Current trends in Economics
(Alkan, Aliprantis, Yannelis ed.)
Springer, 1999

Bargaining in a changing environment*

Clara Ponsatí[1] and József Sákovics[2]

[1] CODE i Departament d'Economia i Història Econòmica, Universitat Autònoma de Barcelona 08193 Bellaterra, Spain
[2] Institut d'Anàlisi Econòmica (CSIC),Campus UAB, 08193 Bellaterra, Spain

Abstract. We study bilateral negotiations where both parties have outside options of uncertain value. First, we argue that the relevant attribute of an outside option is the time by which its value is revealed rather than the time up to which it is available. Second, we show that a random outside option is potentially worth much more – and never less – than its straight expected value. Third, we prove that, rather than by threats, delays might be caused by the bargainers' waiting for their outside option to become available and/or to reveal its value. Finally, we point out that as additional outside options are added to the opportunities of a bargainer, the strategic advantage of the original outside options may decrease.

JEL Classification Numbers: C78

Keywords: Random outside option, bargaining

1 Introduction

Bargaining does not take place in a vacuum, but rather in a changing environment. The opportunities available to players if they decide to quit bargaining change, and the options' known or uncertain characteristics also may vary over time. In this paper we provide insight on how bargaining develops in changing environments by analyzing a bargaining game where both players enjoy outside options of uncertain value that arise and cease to be available over time.

We will first argue that the relevant attribute of an outside option is the time by which its value is revealed rather than the time up to which it is available. Second, we will show that a random outside option is potentially worth much more – and never less – than its straight expected

*We have benefited from the comments and suggestions of two anonymous referees on an earlier version of this work. Support to Ponsatí through research grants PB-96-1192 from DGCYT and SGR-96-75 from CIRIT and to Sákovics through HCM program of the EC contract CHRX-CT94-0489 "Games and Markets" and from DGCYT through grant PB93-0679 is gratefully acknowledged.

value. Third, we will prove that, rather than being caused by threats, delays might be caused by the bargainers' waiting for their outside option to become available and/or to reveal its value. Finally, we will point out that the prospect of several future outside opportunities is not necessary better than the prospect of a single one, since as additional outside options appear the strategic advantage of the original ones may decrease.

Here we propose a sample of four stylized bargaining stories consistent with the distinctive elements of our model, i) that the value of outside opportunities is uncertain, ii) that both players enjoy the possibility of these opportunities, and iii) that strategic capabilities depend crucially on the dates the options appear and mature and on the dates uncertainties are removed.

1) Arbitration: The Teachers Union bargains with the School Board of City A. Each party can unilaterally call for an arbitrated decision. The arbitrator has been nominated recently and has no record on school cases. However she has been called to arbitrate in a similar case in City B and will be announcing a decision in 10 days.

2) Sports: Soccer Star A bargains over his contract with team B while he is also waiting to hear an offer from another team next week. Medical reports on another player are also due next week. Depending on these medical reports, the alternative player will or will not be fit to play as a perfect substitute of A. Agreements must take place before the season starts in two weeks.

3) Home-ownership: Buyer B and seller S are negotiating over the sale of S's home. There is another house that B likes but its owner is away on a trip until next week. S has an appointment with another potential buyer also next week.

4) Exclusive vs. standard designs: Two firms, F and f, bargain over a contract that could make f the provider of an exclusively designed component in F's production. There is a competitive market for standard components where F and f can buy and sell respectively. Both firms can adapt their production to standard or exclusively designed components, but not to both. Both firms have complete information on the costs and benefits of the exclusive design alternative. In contrast, there is uncertainty on how well the standard design adapts to F's needs, and there is uncertainty on f's costs to produce the 'standard' component. The technical departments at both firms are carrying out research to remove this uncertainty and their reports are due on day D.

With uncertainty about the sizes of the outside options, as in all our bargaining stories, the most relevant question is: When does the uncertainty resolve? A moment of reflection suffices to see that if it is never resolved then –in the absence of risk aversion [1]– the uncertain options would be strategically equivalent to fixed options with a value equal to the expected value of the former ones. Therefore, a crucial element of our model is the revelation date, a parameter which gives the time period at the beginning of which the uncertainty is resolved, via a random draw according to the prior distribution. In addition, we introduce two more dates that are significant in an option's life: the first date it is available and the last period in which it can be taken, its maturity date. Moreover, in a bargaining situation which is symmetric – in the sense that both parties can make proposals – we find it more realistic that both players be offered outside opportunities. Moving from one-sided to two-sided outside options has dramatic effects[2] and the combination of uncertainty and two-sided options yields results that are in sharp contrast with the earlier results of Shaked and Sutton (1984) (where the "outside option principle"[3] was formulated) and Shaked (1994)[4], where one-sided, certain outside options are added to a Rubinstein game.

