $F_h^L(x) = \frac{\Pi_l^L}{1 - \Pi_l^L} \cdot \frac{x}{p_{Hh} - \delta(p_{Hl} - c) - x}$ over the interval $[\underline{w}, w_h^c]$; $H$ makes an exclusive offer to $h$, offering zero with probability $Z = \frac{p_{Ll}(1-\delta) + \delta c}{p_{Lh} - \delta(p_{Ll} - c) - \underline{w}}$ and with the remaining probability mixing with $F_h^H(x) = \frac{Z}{1-Z} \cdot \frac{x - \underline{w}}{p_{Lh} - \delta(p_{Ll} - c) - x}$ over the interval $[\underline{w}, w_h^c]$. $h$ accepts the highest offer she receives. $l$ accepts the offer if he receives it. Any firm that does not hire in the first period, hires $l$ for zero in the second.*

(ii) *if $\underline{w} \ge w_h^c$: both firms offer zero to their efficient match and these offers are accepted.*

The two possible equilibrium configurations are strikingly different. One displays both frictional unemployment – as, because of $L$'s mixing over targets, $l$ may not receive an offer in the first period – and mismatch – as, because of the mixed wage offers to $h$, $L$ may end up hiring her. It is reminiscent of the equilibrium of *ex ante* wage setting with uniform wages, as analyzed in Bulow and Levin (2006).[16] The other configuration is fully efficient, but leaves zero surplus for the workers, along the lines of the equilibrium of *ex post* wage setting. Both outcomes give below competitive expected wages to the better worker.

When $\underline{w} = w_h^c$ there are two equilibria (signaling a discontinuity in the equilibrium set). In addition to the efficient equilibrium we have an inefficient one – which would be the limit equilibrium as $\beta \to 1$ in the absence of vetting costs. In this equilibrium the bids for $h$ are fixed ($L$ bids $w_h^c$ and $H$ bids zero) but $L$ is still mixing over the target of its offer – and when it bids for $h$ it wins with positive probability (which tends to one as $\delta \to 1$) – so the inefficiency does not disappear.

**Proof of Proposition 2.** We provide a constructive proof, as it captures the intuition better. We start by noting that in SPE no firm will make an offer that it knows will be rejected for certain. To see this, first note that if the offer is rejected by the targeted worker because it accepts some other firm's offer then the firm is better off bidding for another worker today. If it is because no offer is accepted by the targeted worker, then the worker will expect more later, otherwise she would have accepted today. As a result the firm cannot expect to be better off hiring the same worker later than making her indifferent today. If it expects to trade with another worker, again it would be better off bidding for him today.

Let us assume that in the first period $H$ bids exclusively for its favorite worker, $h$. We will confirm that in any SPE this indeed must be the case later in the proof.

If $L$ bids for $h$ with positive probability, then both $L$ and $H$ must use a mixed strategy for their bids to $h$ (recall that the workers go with the more productive firm in case of equal wage offers). Standard arguments imply that both firms must mix on the same support, which we denote by $[\underline{w}, \overline{w}]$, except that $H$ may also bid zero in the hope that it is the only bidder (because $L$ is bidding for $l$).[17] It is straightforward to see that the only additional possible mass points in the strategies are at $\underline{w}$ for $L$ (if and only if $H$ indeed puts positive probability on zero and $\underline{w}$ is positive) and at $\overline{w}$ for $H$.

We start by hypothesizing that $H$ does not bid zero. In any putative equilibrium, $H$ will obtain the services of $h$ if $L$ either does not bid for her (with probability $\widehat{\Pi}_l^L$) or it offers no more than what $H$ does. If $H$ loses out in the first period, it will hire $l$ in the second period (for zero, as it will face no competition). When $H$ offers the maximum of the common support, $\overline{w}$, then it wins for sure. As it must be indifferent among all bids in the support of its strategy, the following equality must hold for all $x \in [\underline{w}, \overline{w}]$ (we denote the cumulative distribution function of Firm $I$'s bid for Worker $j$ by $\widehat{F}_j^I$): $(p_{Hh} - x)[\widehat{\Pi}_l^L + \widehat{\Pi}_h^L \widehat{F}_h^L(x)] + \widehat{\Pi}_h^L[1 - \widehat{F}_h^L(x)]\delta(p_{Hl} - c) = p_{Hh} - \overline{w}$. Rearranging the equation, we obtain

[14] If the vetting cost is higher than the social cost of mismatch, for $\underline{w} \ge w_h^c$ there exists an additional SPE where there is certain mismatch with monopsony wages.

[15] As it will become clear, this is the continuation value of Worker $h$ when she receives two offers in the first period.

[16] Though there are significant differences, we elaborate on these in Section 5.

[17] If $h$ receives a single bid (from $H$) then $l$ will receive and accept an offer (from $L$), so in the continuation $h$ would be left facing $H$ as the only potential employer, forcing him to accept a zero wage.

$$\widehat{\Pi}_l^L + \widehat{\Pi}_h^L \widehat{F}_h^L(x) = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{p_{Hh} - \delta(p_{Hl} - c) - x}. \tag{1}$$

Now, observe that $\widehat{F}_h^L(\underline{w})$ must be zero, since a bid of $\underline{w}$ could never win as $H$ is bidding with certainty for $h$ and at least $\underline{w}$, leading to

$$\widehat{\Pi}_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{p_{Hh} - \delta(p_{Hl} - c) - \underline{w}}. \tag{2}$$

As $L$ could hire $l$ for free, its bid for $h$ is capped at $p_{Lh} - p_{Ll}$. Consequently, $\overline{w} \le p_{Lh} - p_{Ll} < p_{Hh} - p_{Hl} < p_{Hh} - \delta p_{Hl}$, so $\widehat{\Pi}_l^L > 0$: $L$ makes an offer to $l$ with positive probability as well.

Given that $L$ is making an offer to both workers with positive probability, it must be indifferent between making an offer to either of them. As it faces no competition for $l$, it can hire him for zero, leading to $(p_{Lh} - x)\widehat{F}_h^H(x) + [1 - \widehat{F}_h^H(x)]\delta(p_{Ll} - c) = p_{Ll} \Leftrightarrow \widehat{F}_h^H(x) = \frac{p_{Ll}(1-\delta)+\delta c}{p_{Lh}-\delta(p_{Ll}-c)-x}$ for $x \in (\underline{w}, \overline{w})$. Taking the limit $x \to \underline{w}$ we obtain that $\widehat{F}_h^H(\underline{w}) = \frac{p_{Ll}(1-\delta)+\delta c}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}$. Note that this value is positive, as $\underline{w} \le \overline{w} \le p_{Lh} - p_{Ll} < p_{Lh} - \delta p_{Ll}$. This would mean that $H$ bids weakly less than $\underline{w}$ with positive probability, which rationally can only be an offer of zero, contradicting the hypothesis that it does not bid zero.

We thus know that in SPE $H$ will offer zero to $h$ with positive probability. We drop the "hats" of $F$ and $\Pi$ to capture the change in strategy and denote the probability of $H$ making an offer of zero to $h$ by $Z$. As we have seen above, $H$ must mix, so $Z < 1$.

