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Bertrand and the long run☆

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ABSTRACT

We propose a new model of simultaneous price competition, where firms offer personalized prices to consumers, who then independently decide which offer to accept, if any. Even with decreasing returns to scale, this decentralized market mechanism has a unique equilibrium, which is independent of any exogenously imposed rule for rationing or demand sharing. In equilibrium, the firms behave as if they were price takers, leading to the competitive outcome (but positive profits). Given the unique result for the short-run competition, we are able to investigate the firms' ex ante capital investment decisions. While there is underinvestment in the long-run equilibrium, the overall outcome is more competitive than one-shot Cournot competition.

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1. Introduction

In this paper we take a fresh look at markets where the firms compete in prices to attract consumers. This is a fundamental topic of industrial organization that has been thoroughly investigated, ever since the original contribution of Cournot (1838).[1] Our excuse for re-opening the case is that we offer a new way of modeling price competition, which naturally leads to a unique equilibrium with price equal to marginal cost, even when the latter is increasing. The innovation we propose is to allow the firms to personalize their prices. The resulting conceptual advantage is not the feasibility of first-degree price discrimination – which does not occur in equilibrium –, rather, the flexibility allowed by personalized pricing ensures that competition is cut-throat even when attracting too much demand is harmful (because of increasing marginal costs). The enhanced level of competition leads to a unique (symmetric) equilibrium with all consumers being offered the competitive price. Notably, we need not make arbitrary assumptions about either a rationing rule: each firm serves the very consumers who accept its offer; or a demand sharing rule: when a consumer receives two equal offers she randomizes according to her (endogenously derived) equilibrium strategy. Armed with a solution to price competition, we revisit the question of how competitive the outcome of two-stage competition – first technology choice, then (personalized) price competition – is relative to a one-shot Cournot model. We show that despite the competitive result of the first stage, in the two-stage game there are still distortions: there is underinvestment in the long-run factor. Nonetheless – except if the technology is Leontieff – the overall outcome is more competitive than the Cournot outcome.

1.1. Deconstructing the Bertrand Paradox

Take the standard model of simultaneous price competition between two producers of a homogeneous good at constant and identical marginal cost, commonly referred to as the Bertrand duopoly. This model has a unique equilibrium, where both firms price at marginal/average cost, thereby earning zero profit. While the model itself seems realistic, the result is clearly not: even though there are only two competitors, they have no market power at all. The literature has dealt with this issue by enriching the model, incorporating product differentiation, price-quantity bidding, privately known cost functions or dynamic competition. While these generalized models are useful in their own right, it is nonetheless conceptually relevant to note that actually nothing is amiss in the basic model.

Recall that, assuming that firms are price-takers in the input markets, when average costs are decreasing in output we have a natural monopoly: there is room for only one firm in the market. The "paradoxical" situation with constant marginal cost is the limiting

[1] While Cournot (1838) only discussed quantity competition for the more salient case of substitute goods, he did formalize price competition as well, for the case of perfect complements.

case of this, where two firms can "just" fit. When average costs are *increasing*, marginal costs are above average cost and – as we will discuss below in detail – firms do make positive profits in the Bertrand duopoly, despite still pricing at marginal cost.

The seemingly innocuous "simplifying" assumption of constant marginal costs actually leads to a non-generic, knife-edge situation, just between the cases where a duopoly can make profits or losses. Therefore, it should not come as a surprise that constant marginal costs lead to zero profits in oligopoly: there is no paradox.

Let us re-examine the Bertrand duopoly when marginal costs are increasing. As shown by Dastidar (1995), this scenario is not the *panacea* either, as it leads to multiple equilibria. There exists a range of prices, such that if a firm charges one of them the other firm's best response is to charge the same price.[2] Denoting demand by $D(.)$ and cost by $C(.)$, the lowest equilibrium price, $\underline{p}$, is where the sellers splitting the demand[3] just break even: $\underline{p}D(\underline{p}) = 2C(D(\underline{p})/2)$.[4] The highest one, $\overline{p}$, is where serving the entire demand gives the same profit as splitting it: $\overline{p}D(\overline{p}) - C(D(\overline{p})) = \overline{p}D(\overline{p})/2 - C(D(\overline{p})/2)$. The reason for this plethora of equilibria is the obligation of a deviant firm to serve all comers at the announced price. With constant marginal costs this is not an issue. However, when average costs are increasing, satisfying the entire market demand – what happens if a firm undercuts its competitor – may not be an advantageous proposition. With deviations discouraged, equilibria thrive.

In order to regain a unique equilibrium price, we could make use of Dixon's (1992)[5] modified Bertrand–Edgeworth game, where in addition to their price the firms also announce the maximum quantity they are willing to sell at it. Together with a demand sharing rule[6] and a freely chosen[7] rationing rule, this resolves Dastidar's problem that downward deviations are too costly, and by having firms commit to supply – if needed – more than their share in the competitive equilibrium, it removes the incentive for rivals to increase their price above the competitive one (residual demand is zero), thus destroying the Edgeworth Cycle.

In this paper, we propose an alternative model of price competition – also supporting the competitive outcome as the unique equilibrium outcome – where the firms make a personalized price offer to each consumer. There are a number of reasons for doing this:

[2] The indeterminacy of this result is rather severe. For instance, if demand is $Q = 1 - p$ and cost is quantity squared, the lowest and highest equlibrium prices are $\underline{p} = \frac{1}{3}$ and $\overline{p} = \frac{3}{5}$. The monopoly price would be $\frac{3}{4}$, the Cournot price $\frac{3}{5}$ (it is just a coincidence that it equals $\overline{p}$).