The maturity date makes our model practically finite, since the subgame following it is the original Rubinstein game, which has a unique subgame-perfect equilibrium. This makes possible the use of backward induction to resolve the game, yielding a unique solution despite the two-sided options environment. We show that, apart from this important conceptual effect – as long as it is posterior to the revelation date – the magnitude of the maturity date is irrelevant.

The outside options affect the nature of equilibrium in several ways. Players can take their outside options before their values are revealed if their distributions are sufficiently attractive, for the outside option's value is not discounted after it is selected. Further the players may wait to learn their outside option before negotiating. Finally, even a mid level outside option value may lead to an extreme division of surplus. Once it is credible for the proposer to accept her outside option after the current period, then the responding player accepts any offer equal to her reservation value or

[1] Although the effect risk attitudes on the play of the game is certainly of interest, in this paper we assume risk neutrality, and concentrate on the strategic relevance of an "ill-specified" threat without considering the risks involved.

[2] In Ponsati and Sákovics (1998a) we make this point in a model without uncertainty.

[3] This principle says that unless the solution without the possibility of opting out gives a lower payoff to one of the players than her outside option, the outside options do not affect the outcome of the negotiation. Note that as Sutton (1986) and Dalmazzo (1992) show, the OOP does not hold in such a neat way if the model is augmented by an exogeous risk of breakdown or a shrinking cake. However, it is generically true that the outside options cannot be identified with the disagreement point.

[4] He shows that if it is the proposer who can threaten to take his outside option, the strategic consequences are markedly different because if the outside option exceeds his continuation value in case he does not leave the game he can appropriate the entire surplus, by making a take-it-or-leave-it offer. Thus, there exist a range of outside options (strictly between zero and one) for which there exist multiple equilibria.

The main insight of our model is that an uncertain outside option has many of the properties of a stock option. The strategic value of the outside option is a function of the right-tail of its cumulative distribution. The distinction between outside options and stock options is that an outside option is never exercised (if the values of the outside options leave room for surplus), so that the exact value of the outside option is not important so long as it is above a threshold to make its acceptance a credible threat. The potential for exercising an option at a later date if its value is sufficiently large increases the payoff for a given player in any agreement.

As an extension, we examine the case when a player can choose among several outside options. Our main findings here are that whether an option has an effect on the outcome depends exclusively on the upper end of the support of its distribution and that the presence of an alternative option decreases the strategic value of a given outside opportunity.

A complementary, more general, motivation for our work is the observation that in the real world many "games" are played at the same time (possibly involving the same player in multiple games) and therefore the evolution of game A has direct effects on a player's preferences (and therefore her strategy) in game B. In our Soccer tale, A may expect a negotiation with the alternative team to start after hearing a first offer. In our home-ownership tale, the owner of the alternative house and B (or S and the potential buyer that will visit her next week) may enter a negotiation. In both cases these negotiations will influence and be influenced by the events in our main story. Of course, the straightforward way to deal with such situations would be to model the set of interacting games as a unique game thus internalizing all the externalities. However, we believe that understanding simple, two-person games can provide important insight that will help in understanding of the more complicated games.[5] Therefore, our approach is to concentrate on bilateral bargaining while taking into account at least the most important cross-effects.[6] In this paper, these boil down to the consideration of random outside options.

Finally, a few words on related literature. Models of strategic bargaining with symmetric but imperfect information are very scarce. Vislie (1988) model of Shaked and Sutton (1984) and derives the corresponding unique equilibrium. Avery and Zemsky (1994a) consider a model in which exogenous shocks affect the gains from trade. The outcome of each shock is revealed after the proposer made her offer but before the responder's move. Consequently, the responder can use this "private" information to reduce the proposer's "first mover" advantage. In equilibrium, the first proposal is not accepted for all possible realizations of the shock and therefore there will be delay with positive probability before agreement occurs. This inefficiency, in turn, may cause multiple equilibria, and hence deterministic delay, according to the Money Burning Principle (see, Avery and Zemsky (1994b)). Finally, Merlo and Wilson (1995) also analyze a stochastic model of sequential bargaining, although they only consider the variability of the available surplus and do not allow for (random) outside options, they also obtain equilibria where the players disagree in some states of the world because they both expect to improve their shares in the future.

The paper is organized as follows. Section 2 presents the set up, definitions and instrumental results that are used for characterizing equilibria in Section 3. Section 4 addresses the model with multiple successive outside options. Section 5 concludes.