$H$ has to be indifferent between bidding zero (when it only wins if $L$ does not bid for $h$, and otherwise it hires $l$ next period) and $\overline{w}$ (when it wins for sure), so we must have that $p_{Hh}\Pi_l^L + (1 - \Pi_l^L)\delta(p_{Hl} - c) = p_{Hh} - \overline{w} \Rightarrow$

$$\Pi_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{p_{Hh} - \delta(p_{Hl} - c)} > 0. \tag{3}$$

By the same token, (1) – without "hats" as we have established that $H$ bids zero with positive probability – must also hold for all $x \in [\underline{w}, \overline{w}]$. Solving for the mixing distribution we have

$$F_h^L(x) = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{\overline{w}} \cdot \frac{x}{p_{Hh} - \delta(p_{Hl} - c) - x} \in (0, 1]. \tag{4}$$

Given that $L$ is making an offer to $l$ with positive probability (see (3)), it must be indifferent between making an offer to either worker. As it faces no competition for $l$, it can hire him for zero, leading to $(p_{Lh} - x)(Z + F_h^H(x)(1 - Z)) + (1 - Z)(1 - F_h^H(x))\delta(p_{Ll} - c) = p_{Ll} \Leftrightarrow$

$$(1 - Z)F_h^H(x) = \frac{p_{Ll}(1 - \delta) + \delta c}{p_{Lh} - \delta(p_{Ll} - c) - x} - Z, \tag{5}$$

for $x \in (\underline{w}, \overline{w})$. If there is no mass point at the upper end of $H$'s strategy, $\lim_{x\to\overline{w}} F_h^H(x) = 1$, then the formula still applies and we obtain that $\overline{w} = p_{Lh} - p_{Ll}$. If there were a mass point, then in order to keep $L$ from overbidding it must be that for all $\varepsilon > 0$, $p_{Lh} - \overline{w} - \varepsilon < p_{Ll} \Leftrightarrow \overline{w} \ge p_{Lh} - p_{Ll}$, which when applied to the formula for $\lim_{x\to\overline{w}} F_h^H(x)$, implies again that $\overline{w} = p_{Lh} - p_{Ll}$ and $F_h^H(\overline{w}) = 1$, therefore no mass point is possible. From (3), substituting in for the upper bound, we obtain that $\Pi_l^L = \frac{p_{Hh}-\delta(p_{Hl}-c)-w_h^c}{p_{Hh}-\delta(p_{Hl}-c)}$.

When $L$ bids the lower bound of its support, it can only win if $H$ is bidding zero. Hence, we have that $(p_{Lh} - \underline{w})Z + (1 - Z)\delta(p_{Ll} - c) = p_{Ll}$, from which we can solve for $Z = \frac{p_{Ll}(1-\delta)+\delta c}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}} \in (0, 1)$. Substituting in (5), we obtain

$$F_h^H(x) = \frac{x - \underline{w}}{w_h^c - \underline{w}} \cdot \frac{p_{Ll} - \delta(p_{Ll} - c)}{p_{Lh} - \delta(p_{Ll} - c) - x}.$$

Next, we identify the lower bound of the support of the mixed strategies. Observe that – by the single deviation principle – this has to equal the (discounted) expected continuation value of $h$, *when she receives two offers*[18] and hence expects both firms to be still in the market in the following period.

$$\frac{\underline{w}}{\beta} = \widetilde{Z}\widetilde{\Pi}_h^L\widetilde{F}_h^L(\underline{w})\underline{w} + \int_{\underline{w}}^{\widetilde{w}} x\big[\widetilde{f}_h^H(x)(1 - \widetilde{Z})\big(\widetilde{\Pi}_l^L + \widetilde{\Pi}_h^L\widetilde{F}_h^L(x)\big) + \widetilde{\Pi}_h^L\widetilde{f}_h^L(x)\big(\widetilde{Z} + (1 - \widetilde{Z})\widetilde{F}_h^H(x)\big)\big]dx. \tag{6}$$

Note that the probability distributions (and $\overline{w}$) carry a tilde. This is because following two bids for $h$, no vetting cost will have to be paid to make a new offer to $h$, tilting the competition in favor of $h$ and slightly modifying the formulas. It is crucial to observe that $\underline{w}$ is invariant across periods, as it is only invoked following a history (of any length) where both firms have paid their vetting costs exclusively for $h$.

[18] Whenever $L$ offers to $h$, she will receive two offers, so this is the relevant scenario for the determination of the lower bound of $L$'s bidding distribution.

It is straightforward to see that up to (3) and (4) everything remains the same (except for the substitution of $\widetilde{w}$ for $\overline{w}$) even after a sunk vetting cost for $h$. On the other hand, (5) becomes $(1-\widetilde{Z})\widetilde{F}_h^H(x) = \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-x} - \widetilde{Z}$, which in turn implies that $\widetilde{w} = w_h^c + c$, which then leads to $\widetilde{\Pi}_l^L = \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)}$ and $\widetilde{\Pi}_h^L = \frac{\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)}$. Similarly we have

$$\widetilde{Z} = \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}} \quad \text{and} \quad \widetilde{F}_h^H(x) = \frac{x-\underline{w}}{\widetilde{w}-\underline{w}} \cdot \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-x}. \tag{7}$$

Substituting into (6), we have

$$\begin{aligned}\frac{\underline{w}}{\beta} = {} & \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}} \cdot \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)} \cdot \frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}\underline{w} \\ & + \int_{\underline{w}}^{\widetilde{w}} \frac{(p_{Ll}-c)(1-\delta)x}{(p_{Lh}-\delta(p_{Ll}-c)-x)^2} \cdot \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)-x} dx \\ & + \int_{\underline{w}}^{\widetilde{w}} \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{(p_{Hh}-\delta(p_{Hl}-c)-x)^2} \cdot \frac{(p_{Ll}-c)(1-\delta)x}{p_{Lh}-\delta(p_{Ll}-c)-x} dx.\end{aligned} \tag{8}$$

After a bit of work,[19] this simplifies to the following equation:

$$\begin{aligned}0 = {} & \frac{\underline{w}(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))}{\beta(p_{Ll}-c)(1-\delta)(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})} \\ & - \frac{\underline{w}^2(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c))(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})} \\ & - \frac{(\widetilde{w}-\underline{w})(p_{Lh}-\delta(p_{Ll}-c))}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})} \\ & + \frac{(\widetilde{w}-\underline{w})(p_{Hh}-\delta(p_{Hl}-c))}{(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})} \\ & - \ln \frac{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})}.\end{aligned} \tag{9}$$

The right-hand side of (9) is a continuous function of $\underline{w}$, outside of $[p_{Lh}-\delta(p_{Ll}-c), p_{Hh}-\delta(p_{Hl}-c)]$ where it is not defined. Routine calculations show[20] that it is increasing for $\underline{w} < p_{Lh}-\delta(p_{Ll}-c)$, and that it takes a negative value at $\underline{w} = 0$ and a positive value at $\underline{w} = \widetilde{w}$. Consequently, there is a unique feasible solution. This completes the description of the equilibrium when $\underline{w} \le w_h^c$.

When $\underline{w} \ge w_h^c$, $L$ has no (strict) incentive to bid for $h$, given that $H$ *does* bid for him. Thus we get the efficient matching and, since only one firm bids for each worker, the wages are zero. At the knife-edge case when $\underline{w} = w_h^c$, both equilibria exist.