[3] Dastidar assumes equal sharing of the demand for firms charging the same price.

[4] Ignoring the choke-price root.

[5] See Allen and Hellwig (1986) as well.

[6] He assumes equal sharing, though he also assumes that all firms have the same cost function. In fact, it is straightforward to see from the proofs of his Lemmas 1 and 2 that with asymmetric costs and equal sharing, his model generically has no equilibrium. To regain existence the sharing rule must be in proportion of competitive supply, see below.

[7] Dixon makes the shrewd – but hardly realistic – assumption that individual demand is proportional to income. This ensures that residual demand is independent of the choice of rationing rule. Nonetheless, some rationing rule is still necessary for the operation of the market.

- Firstly, from a conceptual point of view, we feel that our model is closer in spirit to "pure" price competition, as quantities are not explicitly set, and no consumer faces the risk of being rationed after accepting a price offer.
- Second, from the game-theoretic point of view, Dixon's model is subsumed in ours. If we restrict attention to price schedules that take only two values, a sufficiently high one, at which no one buys, and an "interior" price, then such price schedules are equivalent to a single price and a maximum quantity.[8] Thus, Proposition 1 below shows that the larger strategy set does not lead to a different equilibrium price (and neither does it destroy the existence of a deterministic equilibrium price), while it also implies Dixon's result.
- Third, in some applications – like certain services, intermediate goods markets, or Internet commerce, where via cookies sellers can price discriminate – the option of setting personalized prices is more realistic than posted prices. In fact, as we will see, we need not assume that the price schedule be measurable at the individual level. For example, firms with many, geographically distributed, outlets potentially charging different prices would also fit our model, barring "integer problems". We can also think of the personalized offers as proxies for personally negotiated deals, even in labor markets.
- Fourth, our model leads to a decentralized implementation, where each consumer decides individually which price to accept in equilibrium, so there is no need to appeal to demand sharing rules and to an "invisible hand" clearing the market.
- Finally, in the absence of capacity constraints (self-imposed or otherwise) our equilibrium is not hostage to an exogenous choice of rationing rule.

In the remainder of this Introduction we give a brief overview of the most relevant literature. We then present our model in detail in Section 2. Section 3 derives the short-run equilibrium, while Section 4 looks at the long-run consequences. We conclude with a brief discussion of our results.

1.2. A brief literature review

The traditional approach toward the resolution of the Bertrand Paradox – pioneered by Edgeworth (1897) – has been to allow firms to choose the quantity they are willing to sell at the price they set. In its pure form, this leads to an Edgeworth Cycle, or, in modern parlance, a mixed strategy equilibrium (c.f. Levitan and Shubik, 1972): Even if the equilibrium is unique, the range of prices offered is large[9] and the two firms generically set different prices. Allowing firms to set supply functions (complete

[8] Except that the quantity has the names of a subset of consumers on it, which only enriches the set of possible outcomes.

[9] For example, when demand is $Q = 1 - p$ and cost is quantity squared, with the proportional rationing rule proposed by Edgeworth, prices would oscillate between 1/2 (the competitive price) and 2/3.

quantity-price schedules) does not eliminate severe multiplicity either (c.f. Klemperer and Meyer, 1989).[10]

Building on the insights gained from the analysis of Bertrand competition with capacity constraints by Levitan and Shubik (1972), Kreps and Scheinkman (1983) constructed a two-stage model where firms first commit to capacity levels (or simply produce prior to the realization of demand) and then price competition follows. The remarkable outcome is that in the unique sub-game perfect equilibrium prices and quantities (produced and sold) are the same as those that would result in a one-shot Cournot competition. Unfortunately, Davidson and Deneckere (1986) showed that this result is not robust to the choice of rationing rule: Kreps and Scheinkman used the "efficient" or "surplus-maximizing" rule, where the demand is served starting from the highest valuation buyer. As this rule results in the most pessimistic residual demand curve for a firm with the higher price, for any other rule the outcome is more competitive than the Cournot equilibrium.

Looking at competition from the long-run perspective is indeed insightful and it is the main contribution of Kreps and Scheinkman (1983). However, selecting the "fixed factor" to be a choice of capacity is not only unnecessarily restrictive but it is also somewhat misleading. The latter weakness comes from the undue prominence capacity choice gives to rationing. Allowing for the short-run cost curve to be smooth, avoids rationing altogether as the firms are able to supply – within reasonable limits, see below – the entire demand, even if they wished not to. Cabon-Dhersin and Drouhin (2014) were the first to look at a two-stage model with "soft capacity constraints".[11] In the first period the firms choose – at a cost – a Cobb–Douglas production function for the second stage. They use Dastidar's (1995) model of price competition in the second stage, which they refine by selecting the collusive (payoff-dominant) equilibrium.

2. Personalized pricing

Specifically, we assume that there is a set Ω of N producers, indexed by $J = 1, 2, ..., N$, with increasing, strictly convex, twice differentiable costs functions, $C_J(q)$, with $C_J(0) = 0$, and there is a unit mass of consumers, indexed by $i \in [0, 1]$. Consumer i has valuation $v_i \in [0, 1]$ for a single unit. v_i is an i.i.d. draw from the strictly increasing and continuous cumulative distribution function $F(v)$.[12] Note that aggregation results in a deterministic aggregate demand function $D(p) \equiv 1 - F(p) : [0, 1] \to [0, 1]$. We assume that firms do not observe individual buyer valuations (however, see Remark 5 below). Outside options are normalized to zero.