2 The strategic value of a random outside option

As mentioned above, we use the Rubinstein procedure as the base model of our bargaining process. In addition, we assume that there is an outside option[7] available to each player that they can take at the end of every period. The values of the options, $\tilde{a} = (\tilde{a}_1, \tilde{a}_2)$, are random variables, distributed according to the joint cdf $F : [0,1] \times [0,1] \rightarrow [0,1]$, which is common knowledge between the players. The unconditional expected value of the option of Player i is denoted by E_i. In addition, the options are characterized by a three-tuple – which, for simplicity, we assume is the same for both of them: $\{T^a, (\leq), T^r, (\leq), T^m(\infty)\}$. T^a is the first period in which the options are available. If a player takes her option at any time before it is available, her payoff is discounted up to this time. At the beginning of period T^r the uncertainty is revealed, and the realization of the random variables becomes common knowledge. Finally, T^m is the maturity date of the options: if they have not been taken at T^m or before, they cease to be available.

Let us start the analysis by defining the most crucial attribute of a random option:

Definition 1 *The strategic value of the random option of Player i, A_i, is the utility he expects to obtain in the unique subgame-perfect Nash equilib-*

[5] We do not wish to play down the importance of the research carried out on decentralized markets (see Osborne and Rubinstein (1990) for a survey of the pre-nineties literature; see also Hendon and Tranaes (1991), Moldovanu (1993) and Hendon et al. (1994)). However, the primary concern of this literature is to see whether these markets implement Walrasian equilibria and if not how efficient they are, while they treat the fine details of the externalities in a fleeting manner.

[6] One of the obvious effects of this genre is the consequence of bounded rationality: players with limited memory, computing power, etc. will have to "ration" their resources among the games they are simultaneously playing. Without discounting the importance and interest of this type of considerations, in this paper we would like to concentrate on another aspect of the externalities.

[7] For brevity's sake, from here on we will refer to it simply as an option.

rium of the subgame starting at T^r.[8]

As Lemma 1 demonstrates, the strategic value of an option is always well defined, and it is never less, and is often significantly more than its expected value. In order to clarify the intuition behind the lemma, we first present the result for the case where only one player has an option.

Lemma 1 *If only Player 1 has an option, its strategic value to her*[9] *is*
i) If $T^r = T^m$,

$$A_1 = \frac{1}{1+\delta}F_1\left(\frac{\delta^2}{1+\delta}\right) + \int_{\frac{\delta^2}{1-\delta}}^{1} 1dF_1(x) = 1 - \frac{\delta}{1+\delta}F_1\left(\frac{\delta^2}{1+\delta}\right)$$

if 1 proposes at T^r,

$$A_1 = \frac{\delta}{1+\delta}F_1\left(\frac{\delta}{1+\delta}\right) + \int_{\frac{\delta^2}{1+\delta}}^{1} xdF_1(x)$$

otherwise.
ii) If $T^r < T^m$,

$$A_1 = \frac{1}{1+\delta}F_1\left(\frac{\delta^2}{1+\delta}\right) + \int_{\frac{\delta^2}{1+\delta}}^{1} 1dF_1(x) = 1 - \frac{\delta}{1+\delta}F_1\left(\frac{\delta^2}{1+\delta}\right)$$

if 1 proposes at T^r,

$$A_1 = \frac{\delta}{1+\delta}F_1\left(\frac{\delta^2}{1+\delta}\right) + \int_{\frac{\delta^2}{1+\delta}}^{1} xdF_1(x)$$

otherwise.

Proof. It follows from Lemma 2 below. ■

Note that when the maturity date coincides with the revelation of the realization of the option,[10] if there is an arbitrarily small positive probability that the option realizes a value above that period's Rubinstein share (or, if T^r is odd, an even lower value) then, irrespective of the rest of the distribution of her option, Player 1's continuation value will exceed what she would obtain in the Rubinstein game starting in period T^r. Therefore, a random outside option is potentially worth much more – and never less – than its straight expected value; hence it is to be evaluated at its "certainty equivalent": the option's strategic (or conditional) expected value.

[8]Note that this definition refers to the gross effect of the option, taking into account the continuation value that would result in its absence.

[9]Note that when only one player has an option $A_1 + A_2 \equiv 1$.

[10]If the revelation date preceeds the maturity date the situation is less clear cut, but it is qualitatively similar.

It is interesting to observe that while this is effectively an "option value" effect, the parallel with options in financial markets applies with the roles reversed: it is the continuation value not the outside opportunity that plays the role of an option (understood as a derivative security). That is, when a player buys the opportunity to get to an agreement with his bargaining partner after he learns about the value of his outside option, he is effectively buying an option not to have to take his outside option.