Finally, to see that $H$ will not bid for $l$ in any SPE, note that either $L$ is bidding exclusively for $l$ and hence $h$ could be hired for free (as $L$ would hire $l$, so $h$ has no credible threat of rejecting) which is the best possible outcome for $H$; or $L$ bids for $h$ with positive probability. If $L$ is bidding for $h$ only, then $H$ could hire $l$ for free, earning $p_{Hl}$, while bidding $\widetilde{w}$ for $h$, it could obtain its services for certain, yielding a payoff of $p_{Hh}-\widetilde{w}$. As long as $\widetilde{w} \le p_{Hh}-p_{Hl}$ the latter is preferred. As $\widetilde{w} \le w_h^c + c$, $c < p_{Hh}+p_{Ll}-p_{Hl}-p_{Lh}$ is a sufficient condition. Finally, consider the case where $L$ is mixing over the target of its offer. This would weaken $H$'s option of bidding for $l$ – higher wage needs to be paid – and strengthen it for $h$ – as there is not always competition for her. □

Of course, the crucial question is: when, if ever, is $\underline{w} \ge w_h^c$? That is, when can the worker expect more in the continuation than her (lowest) competitive wage? The following corollary gives the answer. We will write $\underline{w}(\delta,\beta)$ for the continuation value of $h$ when both firms bid for her.

**Corollary 1.** *If $\underline{w}(\delta,1) \le w_h^c$, then the unique SPE of the two-firm game is the mixed equilibrium identified in Proposition 2. If $\underline{w}(\delta,1) > w_h^c$ then there exists $\beta^*(\delta) \in (0,1)$ such that*

(i) *if $\beta > \beta^*(\delta)$ (and $c < p_{Hh}+p_{Ll}-p_{Hl}-p_{Lh}$), the unique SPE of the two-firm game is efficient matching, with wages equal to the workers' outside options (zero);*

[19] Details are in Appendix B.

[20] Details are in Appendix B.

(ii) *while if $\beta < \beta^*(\delta)$ then the unique SPE of the two-firm game is the mixed equilibrium identified in Proposition 2.*

*Finally, there exists $\delta^* \in (0,1)$ such that for all $\delta > \delta^*$, $\underline{w}(\delta,1) > w_h^c$.*

**Proof.** Note that the equation defining $\underline{w}$, (9), is of the form $g(\underline{w},\delta,\beta) = 0$. In the range $\underline{w} \in (0, w_h^c + c)$, $g$ is continuous in $\delta$ and $\beta$, implying that so is $\underline{w}(\delta,\beta)$, which is uniquely defined, as shown in the proof of Proposition 2. Observe that if $\delta$ is such that $\underline{w}(\delta,1) > w_h^c$ then, as $\underline{w}(\delta,\beta)$ is increasing in $\beta$ by (6) from $\underline{w}(\delta,0) = 0$, the required $\beta^*$ exists and is uniquely defined by $w_h^c = \underline{w}(\delta,\beta^*)$. Consequently, all we need to show is that $\lim_{\delta\to 1} \underline{w}(\delta,1) > w_h^c$. We actually show that $\lim_{\delta\to 1} \underline{w}(\delta,1) = \widetilde{w} = w_h^c + c$. To see this, assume to the contrary that $\lim_{\delta\to 1} \underline{w}(\delta,1) < \widetilde{w}$. As seen from (7), that would imply that $\lim_{\delta\to 1} \widetilde{Z}(\delta) = 0$. If $H$ never bids zero then an infinitely patient $h$ will never accept a wage below the upper bound of the mixed strategy, leading to a contradiction. □

When at least one side of the market is impatient, the equilibrium is the inefficient one. With patient players we have the efficient equilibrium. In the situations mentioned in the introduction, we would expect the players to be rather patient, so the prediction favors the Diamond equilibrium.

It is interesting to note that whether or not we obtain an efficient (though not Walrasian) outcome in the limit as frictions disappear depends on the order of limits: as we have seen above, if the vetting costs vanish first efficiency is not achievable; nonetheless, if it is the discount factors that hit 1 first efficiency is obtained, even before reaching the limit.

Another interesting limit is when (worker) heterogeneity disappears. The competitive wage clearly tends to zero, while the worker receiving two offers will have a continuation value $\underline{w} = \beta c > 0$, so by Proposition 2 only the Diamond equilibrium survives.

### 3.2. The general case

The characterization of equilibria for a large number of firms is very complicated. As there are multiple offers received by many workers with positive probability, way too many subgames are possible to allow a clean analysis.

Short of a full characterization, what we are really interested in is whether Corollary 1 generalizes to an arbitrary number of firms (and workers). We can answer in the affirmative: indeed, the efficient equilibrium exists if and only if the discount factors are high enough. The intuition for this is that a unilateral deviation from the efficient equilibrium always leads to a worker receiving two offers, just as in the duopsony case analyzed above.

In order to state the precise result, we need to introduce some additional notation. For $M \geq I > j$, let $\underline{w}_{Ij}$ denote the continuation value of Worker $j$ when she receives an offer each from Firms $I$ and $J$ and expects no other firms and no workers with index below $j$ to be in the market in the following period. By Corollary 1, there exists $\delta_{Ij}^* \in (0,1)$, such that $\underline{w}_{Ij}(\delta_{Ij}^*, \beta = 1) = p_{Ij} - p_{Ii}$. Let $\widehat{\delta} = \max_{I>j} \delta_{Ij}^* < 1$. Similarly, let $\widehat{\beta}(\delta) < 1$ be the lowest[21] $\beta$ such that $\underline{w}_{Ij}(\delta,\beta) \geq p_{Ij} - p_{Ii}$ for all $i > j$.

**Proposition 3.** *When*

(i) *either $\delta < \widehat{\delta}$ or $\beta < \widehat{\beta}(\delta)$, there exists no efficient equilibrium.*
(ii) *$\delta > \widehat{\delta}$ and $\beta > \widehat{\beta}(\delta)$, there exists an SPE of the game with efficient matching and wages equal to the workers' outside options.*

**Proof.** Let us start with (i). In an efficient equilibrium no worker can get multiple offers and consequently the wages must equal the outside options. Take an arbitrary pair of firms, $I, (>) J$. In the putative equilibrium Firm $I$ earns $p_{Ii}$, and hence would be willing to pay $p_{Ij} - p_{Ii}$ to hire Worker $j$. By the definition of $\underline{w}_{Ij}$, an offer of $p_{Ij} - p_{Ii} - \varepsilon > \underline{w}_{Ij}$ would be accepted by Worker $j$. Therefore, showing that there exists $I > j$ such that $p_{Ij} - p_{Ii} > \underline{w}_{Ij}$ proves the claim. If $\delta < \widehat{\delta}$ then there exists $I > j$ such that $\delta_{Ij}^* > \delta$, implying that $\underline{w}_{Ij}(\delta,\beta) < p_{Ij} - p_{Ii}$ for any $\beta$. Similarly, if $\beta < \widehat{\beta}(\delta)$ then there exists $I > j$ such that $\underline{w}_{Ij}(\delta,\beta) < p_{Ij} - p_{Ii}$, proving the claim.

For (ii), consider a deviation by Firm $J$, where it makes an offer to worker $k \neq j$. Since the equilibrium wages are zero, this can be only profitable if it prefers $k$ to $j$: $k < j$. As following the equilibrium strategies the rest of the firms will have hired in the first period, Proposition 2 applies, with Firm $K$ playing the role of $H$. Therefore, by the proof of Corollary 1, the continuation value of Worker $k$ exceeds $p_{Jk} - p_{Jj}$, when $\beta > \widehat{\beta}(\delta)$. Thus, for $\beta$ high enough, Firm $J$'s deviation payoff conditional on Worker $k$ accepting is $p_{Jk} - (p_{Jk} - p_{Jj} + \varepsilon) = p_{Jj} - \varepsilon$, less than its equilibrium payoff, $p_{Jj}$. To guarantee that $\widehat{\beta}(\delta) < 1$ we need $\delta > \widehat{\delta}$.