[10] Note that we assume deterministic (aggregate) demand, where Klemperer and Meyer indeed find multiplicity.

[11] Boccard and Wauthy (2000, 2004) look at an extension of Kreps and Scheinkman (1983) where the capacity can be *voluntarily* exceeded, at a linear cost. Previously, Vives (1986) proposed the same model of flexible technology, in a two-stage model where the second stage is *assumed* to be competitive.

[12] The generalization is straightforward to the case where there are several different groups of consumers, whose valuations are drawn from a group-specific distribution.

We denote the inverse of firm J's marginal cost function – its price-taking supply function – by $S_J(p)$, and define the competitive price, p^*, as the price that equates aggregate supply and demand : $D(p^*) = \sum_{J=1}^{N} S_J(p^*)$. Our innovation to the extensive form is to allow firms to simultaneously make *personalized* price offers to each of the consumers**.** Next, consumers observe their offers and accept at most one of them. Formally:

Let $P_J(.)$: [0, 1] → [0, 1], be firm J 's price schedule and let $\boldsymbol{P} = (P_1, P_2, ..., P_N)$ denote the profile of the sellers' strategies. Also, let σ_i denote consumer i's (mixed) strategy, where $\sigma_i = (\sigma_i^1, \sigma_i^2, ..., \sigma_i^N)$ and $\sigma_i^J : \mathbb{R}_+^N \to [0,1]$, with $\sum_{J\in\Omega} \sigma_i^J \leq 1$, represents consumer i's probability of accepting the offer received from firm J: $P_J(i)$. We say that the outcome of ($\boldsymbol{P}$, $\boldsymbol{\sigma}$) is measurable if for each J, $\sigma_i^J(P(i))$ is (Lebesgue) measurable in i.[13]

Note that the individual consumers are full-fledged players in the game and it is their – endogenously derived – equilibrium strategies that determine how demand is shared among firms asking for the same (lowest) price. At the same time, firms are committed to satisfy the (unit) demand of all the consumers who accept their offer, thereby eliminating the need for a rationing rule (see Remark 2 below).

2.1. The short-run equilibrium

The main result of this section is that – assuming that it is not prohibitively costly[14] for any $N-1$ firms to serve the market demand at p^* – our decentralized price setting mechanism leads to the competitive outcome.

We prove the following result in the Appendix:[15]

Proposition 1. *For all J, as long as* $C_I\left(\frac{S_I(p^*)D(p^*)}{D(p^*)-S_J(p^*)}\right) < \infty$ *for all* $I \neq J$*, the unique measurable equilibrium outcome in pure price schedules is such that all trades are at the competitive price and firms sell in proportion to their competitive supply: firm J's offer of* p^* *is accepted by a measure* $S_J(p^*)$ *of consumers.*

While the equilibrium outcome is unique, there are multiple ways of implementing it. The leading contender is the (unique) equilibrium where both firms and consumers use symmetric strategies: all firms post the competitive price, $P_J(.) \equiv p^*$, and the consumers who are willing to buy, $v_i \geq p^*$, use a mixed strategy of acceptance, where Firm J's offer is accepted with probability $S_J(p^*)/D(p^*)$ that equals its share of the aggregate competitive supply.

13 Equivalently, if $\int_0^1 \sigma_i^J(\mathbf{P})di$ exists for all J.
14 See Remark 3.
15 When a continuum of agents each randomize over a common finite set of actions, there is no guarantee that the set of agents that choose a certain action (in this case, accepting trading with seller J) is measurable. In that case, payoffs and best responses cannot be defined. In order to avoid what is but a technical issue, we will only consider strategy profiles where this indeterminacy does not arise. The concept of "equilibrium" implicitly requires measurability of outcomes, anyway.

If there are several firms that are not too unequal, there exists another focal equilibrium strategy profile – which minimizes the number of (serious) offers made – where the firms coordinate so that each consumer receives exactly two offers of p^* (and thus firms make twice as many offers as they wish to sell: $2S_J(p^*)$), which she accepts with 50–50 chance. Note that, despite the different mixed acceptance strategy, demand is still shared in proportion of the competitive supplies. This observation highlights that it is the firms' and consumers' strategies together that determine how demand is shared.

Before developing the intuition for this result, some important observations are in order:

Remark 1. We have seen that in order to achieve endogenous rationing in equilibrium, we need either the consumers to be able to calculate the firms' competitive supplies, or the firms to coordinate on which consumers to target. While, game-theoretically speaking, both of these characteristics are fine, they need some justification from the viewpoint of realism. The first possibility could be rationalized, for example, by firms advertising in proportion to their size/competitive supply. Coordination between firms can be a real-life situation, for example, where the unit-demand consumers are actually aggregated into retailers and the firms can observe which retailer their competitors intend to sell to. In any case, we wish to underscore that this is a conceptual exercise: our goal is to understand price competition in the most abstract setting.

Remark 2. The symmetric equilibrium strategies involve commitment to offer the good[16] for the competitive price to *all* consumers (who – if $v_i \geq p^*$ – then use a mixed strategy of acceptance in proportion to the firms' competitive supply).[17] Thanks to the Law of Large Numbers eliminating uncertainty, the highest realized demand for firm I following a unilateral deviation by firm J is $\frac{S_I(p^*)D(p^*)}{D(p^*)-S_J(p^*)}$, which by assumption is still feasible to satisfy. Following a deviation by a competitor, a firm would prefer to ration consumers. There must be either sufficient reputational concerns or enforced consumer protection regulation in place to ensure compliance.