Lemma 2 *If both players have options, their strategic values to the players are (i being the proposer in the period):*[11]
i)If $T^r = T^m$,

$$A_i = \frac{1}{1+\delta}F\left(\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta}\right) + \iint_{x_j \leq \frac{\delta}{1+\delta}, x_j \leq \frac{\delta^2}{1+\delta}, x_1+x_2 \leq 1} (1-x_j)\, dF(x)$$
$$= \iint_{x_1+x_2>1} x_i dF(x)$$

$$A_j = \frac{\delta}{1+\delta}F\left(\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta}\right) + \iint_{x_j > \frac{\delta}{1+\delta}, x_i > \frac{\delta^2}{1+\delta}, x_1+x_2 \leq 1} x_j dF(x)$$

ii) If $T^r < T^m$,

$$A_i = \frac{1}{1+\delta}F\left(\frac{\delta^2}{1+\delta}, \frac{\delta^2}{1+\delta}\right) + \iint_{x_j, x_i \leq \frac{\delta^2}{1+\delta}, x_1+x_2 \leq 1} (1-x_j)\, dF(x)$$
$$= \iint_{x_1+x_2>1} x_i dF(x)$$

$$A_j = \frac{\delta}{1+\delta}F\left(\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta}\right) + \iint_{x_j, x_i > \frac{\delta^2}{1+\delta}, x_1+x_2 \leq 1} x_j dF(x)$$

Proof. If the sum of the realizations of the options, $a_1 + a_2$, exceeds one, then at least one player will prefer taking her option right away to any other feasible equilibrium payoff, and thus the result obviously holds true. Otherwise, we have two cases to consider, T^m even and T^m odd. If is even, the option matures at a date when Player 2 makes the offer. Now, observe that, since after this date the game reverts to a standard Rubinstein game, in the last period of the options' life Player 1's unique subgame-perfect equilibrium payoff is a_1 if $a_2 \geq \frac{\delta^2}{1+\delta}$, and $\max\{a_1, \frac{\delta}{1+\delta}\}$ otherwise. Similarly, if is odd, Player 1's unique subgame-perfect equilibrium offer (which is always accepted) is $1 - a_2$ if $a_1 \geq \frac{\delta^2}{1+\delta}$ and $\min\{1 - a_2, \frac{1}{1+\delta}\}$ otherwise. To see this, note that if a player can threaten to opt out in case her offer

[11]We adopt the convention that the first argument of F(.,.) is Player i's option.

is not accepted, she can claim the whole surplus – less the other players' option value -, whenever such a threat is credible.[12] Moreover, note that the continuation value of the proposer at T^m in case she stays is her Rubinstein share in the next period, which has a present value of $\frac{\delta^2}{1+\delta}$ for her. Thus opting out is credible if and only if her outside option is larger than this value. If taking the option by the proposer is not credible then, by the Outside Option Principle, that threat simply has no effect and either the responder earns the value of his option or the Rubinstein outcome prevails. This completes the proof of i).

If $T^r < T^m$, using the Shaked-Sutton backward induction argument, we can derive the players' continuation values at T^r. We have two cases to consider, depending on whether the realized options are above or below the threshold value for credibility for the proposer's option, $\frac{\delta^2}{1+\delta}$.

If neither option is credible, then, by the Outside Option Principle, they do not affect the negotiation, since $\frac{\delta^2}{1+\delta} \leq \frac{\delta}{1+\delta}$. Therefore, the players' continuation values at T^r are simply their appropriate (depending on who makes the offer at T^r) Rubinstein shares.

If at least one player's option exceeds $\frac{\delta^2}{1+\delta}$, then in all periods between (and including) T^r and $T^m - 1$, the unique subgame payoff for Player i is $1 - a_j$ if she is the proposer and a_i otherwise. To see this, first note that, if the claim is true a period then it is also true for the previous period, since the options will always be credible for the proposer (he expects a_j in the next period, which has a discounted value of $\delta a_j \leq a_j$). As we saw in the first paragraph of this proof, if the proposer's option is credible, the unique subgame equilibrium is the one posited above. Thus, all we have left to prove is that for period $T^m - 1$ the result holds also. To see this, let j be the proposer in that period. This implies that in period T^m i's continuation values are a_i or $max\{a_i, \frac{\delta}{1+\delta}\}$, when j's option is credible and when it is not, respectively. Then i's option is credible *in* period $T^m - 1$, since $\delta a_i \leq a_i$, and since $\frac{\delta^2}{1+\delta} \leq a_i$, whenever j's option is not credible, by assumption. ■

Lemma 2 is an interesting result on its own right, since in its proof the unique subgame perfect equilibrium of a complete information alternating-offer game with a finite lived outside option for each player is characterized.

Note that the maturity date is only relevant insofar as it is equal or not to the revelation date, independently of the identity of the last proposer before the option expires. Moreover, when the revelation date is different from the maturity date, the only difference is that the responder needs a lower realization of her option to influence the outcome. Otherwise, the maturity date is irrelevant. In fact, the exact date of maturity need not be known to the players to obtain our results, which can be extended for any (common knowledge) distribution over maturity dates that has a finite support.