We still need to check what happens if the deviant offer to Worker $k$ (or indeed, $j$) is unacceptable. In that case, the worker would reject both of his offers. In the continuation, by the proof of Corollary 1, Firm $J$ would end up hiring Worker $j$ for zero, just as in the putative equilibrium, but suffering a delay cost and an extra vetting cost. Hence there exists no profitable deviation for any firm.

[21] Note that unless it is equal to zero, $\underline{w}_{Ij}(\delta,\beta)$ must be strictly increasing in $\beta$.

If a worker rejected his equilibrium offer, next period he would be faced with the same firm, as all the other firms would have hired. He could not improve on his payoff – as any positive continuation payoff could be slightly undercut by the firm, and it would be in the worker's best interest to accept. □

Even in the absence of a general uniqueness result, it is arguable that in a situation where the same firms face each other repeatedly, like the job markets we model, they would coordinate on the efficient equilibrium, which maximizes their aggregate welfare.

What about the other equilibria? We conjecture that there exist variants of the doubly-mixed equilibrium of the two-firm model, where each firm mixes at most over their corresponding worker and the one above.[22] However, there seems to be no tractable way of handling them.

## 4. Variations

### 4.1. Workers' market

In the main text – for simplicity and realism – we have maintained the assumption that the number of firms did not exceed the number of qualified workers looking for a job. Here we show that the existence of the Diamond equilibrium does not require a firms' market, it exists in a workers' market just as well. As before, the main insight comes from the set-up following a unilateral deviation from the Diamond equilibrium: in this case a single worker (and several firms). The generalization follows the same arguments of Corollary 1 and Proposition 3 from there.

Let us denote the firm that is most productive hiring the worker by $H$ and the second most productive firm by $L$. The corresponding outputs are $p_H$ and $p_L$.

**Proposition 4.** *The one-worker–many-firms game has the following set of SPE:*

(i) *if $\beta p_L \le p_L - c$: $L$ with probability $\Pi^L = \frac{p_H - p_L + c}{p_H}$ does not make an offer, while with the remaining probability it mixes its offer with $F^L(x) = \frac{p_H - p_L + c}{p_L - c} \cdot \frac{x}{p_H - x}$ over the interval $[\beta p_L, p_L - c]$; $H$ offers zero with probability $Y = \frac{c}{(1-\beta)p_L}$ and with the remaining probability mixes with $F^H(x) = \frac{x - \beta p_L}{(1-\beta)p_L - c} \cdot \frac{c}{p_L - x}$ over the interval $[\beta p_L, p_L - c]$. The worker accepts the highest offer she receives;*

(ii) *if $\beta p_L \ge p_L - c$: $H$, or any other Firm $i$ such that $\beta p_i \ge p_H - c$, makes the only offer, which is zero and is accepted.*

The proof is in Appendix A.

### 4.2. Multiple vacancies per firm

In the main model we have made the simplifying assumption that each firm has a single vacancy. As shown by Kojima (2007), this assumption is crucial for the result of Bulow and Levin (2006) that firms (workers) are better (worse) off with uniform pricing than with the firms'-best competitive equilibrium. Nonetheless, we can show that in our model the assumption is indeed without loss of generality. Let us relabel firms as vacancies and assume that several vacancies can be controlled by the same company.

**Corollary 2.** *Companies having multiple vacancies would not affect the existence of the Diamond equilibrium.*

**Proof.** First note that no company would try to compete with itself for a worker. So any deviation from the Diamond equilibrium must involve a company poaching a worker which in equilibrium it would not hire. If such a deviation occurs, just as in the main model, all the other vacancies will be filled, so in the continuation there only the two vacancies of different companies will be left. This leaves the continuation value of a worker receiving two offers (out of equilibrium) the same as in the main model. As the equilibrium payoffs of the companies are also unchanged, the incentives to poach continue to be the same. □

Note that Kojima's (2007) result is driven by the fact that with uniform wages and differing firm capacities, some workers would be hired by the same company for any wage vector. The competitive wage of these would be low but the uniform pricing – within companies – forces companies competing for other workers to raise it. In contrast, when companies can offer a personalized wage for each vacancy this externality is absent.

[22] We have worked one out for the three-firm case, when $\delta = \beta = 0$. It has three possible configurations, depending on how the productivities are spaced out.

### 4.3. Batch vetting

Given the apparent importance of vetting taking place worker by worker, it seems reasonable to investigate the consequences of the possibility to vet several workers at a time. Consider the duopsony model. If the weaker firm vetted both workers in the first period, the Diamond equilibrium would indeed cease to exist for impatient players. Note however, that the weaker firm's payoff would be lower with batch vetting, as she would have the same expected gross payoff but would incur two vetting costs.[23] Therefore, batch vetting would not occur in equilibrium. Nonetheless, the workers would have an incentive to have themselves certified by a reliable agency if that were feasible. Such a move has recently happened in the Scottish housing market, where sellers are now obliged to provide a "surveyor's home report" to all interested buyers.

### 4.4. Holding on to an offer

In the main text we have assumed that workers had to respond to each offer immediately and firms thus could only revise their offer once it has been rejected. There are two ways in which this assumption can be relaxed.

First, consider the case where workers can delay the decision on an offer for $n$ periods (and firms can revise their unaccepted offers in every period). It is straightforward to demonstrate that this can only increase the range of parameters for which the Diamond equilibrium exists.

**Corollary 3.** *Workers having several periods to ponder an offer would make the Diamond equilibrium more likely to exist.*

**Proof.** We will show that the continuation value of a worker rejecting two offers can only improve with the workers' option to hold on to an offer. As a result, the incentives for a firm to deviate from the Diamond equilibrium can only decrease. Recall, that in the continuation there are only two workers who receive offers. One of them has no competition for him, so he has no incentive to wait. The other worker is supposed to accept the highest offer in equilibrium. If she decides to hold on to it, she must be better off doing that, increasing her expected payoff – and thus decreasing the poaching firm's. □

The intuition for this result is simple: the only reason to hold on to an offer (rather than accept it right away) is the hope of receiving a better offer in the future. This can only improve a worker's payoff. It does not happen on the equilibrium path as there are no suitors left, while the effect off the equilibrium path only strengthens the equilibrium. The existence of the mixed equilibrium would be affected in exactly the opposite way. The worker's ability to hold on to her offers would make it possible for a firm to improve its offer in the second period in case it lost out in the first. This would lead to the competitive wage ($+c$) in the second period, putting a high lower bound on the mixing interval ($\underline{w}$ would equal $\beta(w_h^c + c)$), and make this interval collapse ($\beta(w_h^c + c) \geq w_h^c$) – ruling out the mixed equilibrium – for lower levels of impatience.

A second way of relaxing the assumption would be to say that workers have to decide instantly, but firms can counter-offer within the same period. This would be similar to having no discounting, leading to the Diamond equilibrium as the unique prediction. However, even if this assumption seems superficially more realistic, say, for the case of the academic job market (offers are exchanged in January–February, while jobs only start in September), we do know that both departments and fresh PhDs *are* impatient during the job market even if less so than if the negotiations were delaying the start of the job.

### 4.5. Full commitment to wage offers

If in addition to not being able to approach another worker while the original target ponders the offer the firm is not allowed to make a second offer to the same worker, the Diamond equilibrium does disappear.

Assume the firms make a single take-it-or-leave-it (TIOLI) offer to the worker of their choice, which she has to accept within $t$ periods.[24]

We start with a general result that equilibria with full commitment must involve simultaneous competition.