Remark 3. For clarity, we have set the limit of feasibility at infinite total cost. One can replace infinity by any other number that determines the limit of feasibility for satisfying residual demand, depending on the circumstances (for example, the bankruptcy constraint in Dixon, 1992). Note that the more firms there are, the lower is the residual demand following a unilateral deviation and the easier is to satisfy the feasibility constraint. If the constraint is not satisfied, it is not possible to avoid rationing.

[16] This may consist of a substitute good or a "rain check". The crucial assumption is that a consumer who has accepted an offer no longer has unsatisfied demand in the market.

[17] This many offers are not needed in general. The necessary condition is that all buyers receive at least two offers.

Remark 4. Unlike in standard Bertrand competition, the strategies sustaining the equilibrium are *not* weakly dominated: making offers to fewer consumers would decrease profits. Nonetheless, it is true that, due to the commitment to serve all accepted offers, a deviating competitor could provoke a serious loss in profits. Thus, a firm might worry about a kind of "strategic risk", in the spirit of risk dominance, even if our equilibrium is unique. However, it is easy to see that, if we actually incorporate such a risk into the analysis, there is no qualitative change in the equilibrium and the – unique and common – price moves continuously with the probability of "mistakes". For example, if each offer got lost with probability ε, the new equilibrium price for a symmetric duopoly would become the solution of $p = C'((1-\varepsilon^2)D(p)/2)$. Alternatively, if all the offers of a competitor could be lost with probability ν, the new price would solve $p(1+\nu) = 2\nu C'(D(p)) + (1-\nu)C'(D(p)/2)$.

Remark 5. Note that, if we assume that consumer valuations are observable, our mechanism allows firms to perfectly price discriminate. Proposition 1 would still apply: marginal cost pricing continues to be the equilibrium outcome, so competition drives out price discrimination. Unlike in the case of monopoly, the lack/presence of the ability to price discriminate has no efficiency consequences.[18]

In the unique equilibrium outcome, all consumers willing to trade do so at the competitive price, and all firms end up with the same marginal cost, equalling it. As a result, firms do not want to undo any sales and, while lowering some price offers would lead to further sales, this would lead to a loss, as marginal cost would exceed this price. This is rather straightforward, and in fact it also holds in the posted price setting: the competitive price is contained in Dastidar's interval.[19] What is more intricate is to see that *all* equilibria must satisfy the condition that (common) price equals (common) marginal cost. Let us break this claim down into two pieces. Assume first, that we have established that in equilibrium all firms have the same marginal cost (as implied by Dastidar's symmetry assumptions). In that case, each inframarginal consumer is equally valuable to each firm, so they all must buy at the same price. Can this price be below marginal cost? No, because then some (in fact, each) firm would prefer to decrease its sales, by raising

[18] This is in contrast to Armstrong and Vickers (1993) but in line with Holmes (1989) and Stole (2007).

[19] The exact description of Dastidar's interval (especially its lower bound) depends on whether there are fixed costs of production (for simplicity we assume not: $C_J(0) = 0$) and on the rule according to which firms charging the same (lowest) price split demand. For consistency with our endogenously derived equilibrium sharing, we adopt the assumption made by Vives (1999) – see Dastidar (1997) as well – that the split is in proportion to their price taking supply ($S_J(p)$): firm J's share as a proportion of the aggregate output if firms in $\Gamma \subseteq \Omega$ set the lowest price (p) is $\alpha_J(p;\Gamma) = \frac{S_J(p)}{\sum_{I\in\Gamma} S_I(p)}$. With this assumption, the Dastidar interval is straightforward to describe (see Vives' note 7 in Chapter 5), as – by construction – in equilibrium all firms must produce: The lowest Dastidar equilibrium price is the lowest commonly charged price where all firms make non-negative profits. As at the competitive price they all have the same marginal cost (p^*) which is above average cost (as $C_J(0) = 0$ and $C_J'' > 0$), $\underline{p} < p^*$. The highest Dastidar price is the highest commonly charged price at which no firm would prefer to serve all the demand. As at the competitive price they all charge at marginal cost, any additional amount sold would strictly decrease their profits, implying that they would strictly prefer not to serve all the demand. Thus, by the continuity of payoffs, $p^* < \overline{p}$.

its price to a *subset* of consumers. Can the common price be above marginal cost? No, because then some (in fact, each) firm would prefer to increase its sales, by lowering its price to a *subset* of consumers. Note that the same arguments fail with posted prices, as then a firm's deviation choices are to undercut all firms for all consumers or to lose out to everyone and sell nothing.

Finally, let us argue why all firms should have the same marginal cost in equilibrium. Assume otherwise: then a lower-marginal-cost (lmc) firm could undercut a higher-marginal-cost (hmc) firm for a subset of its consumers with a price which is higher than lmc's marginal cost and increase its profit – as the hmc firm would not make any offer below its marginal cost and it must make a positive measure of sales in equilibrium.

3. The long run

Now that we have a unique prediction for the outcome of Bertrand competition in the short run, we can turn to the question of the choice of – or investment in – productive technology, which was considered to be fixed in the short run. For simplicity, we will keep the number of firms constant even in the long run.