Observe also that, even though we allow for opting out in every period, we still obtain a unique equilibrium, unlike Shaked (1987/1994). There, one player has an option, and that player's choice to exercise the option can be credible in some but not all periods. Here, since both players have outside options, if the option is a credible choice by one player in a given period, then it is also a credible choice by the other player in the previous period (since he will be offered only the option next period, the option today is a better payoff and thus taking it is a credible threat). Thus if an option is to be credible in the future, then both options are also credible at all earlier stages of the game. This, in principle, does not rule out multiplicity: in Ponsati and Sákovics (1998a) both players can opt out in every period, their options are credible and still there is multiplicity. The difference, however, is that our present model is not stationary (it is effectively finite horizon) and therefore the continuation values are fixed (that is, unique) in period T^r, which, by backward induction, yields a unique outcome in all previous periods too.

3 The SPE with a single outside option for each player

Knowing the value of the subgame starting at T^r, we can derive the solution of the entire game. Let us start with a useful definition:

Definition 2 *Take a Rubinstein game without the possibility of opting out and with exogenously imposed continuation values, A_I and A_{II} ($A_I + A_{II} \leq \frac{1}{\delta}$)*[13] *, for the players in period T. The backward inducted strategic value (BISV) of the period T proposer at t < T is that player's continuation value in period t.*

The BISV is calculated via simple backward induction. Suppose it is Player i who proposes in period T. Then, in equilibrium, in period T-1 Player j has to offer at least δA_i to i, while she is willing to offer no more than a share of $1 - \delta A_j$ to him. Note that, by assumption, $\delta A_i \leq 1 - \delta A_j$, so she will offer $1 - \delta A_j$ to i, giving her a payoff of $1 - \delta A_i$ (since i will accept). In consequence, in period T-2 i has to offer $\delta(1 - \delta A_i)$ to j, and so on. Overall, Player 1's continuation value in period t < T is given by the following table – Player 2's payoff is one minus this value. (Row gives the proposer in period t, Column the proposer in period T):

[12] This argument was first put forward in Shaked (1987/1994).

[13] Note that if at some point in time the discounted sum of the players' strategic values exceeds 1 then at least one of them will always prefer waiting until period T^r to agreeing at any value.

	1	2
1	$A_I\delta^{T-t}+\frac{1-\delta^{T-t}}{1+\delta}$	$1-A_{II}\delta^{T-t}-\frac{\delta-\delta^{T-t}}{1+\delta}$
2	$A_I\delta^{T-t}+\frac{\delta-\delta^{T-t}}{1+\delta}$	$1-A_{II}\delta^{T-t}-\frac{1-\delta^{T-t}}{1+\delta}$

Table 1

Let us now interpret A as the strategic value of an option in period T. Observe that the option is effective (that is, it modifies the partition that would result in its absence) if and only if A is larger than its owner's Rubinstein share in period T, independent of the size of T (c.f. the remark following Lemma 1). This observation should be interpreted as a generalization (and strengthening) of the Outside Option Principle.[14] The important fact here is that even though the option is to be discounted in case it is taken, it is its undiscounted value what matters when it is used as a threat. In a parallel way, the value of the outside option at the beginning of the bargaining game is significantly more than simply its discounted strategic value. Whenever the option is effective, its (strategic) value at time zero is the sum of two terms: its discounted strategic value and an increasing (!) proportion $\left(1-\delta^T\right)$ of the Rubinstein share.[15] By the proof of Lemma 2 it is quite clear why is it A what matters instead of the straight expected value of the option. The other term is a direct consequence of the Rubinstein formulation.

How does the strategic advantage derived from having the option trade off against the "first mover advantage"? The direct comparison of the four different situations with different bargaining power (1 or 2 makes the first/last offer) is not feasible because the different delays that necessarily arise between the first offer and the revelation date. Instead, we propose the comparison of two hypothetical situations, where both the strategic value and the revelation date, T-t, are assumed to stay constant while we change the identity of the first proposer. In this case, the difference between being a first or a second mover is $\frac{1-\delta}{1+\delta}$ just as in the game without outside options.

Let us now return to the characterization of the equilibria of our game with stochastic outside options. The equilibrium behavior from period T^τ on we have already settled. To fully characterize the equilibrium we use backwards induction, simply observing that the equilibrium moves in previous periods are more restricted depending on which is the unique[16] subgame equilibrium in some period. As the revelation date increases there are four possible phases in the equilibrium play. These must follow each other in a monotone order. However, depending on the parameters, some of the phases may not come about. The phases as the revelation date moves into the future are i) "dealing out" the opponent at the discounted expected value of his option; ii) taking the options before their value is known; iii) "dealing out" at the discounted expected value of the other's continuation (which includes an "option value"); iv) waiting for next period. We summarize this exercise in the following straightforward proposition.