**Proposition 5.** *With TIOLI offers, in any SPE some worker must receive two simultaneous offers with positive probability.*

**Proof.** Assume to the contrary, that there exists an SPE where each worker receives a maximum of one offer on the equilibrium path. Then all these offers would have to be simultaneous, as they would be accepted immediately and hence any delay in making them would be suboptimal. If all offers are simultaneous and one per worker, then they must be zero. But then there is an incentive to deviate and bid $\varepsilon$ for a better worker. The firm whose worker is "poached" cannot react, while the others hire their equilibrium worker, so the worker would be compelled to accept. □

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[23] Even if there were economies of scale in vetting, as long as vetting two workers is more expensive than vetting one, the result would be the same.

[24] Konishi and Sapozhnikov (2008) make this assumption, with $t = \infty$ (and $\delta = \beta = 1$).

Note that Proposition 5 rules out both Diamond-type equilibria and the sequential-move equilibria of Konishi and Sapozhnikov (2008), where firms make offers one after the other. This shows that the assumption that leads to their results is the absence of discounting and not the non-explosive nature of the offers.

In order to get a better feel for what equilibria with full commitment look like, we discuss the case of a duopsony. When $t$ is zero (exploding offers) then the equilibrium is the same as in the case without commitment (and low $\delta$), except that the mixing interval starts from zero, as the continuation value of a worker is zero, since the offer explodes and next period she would face a monopsony situation. When $t > 0$, the better firm would sometimes (for $\delta$ high enough) prefer to wait and see what the other firm has offered to the better worker, as matching that offer it would hire the worker for sure. However, anticipating this, the worker would accept the first offer she received, thereby bringing trade forward by one period. Consequently, $t > 0$ does not affect equilibrium behavior.

As the only change is the zero lower bound for the mixing interval, the expected wage of the better worker is lower with commitment than without it (as long as in the absence of commitment the mixed equilibrium would prevail). However, the mismatch probability is increased: note that the weaker firm before had a mass point at $\underline{w}$. With that offer it won if and only if the better firm bid zero. Now this same mass is distributed over $(0, \underline{w}]$, while the better firm redistributes the mass he had on $(\underline{w}, \overline{w}]$ on to $(0, \overline{w}]$. As a result, the weaker firm sometimes will win when it bids in $(0, \underline{w}]$, and it will win more often than before when it bids in $(\underline{w}, \overline{w}]$. Consequently, the weaker firm and the weaker worker expect the same as without commitment, the better worker is clearly worse off, while the effect on the better firm is ambiguous.

With more firms, the situation is less clear cut. If with positive probability there was competition for a worker in the second period, she would consider "sitting" on her offer (when $t > 0$). Of course, to keep the first period offer being mixed – otherwise there would be no reason to wait and see what the offer was going to be – we would need competition with positive probability in the first period as well. An additional factor is that a firm may decide to wait, not in order to learn the realization of a mixed wage offer, but to learn the realization of mixed targeting: a low productivity firm may want to wait and see if there was a coordination failure, leaving some high productivity worker without suitors.

## 5. A comparison of pricing schemes

As we have seen, in our market efficiency can always be achieved if the firms offer menus of personalized wages. As we do not observe this in practice, it is worthwhile to see how the equilibrium outcome changes if we put restrictions on the wages that can be offered. Bulow and Levin (2006) provide an alternative benchmark, where a firm can only make a one-time offer of a single wage, which can be accepted by any worker.[25] An alternative procedure is to retain the personalized nature of wages from the competitive set-up but to restrict each firm to a single such offer, as in this paper.[26] The comparison is more meaningful if we restrict our model to its static variant: $\delta = \beta = 0$. Konishi and Sapozhnikov (2008, Proposition 1) give a partial characterization result that we can directly employ.

Both games lead to very similar mixed strategy[27] equilibria. The targeted game has a lower upper end of the supports for each worker and it also puts a higher weight on offering a zero wage. This points in the direction of expected wages being lower under targeting, though we do not have a general proof for that.[28] The fact that workers other than the bottom one can be held to their outside options with positive probability is a novel feature in the entire literature.

The targeted game leads to the firm optimal competitive profits, while the uniform wage game leads to higher ones. Overall efficiency is likely to be higher for the uniform wage scheme. A partial explanation for that is that it does not generate unmatched pairs. Note however, that Mailath et al. (2013) show that, in a setting where investments are made prior to matching in a *competitive* market, uniform prices lead to additional inefficiencies at the investment stage.

It is interesting to observe that the main driving forces behind the structures of the two equilibria are two distinct impossibility results. In the uniform wage model no two firms can have the same support for their mixed strategy. In the personalized wage model no two firms can compete for the same two workers.

## 6. Conclusion

This paper is about the nature of endogenous competition when agents on one side of the market have to decide at the same time which agent on the other side to compete for and what to offer her. We have found that synchronous wage setting sits roughly in between its *ex ante* and *ex post* variants: if agents (especially the passive ones) are impatient the outcome is reminiscent of directed search, otherwise it is more like matching and bargaining models. Despite the superficial similarities, we have also identified that the underlying reasons are quite different.

[25] Once the offers are made public, there is either a sequential procedure where workers decide which offer to accept in decreasing order of their productivity or, equivalently, one can simply assume that a stable matching will result.

[26] Kawamura and Sákovics (forthcoming) look at an intermediate scenario, where the firms are forced to use uniform wages for only a subset of the workers, while they can make personalized offers to the rest of them. They find that the accepted personalized wages maintain their competitive distance, while the uniform ones are determined *à la* Bulow and Levin (2006). There are upwards externalities: as the wages in the uniform range are compressed, personalized wages "above" a uniform range are lower than the competitive ones.

[27] In the absence of vetting fees pure strategy equilibria can exist in the targeted-wage regime. A typical example is two firms and one worker where the weaker firm makes a potentially loss making offer, which forces the better firm to match it.

[28] A countervailing tendency is that in the uniform wage setup the number of firms mixing over the same interval is higher.

In the presence of heterogeneity, efficient matching requires the absence of direct competition, but the latter would lead to monopsony rents, making the incentives to compete too strong to resist. So, what can be done to drive such a market towards efficiency? The surprising answer is to differentially increase the bargaining power of the passive side of the market: a local monopsonist retains all of her bargaining power in equilibrium, but if her preferred seller becomes the target of a "raider" – off the equilibrium path – the ensuing price competition drives the raider's profits down. Thus, paradoxically, the increased bargaining power has an adverse effect on the passive side of the market, as it scares off the competition for them. Nothing untoward is required to achieve the above effect: all we need is a vetting cost, together with a dynamic set-up where (patient) bid takers can reject all their bids and send the game to the next period.

Despite its efficiency, the Diamond equilibrium suffers from a drawback: the workers are held to their outside options, where the latter are what they would be able to make outside the market and hence are likely to be very low. In many applications – and also from a normative point of view – this is not quite appropriate.[29] It is therefore of interest to extend the model in a such a way that despite the firms' making the initial wage offers, the workers retain some bargaining power. Assume, for example, that if (and only if) a worker receives a single offer, a bilateral bargaining game ensues. With homogeneous bargaining powers, such a modification would not affect the existence of the efficient equilibrium,[30] but would clearly increase the wages – as the workers would only accept their discounted continuation value in the bargaining game.

Finally note that other frictions, like (small) uncertainty about productivities, or non-pecuniary preferences on part of the workers, would neither be substitutes for the vetting cost, nor would they destroy the efficient equilibrium (in the presence of vetting). Their effect would be the same on the two-firm continuation game as in the main game, leaving the incentives to deviate unaffected. The vetting cost is very special in this sense as it has a different effect on and off the equilibrium path.