We assume that all firms have access to the same technology and input prices, and so the same differentiable, sub-additive production function $f(K, L)$ which satisfies $f_K, f_L > 0$, $f_{LL}, f_{KK} < 0$, and $f_{KL} > 0$. Here K, say capital, priced at r, is considered to be the fixed factor while L, say labor, priced at w, is the short-run decision variable.[20] When K is fixed, the production function results in a cost function $C^{SR} = wL(q; K)$, where $L(q; K)$ is the short-run input demand for L implicitly defined by

$$f(K, L(q; K)) \equiv q. \tag{1}$$

Differentiating both sides with respect to q we obtain $f_L L'(q;\ K) \equiv 1$, implying that $MC^{SR}(q; K) = wL'(q; K) = \frac{w}{f_L}$. Differentiating this with respect to q, we have that marginal cost is increasing and thus it can be inverted to yield the short-run supply function, $S(p; K) = f_L^{-1}(w/p; K)$, assumed in the previous section:

$$\frac{\partial MC^{SR}(q; K)}{\partial q} = -\frac{f_{LL} \cdot w}{f_L^3} > 0.$$

Differentiating the short-run marginal cost with respect to K, we have that at any given quantity, marginal cost is reduced by investment:

[20] Brander and Spencer (1983) were the first to discuss Cournot competition following R&D investment, which, from the production and cost functions points of view is equivalent to our model. They also allowed for "flexible" production functions. As Cournot and Bertrand embody opposite incentives to use the strategic variable (R&D or capital) our results in this respect will mirror theirs. Needless to say, this is not the focus of this paper: rather, we are investigating the sources of inefficiencies when pricing is efficient, and also to what extent is Cournot a good reduced form of this pricing model.

$$\frac{\partial MC^{SR}(q;K)}{\partial K} = -\frac{f_{KL}\cdot w}{f_L^2} < 0. \tag{2}$$

Thus, firms indeed have an incentive to sink capital into their technology.

We model the long-run competition as follows: In the first stage, firms simultaneously install their "fixed" inputs, K_J, $J = 1, 2, ..., N$; these are publicly observed before the second stage, where they engage in simultaneous personalized price setting. By our result in the previous section – and sequential rationality – it is common knowledge that the second stage leads to a unique equilibrium outcome, price p^* and quantity $S(p^*)$, parametrized by the capital structure, $\boldsymbol{K}$, installed in the first stage.

Thus, given the capital choices of all other firms, $\mathbf{K}_{-J}$, firm J's best response in the first stage solves

$$\max_{K_J}\left\{p^*(\mathbf{K})S_J(p^*(\mathbf{K});K_J) - C^{SR}(S_J(p^*(\mathbf{K});K_J);K_J) - rK_J\right\}. \tag{3}$$

The first-order condition for this maximization problem is

$$\frac{\partial p^*}{\partial K_J}S_J + \left(p^* - MC^{SR}\right)\frac{dS_J}{dK_J} = r + \frac{\partial C^{SR}}{\partial K_J}, \tag{4}$$

where we have omitted the arguments of the functions for compactness. As we have discussed in the previous section, $p^* = MC^{SR}$, so the above condition simplifies to

$$\frac{\partial p^*}{\partial K_J}S_J = r + \frac{\partial C^{SR}}{\partial K_J}. \tag{5}$$

This leads to the following immediate result.

Proposition 2. *In equilibrium all firms underinvest, not only relative to the first best but even conditional on their equilibrium output.*

Proof. Note that in equilibrium the right-hand side of (5) has the same sign as $\frac{\partial p^*}{\partial K_J}$. We will show that $\frac{\partial p^*}{\partial K_J} < 0$, implying that in equilibrium $0 > r + \frac{\partial C^{SR}}{\partial K_J}$. In other words, an extra unit of capital would decrease short-term costs by more than its price. That is, K is below the cost-minimizing level for the equilibrium output level, just as claimed in the proposition.

Recall that in short-run equilibrium $\sum_{I\neq J} S(p^*;K_I) = D(p^*) - S(p^*;K_J)$. Totally differentiating both sides with respect to K_J we obtain

$$\sum_{I\neq J} S'(p^*;K_I)\cdot\frac{dp^*}{dK_J} = D'(p^*)\frac{dp^*}{dK_J} - S'(p^*;K_J)\frac{dp^*}{dK_J} - \frac{\partial S(p^*;K_J)}{\partial K_J}. \tag{6}$$

Solving for $\frac{dp^*}{dK_J}$ we have

$$\frac{dp^*}{dK_J} = \frac{\frac{\partial S(p^*;K_J)}{\partial K_J}}{D'(p^*) - \sum_{I\in\Omega} S'(p^*;K_I)}. \quad (7)$$

By (2), $\frac{\partial S(p^*;K_J)}{\partial K_J} > 0$. Moreover, we have established already that marginal costs are increasing and thus $S'(p^*;\ K_J) > 0$ for all firms. The fact that demand is downward sloping completes the proof. □

Note that Proposition 2 points to an effect beyond the hold-up problem: It is not that firms restrict investment because they will not reap its full benefit. Rather, there is a market power effect: taking into account that the final price decreases in their investment, the firms have an additional reason to invest too little in "capital". This is an example of the "puppy dog" ploy in Tirole's (1988) terminology. The following result provides further insight.

Corollary 1. *If firm I deviates in the first stage and increases its capital investment, the rest of the firms will all reduce production:* $\frac{dS(p^*;K_J)}{dK_I} < 0$, *for* $J \neq I$.

Proof. By the proof of Proposition 2, $\frac{dp^*}{dK_J} < 0$. As $S'(p^*;\ K_J) > 0$, the result follows. □

That is, upon a unilateral increase in investment, the resulting decrease in price of course increases demand but it *decreases* the output of the competing firms, thereby limiting the price drop. We will build on this insight in the following section.