Proposition 3 *The game with an outside option for each player has a unique SPE. It can be computed by backwards induction as follows:*

i) Compute the unique equilibrium behavior from period T^τ on (see Lemma 1). Using these strategic values, the equilibrium action profiles at $T^\tau-1$ can be derived easily. Depending on the parameters, these must be one of the following:

a) waiting for next period's payoff;

b) agreeing at a value holding the responder to the discounted expected value of her continuation payoff next period;

c) taking their respective outside options (without knowing their realization);

d) agreeing at a value holding the responder to the (possibly) discounted expected value of her option.

ii) For all $t \le T^\tau-1$, if the equilibrium action profiles at t are

a) then the equilibrium action profiles at t -1 are (a), (b), (c) or (d);

b) then the equilibrium action profiles at t -1 are (b), (c) or (d);

c) then the equilibrium action profiles at t -1 are either (c) or (d);

d) then the equilibrium action profiles at t -1 must be (d).

Let us now interpret what the proposition tells us (this will also provide a sketch of the proof). First of all, it formalizes the intuition that if an outside option has high option value (that is, its prior distribution has large variance) then the players may find it in their interest to delay agreement until their information about their outside opportunities improves. If the revelation date is "too far" in the future then the players will prefer to end the game immediately. If neither player prefers her option to it they will agree at the corresponding BISV (b). Note that if in some period $t < T^\tau$ both players prefer agreeing at the BISV of one of them to waiting,

[14] Rubio (1994) has shown a similar result, for a time-varying outside option with ex ante known value.

[15] Note, however, that the equilibrium share still has a decreasing trend in T.

[16] We assume that when players are indifferent between two actions they choose the one that ends the game earlier.

then they do so in all previous periods as well.[17] If at least one does then they either both take the options (c) or they agree at the value which gives the responder exactly as much as he would have obtained (in expectation) opting out (d). Now, if the options become available sufficiently before their revelation date then it may be the case that in periods previous to ones where they would agree, they take their options (since the options do not get discounted). Finally, if in a period before T^a they both take their options then in (sufficiently) earlier periods agreement will be the unique equilibrium, since the discounted sum of the expected values of the options decreases below one.

Having described the nature of the bargaining process when the negotiators have a single outside option, we now turn our attention to the case where the same player may choose among several outside opportunities.

4 Multiple options

In this section we analyze the case when Player 1 (the first proposer) has several options available to her if she leaves the bargaining game. Specifically, assume that there are N options, and option i matures at T_k and its value is distributed according to F_k. Without loss of generality, we assume that $T_1 \leq T_2 \leq ... \leq T_N$. In order to be able to concentrate on the issues arising from the multiplicity of options, in this section we assume that Player 2 has no outside option and that $T_k^a = T_k^r = T_k^m$ for all k.

Let us start by a generalization of the concept of backward inducted strategic value for the multi-option setting.

Definition 3 *The backward inducted strategic value of the set of options {j,j+1,...,N} at time T_k, denoted by $B_k(j)$, is the equilibrium continuation value of Player 1 in that period, only taking into account the existence of options {j,j+1,...,N}.*

The way to calculate the BISVs is presented in the following lemma:

Lemma 4 *The backward inducted strategic values of the options are interrelated according to the following system of equations:*[18]

$$B_k(k) = \begin{cases} B_k(k+1)\,F^k(B_k(k+1)) + \int_{B_k(i+1)}^{1} y dF^k(y), & \text{if } T_k \text{ is even} \\ 1 - (1 - B_k(k+1)\,F^k(B_k(k+1) + \delta - 1)), & \text{if } T_k \text{ is odd} \end{cases}$$

$$B_k(k+1) = \begin{cases} B_{k+1}(k+1)\,\delta^{t(k)} + \frac{1-\delta^{t*(k)}}{1+\delta}, & \text{if } T_k \text{ is odd} \\ B_{k+1}(k+1)\,\delta^{t(k)} + \frac{\delta-\delta^{t*(k)}}{1+\delta}, & \text{if } T_k \text{ is even} \end{cases}$$

where $t(k) = T_{k+1} - T_k$ and $t*(k) = t(k)$ if $t(k)$ is odd and $t(k)+1$ otherwise.

Proof. The first equation follows from the same argument as the proof of Lemma 1 (2). The second equation is a straightforward application of Table 1. ■

Now we are ready to state the main result of this section:

Proposition 5 *The unique outcome of the multi-option game, which is supportable by a subgame-perfect equilibrium, is immediate agreement, with Player 1 obtaining $B_1(1)\delta^{T_1-1} + \frac{1-\delta^{T*}}{1+\delta}$ (where $T* = T_1$ if T_1 is even and $T_1 - 1$ otherwise), unless $T_1 = 1$, in which case she obtains the whole surplus if $B_1(2) \leq a_1$ (the realized value of the first option) and $B_1(2)$ otherwise.*