Let us wrap up with a thought about public intervention. As we have seen, increasing the vetting cost makes the efficient, though very skewed, outcome more likely. It is immediate that artificially inflating the vetting cost (but not too much, so that it does not overly restrict entry) via a tax and then transferring the revenue to the workers (say, via tax credits) would be a move in the right direction.

## Appendix A

**Proof of Proposition 1.** First, note that as all matches are productive, we cannot have the coexistence of a vacancy and an unemployed worker in competitive equilibrium. Next, note that no firm will hire a worker with index higher than $N$. To see this, note that otherwise there would be a worker with index lower than $N$, who did not get hired. This worker and the firm who hired the worker with index lower than $N$ would both be better off (the firm strictly so) trading with each other at the wage paid to the worker with index higher than $N$. Next, we show that the matching must be positively assortative (PAM). Assume to the contrary that Firm $I$ hires Worker $j < I$. Then there must exist a Firm $K < I$ that hires Worker $l > j$. For this to be an equilibrium, we would need that no traders would like to switch partners at the going wages: $p_{Kj} - w_{Ij} \le p_{Kl} - w_{Kl}$ and $p_{Ij} - w_{Ij} \ge p_{Il} - w_{Kl}$, implying $p_{Ij} - p_{Il} \ge w_{Ij} - w_{Kl} \ge p_{Kj} - p_{Kl}$, contradicting (reverse) supermodularity. Similarly, if Firm $I$ hired Worker $j > I$, then there would exist a firm $K > I$ that hired a Worker $l < j$, leading to the same contradiction. Hence we must have PAM in equilibrium. Using the equilibrium conditions for PAM yields

$$p_{I,i} - p_{I,i+1} \ge w_i^c - w_{i+1}^c \ge p_{I+1,i} - p_{I+1,i+1}. \tag{10}$$

Noting that the lowest individually rational salary for a worker is zero and that firms prefer low wages completes the proof. □

**Proof of Proposition 4.** Let us begin the analysis assuming that there are only two firms. Consider the subgame where both firms have made an offer. If the worker rejects both, in the continuation we have the equivalent of an asymmetric Bertrand competition. This leads to both firms offering $p_L$ with probability one,[31] and the worker taking $H$'s offer. Consequently, the worker's continuation value in this subgame is $\beta p_L$.

Let us return to the main game now (maintaining the two-firm assumption). If $L$ does not bid, then $H$'s best response is to bid zero. This can form part of an equilibrium if and only if any wage that $L$ would be willing to pay – namely, $s_L \le p_L - c$ – would be rejected by the worker.

If $L$ bids with positive probability then both $L$ and $H$ must use a mixed strategy for their wage offers (recall that the workers go with the more productive firm in case of equal wage offers). Standard arguments imply that both firms must

[29] On-the-job-search is a realistic assumption, which has been used to get around the Diamond paradox (see, for example, Burdett and Mortensen, 1998). We are looking at a market here where the workers come directly from training. But even if we incorporated on-the-job search, it would not affect the existence of the Diamond equilibrium: by construction, the wages on the job would play the same role as the outside options.

[30] Of course, it would change the mixing distributions in the inefficient equilibrium, but not its qualitative features.

[31] We do not have the mixed strategy equilibrium of Blume (2003), because we have an asymmetric rationing rule, instead of the standard fifty–fifty used by him.

mix on the same support, which we denote by $[\underline{s}, \bar{s}]$, except that $H$ may also bid zero – possibly outside of this interval – in the hope that it is the only bidder. It is straightforward to see that the only additional possible mass points in the strategies are at $\underline{s}$ for $L$ (and only if $H$ puts positive probability on zero) and $\bar{s}$ for $H$ (as a mass point there for $L$ could be simply outbid by $H$).

We start by hypothesizing that $H$ strictly prefers not to bid zero. In equilibrium, $H$ will obtain the services of the worker, if $L$ either does not bid for her (what happens with probability $\widehat{\Pi}^L$) or it offers no more than what $H$ does. If $H$ loses out in the first period, it earns zero. When $H$ offers the maximum of the common support, $\bar{s}$, then it wins for sure. As it must be indifferent among all bids in the support of its strategy, the following equality must hold for all $x \in [\underline{s}, \bar{s}]$: $(p_H - x)[\widehat{\Pi}^L + (1 - \widehat{\Pi}^L)\widehat{F}^L(x)] = p_H - \bar{s}$. Rearranging the equation, we obtain

$$\widehat{\Pi}^L + \left(1 - \widehat{\Pi}^L\right)\widehat{F}^L(x) = \frac{p_H - \bar{s}}{p_H - x}. \tag{11}$$

Now, observe that $\widehat{F}^L(\underline{s})$ must be zero, since a bid of $\underline{s}$ could never win against $H$, leading to

$$\widehat{\Pi}^L = \frac{p_H - \bar{s}}{p_H - \underline{s}} > 0. \tag{12}$$

As $L$ is assumed to make an offer with positive probability (12) implies that it must be mixing between making an offer or not, and hence it must be indifferent. Therefore, $(p_L - x)\widehat{F}^H(x) - c = 0 \Leftrightarrow \widehat{F}^H(x) = \frac{c}{p_L - x}$. Substituting $x = \underline{s}$ we obtain that $\widehat{F}^H(\underline{s}) = \frac{c}{p_L - \underline{s}}$. Note that this value is positive, as $\underline{s} < \bar{s} \le p_L - c$. This would mean that $H$ makes an offer no greater than $\underline{s}$ with positive probability, which rationally can only be an offer of zero, contradicting the hypothesis that it strictly prefers not to offer zero.

We thus know that in equilibrium $H$ weakly prefers to offer zero. We drop the "hats" of $F$ and $\Pi$ to capture the change in strategy and denote the probability of making an offer of zero by $Y$. As we have seen above, $H$ must mix, so $Y < 1$.

$H$ has to be indifferent between bidding zero (when it only wins if $L$ does not bid) and $\bar{s}$ (when it wins for sure), so we must have that $p_H \Pi^L = p_H - \bar{s} \Rightarrow$

$$\Pi^L = \frac{p_H - \bar{s}}{p_H} > 0. \tag{13}$$

By the same token, (11) – without "hats" as we have established that $H$ bids zero with positive probability – must also hold for all $x \in [\underline{s}, \bar{s}]$. Solving for the mixing distribution we have

$$F^L(x) = \frac{p_H - \bar{s}}{\bar{s}} \cdot \frac{x}{p_H - x} \in (0, 1]. \tag{14}$$

Given that $L$ is not making an offer with positive probability (see (13)), it must be indifferent between making an offer or not. Thus we have $(p_L - x)(Y + F^H(x)(1 - Y)) - c = 0 \Leftrightarrow$

$$(1 - Y)F^H(x) = \frac{c}{p_L - x} - Y, \tag{15}$$

for $x \in (\underline{s}, \bar{s})$. If there is no mass point at the upper end of $H$'s strategy, $\lim_{x \to \bar{s}} F^H(x) = 1$, then the formula still applies and we obtain that $\bar{s} = p_L - c$. If there were a mass point, then in order to keep $L$ from overbidding it must be that for all $\varepsilon > 0$, $p_L - \bar{s} - \varepsilon - c < 0 \Leftrightarrow \bar{s} \ge p_L - c$, which when applied to the formula for $\lim_{x \to \bar{s}} F^H(x)$, implies again that $\bar{s} = p_L - c$ and $F^H(\bar{s}) = 1$, therefore no mass point is possible. From (13), substituting in for the upper bound, we obtain that $\Pi^L = \frac{p_H - p_L + c}{p_H}$.