It is worth noting that the proposition implies that the short-run marginal cost is strictly larger than the long-run marginal cost for the equilibrium level of output. That is, even though the price equals the short-run marginal cost, the equilibrium is not efficient: the price is higher than the long-run marginal cost (market power effect) and firms do not minimize costs (cost inefficiency). This cost inefficiency consequence of oligopoly is ignored when K is thought of as "capacity", notably as in Kreps and Scheinkman (1983) and Davidson and Deneckere (1986). Interpreting K as capacity is a particular case of our model, assuming that $f(K,\ L)$ is a Leontieff production function, except for the lack of differentiability.

3.1. Cournot or not, revisited

We can now check what our two-stage model has to say in the discussion of whether the (long-run) Cournot model is a good proxy for a two-stage market model, where firms first take decisions that affect the cost of output, and which they take as given when they set their prices.

Using (7), we first show that, given $\mathbf{K}_{-J}$, the function $q_J(\mathbf{K}) = S(p^*(\mathbf{K}); K_J)$ is increasing in K_J:

$$\frac{\partial q_J}{\partial K_J} = S'_J \frac{dp^*}{dK_J} + \frac{\partial S_J}{\partial K_J} = \frac{\partial S_J}{\partial K_J}\left(\frac{S'(p^*;K_J)}{D'(p^*) - \sum_{I\in\Omega} S'(p^*;K_I)} + 1\right) > 0.$$

Thus, we can invert $q_J(K_J, \mathbf{K}_{-J})$ and define $K^{2s}(q; \mathbf{K}_{-J})$ as the level of K that a firm would choose in the first stage if in the second stage equilibrium it sold q, given its rivals' capital choice, $\mathbf{K}_{-J}$. Therefore, the first-stage best response problem, (3), can be rewritten as a choice of q_J instead of K_J:

$$\max_{q_J} \left\{p^*(\mathbf{K}_{-J}, K_J^{2s})q_J - C^{SR}\left(q_J; K_J^{2s}\right) - rK_J^{2s}\right\}$$

The first-order condition for this problem is

$$\frac{dp^*}{dq_J} q_J^{2s} + p^* - MC^{SR}\left(q_J^{2s}; K_J^{2s}\right) - \left(\frac{\partial C^{SR}\left(q_J^{2s}; K_J^{2s}\right)}{\partial K_J} + r\right)\frac{dK_J^{2s}}{dq_J} = 0. \tag{8}$$

Let us turn to the best response in the one-shot Cournot model:

$$\max_{q_J} \left\{P^C q_J - rK_J^C - C^{SR}\left(q_J; K_J^C\right)\right\}.$$

The first-order condition for this problem is

$$\left(P^C\right)' q_J^C + P^C - MC^{SR}\left(q_J^C; K_J^C\right) - \left(\frac{\partial C^{SR}\left(q_J^C; K_J^C\right)}{\partial K_J} + r\right)\frac{dK_J^C}{dq_J} = 0,$$

but observing that the last term on the left-hand side must be zero (investment must be efficient conditional on output) we have

$$\left(P^C\right)' q_J^C + P^C - MC^{SR}\left(q_J^C; K_J^C\right) = 0. \tag{9}$$

As, by Proposition 2, the last term on the left-hand side in (8) is positive, (8) and (9) together imply that

$$\frac{dp^*}{dq_J} q_J^{2s} + p^* - MC^{SR}\left(q_J^{2s}; K_J^{2s}\right) < \left(P^C\right)' q_J^C + P^C - MC^{SR}\left(q_J^C; K_J^C\right). \tag{10}$$

Assume that it were the case that $q_J^{2s} \le q_J^C$. In that case we would necessarily have that $p^* \ge P^C$. Also, observe that $\frac{dp^*}{dq_J}$ here refers to the change in the equilibrium price of our two-stage model that would result from an increase in K_J so that, also in equilibrium, firm J sells one more unit of output, while P' is the change in the market clearing price if – starting from the Cournot equilibrium – aggregate production is increased by one unit. Note that, evaluated at the same output levels, say q_J^{2s}, $\left|\frac{dp^*}{dq_J}\right| < |P'|$, since by Corollary 1 in our two stage model the increase in K_J would result in a reduction of all other firms' outputs. Therefore, we would also have that $\frac{dp^*}{dq_J} q_J^{2s} > P'(q_J^{2s}) q_J^{2s}$. Consequently, if the demand function is not too concave $\frac{dp^*}{dq_J} q_J^{2s} \geq \left(P^C\right)' q_J^C$. Putting these inequalities together with (10) we obtain $MC^{SR}\left(q_J^{2s}; K_J^{2s}\right) > MC^{SR}\left(q_J^C; K_J^C\right)$. As we have assumed $q_J^{2s} \leq q_J^C$, this can only hold if $K_J^{2s} < K_J^C$, which, by Corollary 1, cannot be. Thus we have proved the following.

Proposition 3. *When the demand function is not too concave,*[21] *equilibrium output is higher (equilibrium price is lower) in the two-stage model with price competition than in the one-stage quantity competition model.*

Proposition 3 implies that the Cournot outcome is an overestimation of the market power that oligopolistic firms enjoy (thereby giving a non-cooperative foundation for Vives (1986), who *assumes* price taking behavior in the second stage). This is consistent with Davidson and Deneckere's (1986) critique of Kreps and Scheinkman's (1983) rendering of first long-run quantity, and then short-run price competition. However, our result is not based on the plausibility of one or another rationing rule, but rather on a basic, but quite different, strategic interaction taking into account input substitutability.