Proof. Note that $B_1(1)$ can be considered as the strategic value of a single outside option whose value realizes at time T_1. The rest follows from Table 1. The results for $T_1 = 1$ follow from the same argument as the proof of Lemma 1 (2). ■

An immediate observation to make is that the upper end of the support of an option's distribution function is a sufficient statistic for the entire distribution.[19] The following corollary is then straightforward:

Corollary 6 *If and only if the upper end of the support of all the options maturing in odd (even) periods is no greater than $\frac{\delta^2}{1+\delta}$ $\left(\frac{\delta}{1+\delta}\right)$ the outside options have no influence on the bargaining outcome.*

As a consequence, the availability of several options over time gives rise to a Coasian result: whenever one of the later options is effective, in periods where she is the proposer Player 1's bargaining power is reduced with respect to the case where she only has option 1, just as a monopolist seller of a durable good drives down the price competing against herself. This reduction takes the form that some outside options that on their own would give a strategic value of one, will cease to be effective because of the later option's presence.

To illustrate how the potential effectiveness of earlier options is affected by a later option being effective, we present the following example:

Example 1 *Assume that the last option that is locally effective (that is, it has a strategic value that is different from the Rubinstein continuation),*

[17] Simply observe that all four elements in Table 1 satisfy the relationship that decreasing t by one gives a value that is strictly greater than multiplying that element by δ.

[18] Note that BN(N) is the strategic value of the option maturing last.

[19] By the first equation in Lemma 1 it follows that this is so for "local" effectiveness. But "global" effectiveness is just the local one checked sequentially.

matures in period 6 and it yields a strategic value of $A(6) = \frac{1}{2}$. We are now going to calculate what the maximal upper ends of the supports of the distributions of the (potential) options in all the previous periods are so that the last option remains the one determining the terms of agreement.

Since period 5 is odd, by Lemma 1 that period's option is effective iff it is greater than $\delta A(6)=\frac{\delta}{2}$. If the option is not effective then Player 1's subgame-perfect share in that period is $1-\delta(1-\frac{1}{2}) = \frac{2-\delta}{2}$. Then in period 4 the option has to give Player 1 $\delta\frac{2-\delta}{2}$ in order to be credible (as well as effective, since we are in an even period). This will also be her subgame-perfect share if the option is not effective. In period 3 then the effectiveness limit is $\delta^2\frac{2-\delta}{2}$, while Player 1's subgame-perfect share is$1-\delta\left(1-\delta\frac{2-\delta}{2}\right) = 1-\delta+\delta^2\frac{2-\delta}{2}$. In period 2 then the effectiveness limit, as well as Player 1's share, is $\delta-\delta^2+\delta^3\frac{2-\delta}{2}$. Finally, in the first period the effectiveness limit is $\delta^2-\delta^3+\delta^4\frac{2-\delta}{2}$, while Player 1's share is $1-\delta\left(\delta-\delta^2+\delta^3\frac{2-\delta}{2}\right) = 1-\delta+\delta^2-\delta^3+\delta^4\frac{2-\delta}{2}$. In Table 2 we can appreciate that while the effect of the option is diminishing the further it is, it never vanishes: in every preceding period it increases both the effectiveness limit of the current option and Player 1's share, with respect to the case with no effective options.

Period	1	2	3	4	5
Rub. share	128	64	128	64	128
Eq. share	129	66	132	72	144
Indiv. eff. lim	32	64	32	64	32
Eq. eff. lim.	33	66	36	72	48

Table 2. ($\delta = \frac{1}{2}$ and the unit of measure is $\frac{1}{192}$)

5 Final remarks

In many bargaining situations outside options are uncertain and they arise and cease over time. Proposals that may seem attractive (unattractive) at the start of the game may cease to be so as the uncertainty disappears or as time approaches the maturity date of the outside opportunities. We have analyzed the effects of this kind of uncertainty in the outcome of a negotiation.

The crucial influence of the outside opportunities available to the players on the outcome of a bargaining game has long been recognized (Nash (1950)). However, once the confusion among the definitions of "status quo point," "disagreement point" and "threat point" was first pointed out and then satisfactorily settled (Sutton (1986), Binmore, Shaked and Sutton (1989)), outside options have seemingly exited from the research agenda. The implicit argument behind this neglect was that, since we know exactly their effects, why should we unnecessarily complicate our models with those extra details?

We have argued here and in related work (see Ponsatí and Sákovics (1998a,1998b)) that there are still interesting questions that can be addressed and new insight to be gained in the context of bargaining games à la Rubinstein-Stahl with outside options. In this paper we have claimed that considering uncertainty about the outside options is a natural extension to the literature that deserves attention. Moreover, since allowing both players to take an outside option has important effects (compared to models with only one-sided outside options), the combination of these two elements yields novel insights on the role of outside opportunities in bargaining.