When $L$ bids the lower bound of its support, it can only win if $H$ is bidding zero. Hence, we have that $(p_L - \underline{s})Y - c = 0$, from which we can solve for $Y = \frac{c}{p_L - \underline{s}} \in (0, 1)$. Substituting in (15), we obtain

$$F^H(x) = \frac{x - \underline{s}}{p_L - c - \underline{s}} \cdot \frac{c}{p_L - x}.$$

All we have left to do is to identify the lower bound of the support of the mixed strategies. Observe that – by the single deviation principle – this has to equal the (discounted) expected continuation value of the worker when she receives two offers[32] and hence expects both firms to be still in the market in the next period. We have already established that this value is $\beta p_L$. When $p_L - c \le \beta p_L$, it is not profitable for $L$ to make a bid when $H$ is bidding for the worker. However, we also have to consider the case that $H$ is not bidding. By the same token as above, when $p_H - c \le \beta p_L$, it is not profitable for $H$ to bid when $L$ is bidding for the worker. Thus, when $p_H - c \le \beta p_L$, we have both equilibria.

[32] Whenever $L$ makes an offer, the worker will receive two offers, so this is the relevant scenario for the determination of the lower bound of $L$'s bidding distribution.

Let us consider now the case with more than two firms. We proceed in three steps. First, we show that the above equilibria continue to be equilibria. Second, we show that no equilibrium exists with more than two firms bidding with positive probability. Finally, we check whether the firms bidding can be different from $H$ and $L$.

Note that in the two-firm equilibrium $L$ always expects zero net profit. When $p_L - c \geq \beta p_L$, by making a bid that $L$ also makes in equilibrium, any firm with a lower productivity can only fare worse than $L$. By making a bid below $\beta p_L$ the entrant would win with probability $Y\Pi^L$ and it would need to offer at least $\beta p_i$ to be accepted. This leads to an expected gross profit of $\frac{p_H - p_L + c}{p_H} \cdot \frac{c}{(1-\beta)p_L}(1-\beta)p_i = \frac{p_H - p_L + c}{p_H} \cdot \frac{p_i}{p_L} \cdot c < c$. When $p_L - c \leq \beta p_L$, $p_i - c \leq \beta p_i$ so there is no room for a profitable bid for the worker.

Next note that $H$ can guarantee itself $p_H - p_L$, the amount it makes in the two-firm equilibrium (for low $\beta$). Any other player who bids, must expect to recover the vetting cost, $c$. Thus, if we had more than two bidders, the worker should expect a lower wage than with two bidders, what is clearly impossible.

It is straightforward to see that if the two firms bidding were not $H$ and $L$ then the one left out could outbid the intruder and expect strictly more than $c$. Finally, as we have seen before, Firm $i$ could be the only bidder as long as $p_H - c \leq \beta p_i$. $\square$

## Appendix B. Mathematical appendix

### *B.1. Intermediate steps to get to (9) from (8)*

Dividing across by the common factor in (8), we have

$$\begin{aligned}
&\frac{\underline{w}}{\beta(p_{Ll}-c)(1-\delta)(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})} \\
&\quad = \frac{1}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}} \cdot \frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)} \cdot \frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}} \\
&\qquad + \int_{\underline{w}}^{\widetilde{w}} \frac{x}{(p_{Hh}-\delta(p_{Hl}-c)-x)(p_{Lh}-\delta(p_{Ll}-c)-x)^2} dx \\
&\qquad + \int_{\underline{w}}^{\widetilde{w}} \frac{x}{(p_{Hh}-\delta(p_{Hl}-c)-x)^2(p_{Lh}-\delta(p_{Ll}-c)-x)} dx.
\end{aligned}$$

Using that $\int \frac{x}{(a-x)^2(b-x)}dx = \frac{b \ln \frac{a-x}{b-x}}{(a-b)^2} - \frac{a}{(a-b)(a-x)}$ the equation becomes

$$\begin{aligned}
&\frac{\underline{w}}{\beta(p_{Ll}-c)(1-\delta)(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})} - \frac{\underline{w}^2}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c))(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})} \\
&\quad = \frac{p_{Hh}-\delta(p_{Hl}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\left(\ln\frac{p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}} - \ln\frac{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}\right) \\
&\qquad + \frac{p_{Lh}-\delta(p_{Ll}-c)}{p_{Lh}-p_{Hh}+\delta(p_{Hl}-p_{Ll})} \cdot \left(\frac{1}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}} - \frac{1}{p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w}}\right) \\
&\qquad + \frac{p_{Lh}-\delta(p_{Ll}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\left(\ln\frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w}} - \ln\frac{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}\right) \\
&\qquad + \frac{p_{Hh}-\delta(p_{Hl}-c)}{p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll})} \cdot \left(\frac{1}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}} - \frac{1}{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}\right) \\
&\quad = \frac{p_{Hh}-\delta(p_{Hl}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\ln\frac{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})} \\
&\qquad + \frac{p_{Lh}-\delta(p_{Ll}-c)}{p_{Lh}-p_{Hh}+\delta(p_{Hl}-p_{Ll})} \cdot \frac{\underline{w}-\widetilde{w}}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})} \\
&\qquad + \frac{p_{Lh}-\delta(p_{Ll}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\ln\frac{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})}{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})} \\
&\qquad + \frac{p_{Hh}-\delta(p_{Hl}-c)}{p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll})} \cdot \frac{\underline{w}-\widetilde{w}}{(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})}
\end{aligned}$$

$$= \frac{1}{p_{Hh} - p_{Lh} - \delta(p_{Hl} - p_{Ll})} \ln \frac{(p_{Lh} - \delta(p_{Ll} - c) - \widetilde{w})(p_{Hh} - \delta(p_{Hl} - c) - \underline{w})}{(p_{Hh} - \delta(p_{Hl} - c) - \widetilde{w})(p_{Lh} - \delta(p_{Ll} - c) - \underline{w})}$$
$$- \frac{\underline{w} - \widetilde{w}}{p_{Hh} - p_{Lh} - \delta(p_{Hl} - p_{Ll})} \left( \frac{p_{Lh} - \delta(p_{Ll} - c)}{(p_{Lh} - \delta(p_{Ll} - c) - \underline{w})(p_{Lh} - \delta(p_{Ll} - c) - \widetilde{w})} \right.$$
$$\left. - \frac{p_{Hh} - \delta(p_{Hl} - c)}{(p_{Hh} - \delta(p_{Hl} - c) - \underline{w})(p_{Hh} - \delta(p_{Hl} - c) - \widetilde{w})} \right).$$

Moving everything to the LHS and multiplying across by $p_{Hh} - p_{Lh} - \delta(p_{Hl} - p_{Ll})$, we obtain (9).