In fact, if we assumed Leontieff technologies, then our model would result in the same equilibrium output as in the Cournot model! That is, under their common technological assumptions, Kreps and Scheinkman, not Davidson and Deneckere, would be "right". Indeed, if the production function were Leontieff, then in equilibrium firms would always mix inputs efficiently in our model. The marginal cost when the fixed input is used strictly below the efficient level is infinity, and so the demand would never cross total supply at such price. Also, and for the same reason, $|\frac{dp^*}{dq_J}| = |P'|$ in that case. Indeed (and as long as the price is above the constant marginal cost below capacity), increasing capacity by a unit will increase aggregate output in exactly one unit. The rest of the firms would still produce up to capacity in our model. Thus, with Leontieff technology our two-stage model would also predict, as the only symmetric equilibrium, the Cournot outcome.

3.2. Discussion

Price competition should lead to marginal cost pricing even when firms enjoy market power. As price competition is perhaps the best description of market behavior in the short run, we should expect that the price is indeed close to the marginal cost of firms. However, we have been familiar with the distinction between long and short run since the days of our first college studies of Microeconomics. Certain decisions, input decisions in

[21] Note that convexity of the demand simply means that the density of consumers is decreasing in their valuation: there are more poor people than rich. Alternatively, for an unconstrained demand function, but either a homothetic production function (proof available on request), or for the Vives (1986) technology – zero short-run marginal cost up to capacity, constant marginal cost thereafter – Proposition 3 also holds.

particular, are mostly taken as given, when prices are chosen, as Kreps and Scheinkman (1983) argued. Fixed production factors typically result in decreasing returns even when the technology is constant returns in the long run. Thus, marginal cost pricing and extraordinary profits are compatible. What is important to understand is not so much the difference between price and (short-run) marginal cost, but the incentives for the choice of levels and mix of inputs – and as a result, the level of output – arising from the strategic considerations present when firms do have market power: when firms' decisions affect market output and price.

This is the main message of this paper. We have shown how these strategic considerations typically lead to both an inefficient mix of inputs, with long-run decisions resulting in too low levels of these long-run determined inputs; and result in prices above long-run marginal cost.

We have also shown that, from a long-run point of view, and as argued by Davidson and Deneckere (1986), (short-run) price competition results in more output than predicted by the Cournot model. According to our analysis, the discrepancy comes from the strategic interaction between the long-run decisions of the different firms. When a firm determines its own short-run cost function by investing in the long-run factor of production, it takes into account how these decisions will affect future output decisions of rivals. A lower short-run marginal cost will be answered by rivals with a reduction in their own output. Thus, investments in these production factors have a lower impact on prices than what is predicted by the Cournot model. The result is a stronger incentive on short-run cost reduction and therefore, a larger output.

Despite this stronger incentive to invest in the long-run factor, the equilibrium input mix shows inefficiently low levels of it. As we have shown, this is associated with the effect of the long-run factor on prices, and is a well-understood phenomenon in price competition: at the cost minimizing mix of inputs, a small reduction in the use of the long-run input increases the equilibrium price. When marginal units are sold at marginal (short-run) cost, this effect dominates the second order effect on cost minimization.

Both the departure from the efficient mix of inputs and the departure from long-run marginal cost pricing are, therefore, consequences of market power. Indeed, from (2) if symmetric firms behave symmetrically and there are N active firms in the market, $\left|\frac{\partial p^*}{\partial K_J}\right| < \frac{1}{N-1}$. Thus, as N gets large $\frac{\partial p^*}{\partial K_J}$ approaches zero and, by (5), the input mix approaches efficiency. Moreover, market clearing (and efficient input mix) implies output per firm approaching 0, at which point long-run marginal cost equals short run marginal cost (and then price).[22]

[22] This is not an artifact of our assumption of always increasing average cost, and so marginal cost of 0 at $q = 0$. Indeed, assume the more standard, "U-shaped" average cost in the long run, and define the minimum efficient scale

$$q^* = \arg\min_q \{\frac{C(q)}{q}\}.$$

4. Concluding remarks

In this paper we have presented a novel way of modeling price competition, which leads to marginal cost pricing – but positive profits – as the unique equilibrium, without the need to specify rationing (when demand exceeds supply) or sharing (when supply exceeds demand) rules. It should therefore be a useful off-the-shelf workhorse model to embed in more complex scenarios.

We have also developed the most direct implications in a set-up with long-run competition, underlining the consequences of market power as inefficient investments in the fixed factor. This analysis has also shed more light on the literature on two-stage, Bertrand–Edgeworth competition.

Appendix

Proof of Proposition 1. First, we show that charging p^* to (almost) everyone is indeed an equilibrium. Suppose that consumers use a mixed strategy of acceptance such that if they receive the lowest offer, p, from the firms in $\Gamma \subseteq \Omega$, they probabilistically accept them in proportion of the firms' competitive supplies at p: they accept firm J 's offer with probability $\alpha_J(p;\Gamma) = \frac{S_J(p)}{\sum_{I\in\Gamma} S_I(p)}$.