There are a number of ways in which our models could be extended. The most obvious generalization would be to add asymmetric information, players could learn privately the value of their options, about the probabilities, about the revelation date etc. Another interesting scenario is when the revelation of information is not instantaneous but the players learn over time about the prospects of their outside opportunities. In these cases, we would also have the additional consideration of how much a player's actions (offers) reveal about what she knows at the time.

References

Avery, C., and Zemsky, P. (1994a) "Option Values and Bargaining Delays," *Games and Economic Behavior* 7, 139-153.

Avery, C., and Zemsky, P. (1994b) "Money Burning and Multiple Equilibria in Bargaining," *Games and Economic Behavior* 7, 154-168.

Binmore, K., Shaked, A. and Sutton, J., (1989) "An Outside Option Experiment," *Quarterly Journal of Economics* 104, 753-770.

Dalmazzo, A. (1992) "Outside Options in a Bargaining Model with Decay in the Size of the Cake," *Economics Letters*, December, 417-421.

Hendon, E., Sloth, B. and Tranaes, T., (1994) "Decentralized Trade with Bargaining and Voluntary Matching," *Economic Design* 1, 55-78.

Hendon, E. and Tranaes, T., (1991) "Sequential Bargaining in a Market with One Seller and Two Different Buyers," *Games and Economic Behavior* 3, 453-466.

Merlo, A. and Wilson, C., (1995) "A Stochastic Model of Sequential Bargaining with Complete Information," *Econometrica* 63, 371-399.

Moldovanu, B. (1993) "Price Indeterminacy and Bargaining in a Market with Indivisibilities," *Journal of Mathematical Economics* 22, 581-597.

Nash, J., (1950) "The Bargaining Problem," *Econometrica* 18, 155-162.

Osborne, M., and Rubinstein, A. (1990) Bargaining and Markets, Academic Press, San Diego.

Ponsatí, C., and Sákovics, J. (1998a) "Rubinstein Bargaining with Two-

Sided Outside Options," *Economic Theory*, forthcoming .
Ponsatí, C., and Sákovics, J. (1998b) "Two-Sided Options and Stationary Uncertainty in Bargaining," *mimeo*, Universitat Autònoma de Barcelona, March.
Rubio, J., (1994) "Time-dependent outside option in an alternating offers bargaining model," *mimeo*, Euskal Herriko Unibertsitatea, October.
Shaked, A., (1994) "Opting Out: Bazaars versus 'Hi Tech' Markets," *Investigaciones Económicas* 18(3), 421-432. (previously circulated as LSE discussion paper 1987/159)
Shaked, A. and Sutton, J., (1984) "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model," *Econometrica* 52. 1351-1364.
Sutton, J., (1986) "Non-Cooperative Bargaining Theory: An Introduction," *Review of Economic Studies* 53, 709-724.
Vislie, J., (1988) "Equilibrium in A Market with Sequential Bargaining and Random Outside Options," *Economics Letters* 27, 325-328.

An extension of the nonatomic assignment model*

Doraiswamy Ramachandran[1] and L. Rüschendorf[2]

[1] Department Of Mathematics and Statistics, California State University, 6000 J Street, Sacramento, CA 95819-6051, U.S.A
[2] Institut Für Mathematische Stochastik, Albert-Ludwigs-Universität, Eckerstrasse 1, D-79104 Freiburg, Germany.

Abstract. We investigate the nonatomic assignment model introduced by Gretsky, Ostroy and Zame (1992) under general conditions on the economic agents and utility functions. This extension allows considering various function spaces (price processes and risk curves) as models for the market. We show that the assignment problem based on "inequality" constraints is equivalent to a corresponding transportation problem under "equality" constraints. This equivalence leads to a general duality theorem in this setting. In order to model some external regulations on the market requiring at least a certain minimum level of activity on the part of the agents, we then introduce a modified assignment model with constraints on the agents. We establish duality theorems for this modified assignment problem and existence results for optimal solutions.

JEL Classification Numbers: C71, C78

Keywords: Assignment problem, duality theorem

1 Introduction

The assignment model of Shapley and Shubik (1972) has been extended to the following version with a continuum of buyers and sellers by Gretsky, Ostroy and Zame (1992) called the nonatomic assignment model:

Consider the set X_1 of buyers and the set X_2 of sellers each having a distinct house to sell, and associate two probability spaces $(X_i, \mathcal{A}_i, P_i), i = 1, 2$ with the buyers and sellers respectively, where P_i represent the population distributions. If buyer x_1 and seller x_2 were to transfer ownership of the house x_2, then the monetary value of this transfer between the pair (x_1, x_2) can be represented by a measurable function $h(x_1, x_2)$ on

*The work is supported by the U.S. - Germany Cooperative Science Program, NSF grant INT-9513375 and DAAD grant 315/PRD/fo-ab.