*B.2. RHS of (9) is increasing in $\underline{w}$ on $(-\infty, p_{Lh} - p_{Ll} + c)$*

To enable Scientific Workplace, we eliminate the subindices, denoting $p_{Hh}$ by $H$, $p_{Lh}$ by $h$, $p_{Ll}$ by $L$ and $p_{Hl}$ by $l$.

$$\frac{x(H-h-\delta(l-L))}{\beta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)} - \frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}$$
$$-(h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)} - \frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)$$
$$-\ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}.$$

$$\frac{d\left(\frac{x(H-h-\delta(l-L))}{\beta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)}\right)}{dx} = -\frac{1}{\beta(\delta-1)(L-c)}\frac{H-h+L\delta-l\delta}{H+L-c-h+c\delta-l\delta}$$

$$\frac{d\left(-\frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}\right)}{dx} = \frac{x}{H+c\delta-l\delta}\frac{H-h+L\delta-l\delta}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2}$$
$$\left(Hx-2Hh-2c^2\delta^2+hx+2HL\delta-2Hc\delta-Lx\delta-2ch\delta+2hl\delta+2cx\delta-lx\delta+2Lc\delta^2-2Ll\delta^2+2cl\delta^2\right)$$

$$\frac{d\left(-(h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)} - \frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)\right)}{dx}$$
$$= \frac{H-h+L\delta-l\delta}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2}$$
$$\left(c^2\delta^2+Hh-x^2-HL\delta+Hc\delta+ch\delta-hl\delta-Lc\delta^2+Ll\delta^2-cl\delta^2\right)$$

$$\frac{d\left(-\ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}\right)}{dx} = -\frac{H-h+L\delta-l\delta}{(H-x+c\delta-l\delta)(h-x-L\delta+c\delta)}.$$

Putting the terms together and dividing by $H-h+L\delta-l\delta>0$ (recall that $H-h>l-L$ by submodularity of the production function):

$$-\frac{1}{\beta(\delta-1)(L-c)}\frac{1}{H+L-c-h+c\delta-l\delta} + \frac{x}{H+c\delta-l\delta}\frac{1}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2}$$
$$\left(Hx-2Hh-2c^2\delta^2+hx+2HL\delta-2Hc\delta-Lx\delta-2ch\delta+2hl\delta+2cx\delta-lx\delta+2Lc\delta^2-2Ll\delta^2+2cl\delta^2\right)$$
$$+\frac{c^2\delta^2+Hh-x^2-HL\delta+Hc\delta+ch\delta-hl\delta-Lc\delta^2+Ll\delta^2-cl\delta^2}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2} - \frac{1}{(H-x+c\delta-l\delta)(h-x-L\delta+c\delta)}.$$

Note that the last term is decreasing in $x$. Therefore we can bound it from below by substituting the largest possible $x = h-L+c$. The last term then becomes $-\frac{1}{(H+L-c-h+c\delta-l\delta)(1-\delta)(L-c)}$. Adding it to the first term, we have $\frac{1-\beta}{\beta(1-\delta)(L-c)(H+L-c-h+c\delta-l\delta)}$. This is positive as long as $H+L-c-h+c\delta-l\delta>0$, which holds by submodularity and the fact that $c<\min\{H,L,h,l\}$. We can multiply the rest of the terms by $(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2$:

$$\frac{x}{H+c\delta-l\delta}\cdot\left[2\delta^2(L-c)(c-l)+2\delta\big(H(L-c)+h(l-c)\big)+x\delta(2c-l-L)+x(h+H)-2hH\right]$$
$$+\delta^2(L-c)(l-c)-\delta\big(H(L-c)+h(l-c)\big)+Hh-x^2$$
$$=\left[\delta^2(L-c)(l-c)-\delta\big(H(L-c)+h(l-c)\big)+Hh\right]\left[1-\frac{2x}{H+c\delta-l\delta}\right]+x^2\left[\frac{\delta(2c-l-L)+h+H}{H+c\delta-l\delta}-1\right]$$
$$=\big(h-\delta(L-c)\big)\left(H-\delta(l-c)-2x+\frac{x^2}{H+c\delta-l\delta}\right).$$

The first term is positive, the second is positive if $x < H - \delta(l - c)$. Finally, note that $H - \delta(l - c) > H - l + c > h - L + c$, by submodularity. □

*B.3. RHS of (9) is negative at $\underline{w} = 0$*

$$\Bigg[\frac{x(H-h-\delta(l-L))}{\beta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)} - \frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}$$
$$- (h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)} - \frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)$$
$$- \ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}\Bigg]_{x=0}$$
$$= \left(\frac{1}{(\delta-1)(L-c)} + \frac{1}{H+L-c-h+\delta(c-l)}\right)(c-L+h) - \ln\left(-\frac{H+\delta(c-l)}{h-\delta(L-c)}\frac{c-L+\delta(L-c)}{H+L-c-h+\delta(c-l)}\right).$$

Now recall that $\ln y \geq 1 - 1/y$. Hence, the above is no more than

$$\left(\frac{1}{(\delta-1)(L-c)} + \frac{1}{H+L-c-h+\delta(c-l)}\right)(c-L+h) - \left(1 + \frac{h-\delta(L-c)}{H+\delta(c-l)}\frac{H+L-c-h+\delta(c-l)}{(\delta-1)(L-c)}\right)$$
$$= \frac{c-L+h}{(\delta-1)(L-c)} - \frac{H+\delta(c-l)}{H+L-c-h+\delta(c-l)} - \left(\frac{h-\delta(L-c)}{H+\delta(c-l)}\frac{H+L-c-h+\delta(c-l)}{(\delta-1)(L-c)}\right).$$

Multiplying across by $(L-c)(1-\delta) > 0$ we get

$$L-c-h - \frac{(H+\delta(c-l))(L-c)(1-\delta)}{H+L-c-h+\delta(c-l)} + \frac{h-\delta(L-c)}{H+\delta(c-l)}\big(H+L-c-h+\delta(c-l)\big)$$
$$= (L-c)(1-\delta)\left[1 - \frac{H+\delta(c-l)}{H+L-c-h+\delta(c-l)}\right] + \frac{h-\delta(L-c)}{H+\delta(c-l)}(L-c-h)$$
$$= (L-c)(1-\delta)\frac{L-c-h}{H+L-c-h+\delta(c-l)} + \frac{h-\delta(L-c)}{H+\delta(c-l)}(L-c-h)$$
$$= (L-c-h)\left[\frac{(L-c)(1-\delta)}{H+L-c-h+\delta(c-l)} + \frac{h-\delta(L-c)}{H+\delta(c-l)}\right],$$

the first term is clearly negative, while the second is positive. □

*B.4. RHS of (9) is positive at $\underline{w} = p_{Lh} - p_{Ll} + c$*

$$\Bigg[\frac{x(H-h-\delta(l-L))}{\beta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)} - \frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}$$
$$- (h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)} - \frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)$$
$$- \ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}\Bigg]_{x=h-L+c}$$
$$= \frac{1}{H+\delta(c-l)}\frac{(c-L+h)^2}{c-L+\delta(L-c)}\frac{H-h+\delta(L-l)}{H+L-c-h+\delta(c-l)} - \frac{1}{\beta(\delta-1)(L-c)}(c-L+h)\frac{H-h+\delta(L-l)}{H+L-c-h+\delta(c-l)}.$$

Dividing by the common positive term $\frac{(c-L+h)(H-h+\delta(L-l))}{H(h-l\delta)+c(\delta-1)-L(h-l)}$:

$$\frac{1}{\beta(\delta-1)}\cdot\frac{1}{c-L} + \frac{1}{H+\delta(c-l)}\frac{c-L+h}{(\delta-1)(L-c)}.$$

Multiplying by $(L-c)(1-\delta)$:

$$\frac{1}{\beta} - \frac{c-L+h}{H+\delta(c-l)} = \frac{H+\delta(c-l)-\beta(c-L+h)}{(H+\delta(c-l))\beta} > \frac{H-(c-L+h)}{(H+\delta(c-l))\beta} = \frac{H-h+L-c}{(H+\delta(c-l))\beta} > 0. \qquad \square$$

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