Assume all firms but J make a price offer p^* to all consumers, and consider the best response of firm J: $P_J(.)$. Let Q_1 be the (Lebesgue) measure of the set $\{i$: $P_J(i) < p^*\}$ and Q_2 be the measure of the set $\{i : P_J(i) = p^*\}$. Then, the profits of firm J are not larger than $p^*(Q_1 + \alpha_J(p^*;\Omega)Q_2) - C_J(Q_1 + \alpha_J(p^*;\Omega)Q_2)$. Indeed, a mass of consumers Q_1 accept J's offer of a price below p^*, and a mass of consumers Q_2 receive N offers of p^*, and proportion[23] $\alpha_J(p^*;\ \Omega)$ of these accept firm J's offer. The rest of consumers receive better offers than firm J's offer to them so – since, by assumption, the other firms can satisfy all the demand at p^* – they do not buy from it. Now, note that $Q = \alpha_J(p^*;\Omega)D(p^*) = S_J(p^*)$ solves

$$\max_Q p^*Q - C_J(Q),$$

and therefore, $p^*(Q_1 + \alpha_J(p^*;\Omega)Q_2) - C_J(Q_1 + \alpha_J(p^*;\Omega)Q_2) \leq p^*S_J(p^*) - C_J(S_J(p^*))$. Observing that by using the price schedule $P_J(i) \equiv p^*$, firm J sells exactly $S_J(p^*)$, we can see that $P_J(i) \equiv p^*$ is indeed a best response. Finally, as the

Let $p^* = \frac{C(q^*)}{q^*}$, i.e., the average cost at that level of output. If

$$N^* = \frac{D(p^*)}{q^*}$$

is large, as N approaches N^*, market clearing and efficient input mix implies output per firm approaching q^*, at which point, again, long run marginal cost equals short term marginal cost and so price.

[23] By the Law of Large Numbers this proportion is deterministic.

consumers are indifferent, they are clearly happy mixing in the prescribed proportions. Note that the σ_i^J thus defined is indeed measurable.

We now show that there exists no other measurable equilibrium outcome with pure strategy price schedules. Assume the Law of Large Numbers is satisfied for a continuum – in the index i – of independent random variables – on Ω – with bounded variance, so that the quantity that firm J sells in a hypothetical alternative equilibrium is $q_J(\mathbf{P}, \boldsymbol{\sigma}) = \int \sigma_i^J(\mathbf{P}(i))di$ almost surely.

Note that for $\mathbf{P}$ to be part of an equilibrium it has to be that $P_J(i) \geq C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma}))$ for almost all i such that $\sigma_i^J(\mathbf{P}(i)) > 0$. Indeed, otherwise firm J could profit by increasing her offer (up to, say, $P_J(i) = 1$) to a positive measure of these consumers so as not to sell to them.

Next, note that each firm must sell a positive amount in equilibrium. Indeed, consider a small $\delta > 0$. Since the marginal cost is increasing, there could be no more than $(N-1)\delta$ consumers that receive a price offer below $\min_{J' \neq J} C'_{J'}(\delta) > C'_J(0) = 0$: otherwise, some producer J' would be selling units below marginal cost and so would profit from withdrawing the corresponding offers. Thus, there are at least $D(\min_{J' \neq J} C'_{J'}(\delta)) - (N-1)\delta$ (> 0, for δ small enough) that are willing to pay $\min_{J' \neq J} C'_{J'}(\delta) > C'_J(0)$ and either do not buy or buy at higher prices from the other firms. Thus, if $q_J(\mathbf{P}, \boldsymbol{\sigma})$ were 0, J could gain by offering to a small measure of those consumers the price $\min_{J' \neq J} C'_{J'}(\delta)$.

Next, observe that in equilibrium we must have $C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) = C'_I(q_I(\mathbf{P}, \boldsymbol{\sigma}))$ for all J, I. Otherwise, if $C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) > C'_I(q_I(\mathbf{P}, \boldsymbol{\sigma}))$ then firm I could profit by deviating and making a (unique winning) offer $P_J(i) - \mu$ to some arbitrarily small but positive measure υ of consumers i such that $\sigma_i^J(\mathbf{P}(i)) > 0$, for some μ satisfying $(P_J(i) - \mu \geq)$ $C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) - \mu > C'_I(q_I(\mathbf{P}, \boldsymbol{\sigma}) + \upsilon)$.[24]

Similarly, $P_J(i) = C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma}))$ for all J, and for almost all i such that $\sigma_i^J(\mathbf{P}(i)) > 0$. Indeed, if $P_J(i) > C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma})) = C'_I(q_I(\mathbf{P}, \boldsymbol{\sigma}))$ for a positive measure of i such that $\sigma_i^J(\mathbf{P}(i)) > 0$, then firm I could profit by reducing her price to $P_J(i) - \epsilon$, to a small but positive measure of these consumers and for a small enough ϵ. That would increase the sales of firm I by a positive measure, at a price above its marginal cost. It follows that in any equilibrium almost all consumers must buy at the same price in equilibrium. We have left to show that this price must be the competitive price. That is, we need to show that the total sales must be equal to the demand at the price common to all transactions. It cannot be higher, since then some consumers would be buying at a price higher than their willingness to pay. It cannot be smaller either. Indeed, in such a case a positive measure of consumers with willingness to pay higher than $p = C'_J(q_J(\mathbf{P}, \boldsymbol{\sigma}))$ would not buy. Some firm could profit by deviating and offering to a small measure of them a price equal to their willingness to pay (and above its marginal cost). □

[24] If for some of these i, $\sigma_i^I(\mathbf{P}(i)) > 0$ as well, then the condition is even easier to satisfy.

References

Allen, B., Hellwig, M., 1986. Price-setting firms and the oligopolistic foundations of perfect competition. The American Economic Review 76 (2), 387–392.

Armstrong, M., Vickers, J., 1993. Price discrimination, competition and regulation. The Journal of Industrial Economics 41 (4), 335–359.

Boccard, N., Wauthy, X., 2000, 2004. Bertrand competition and Cournot outcomes: further results,and correction. Economics Letters 68, 279–285. 84, 163–166

Brander, J.A., Spencer, B.J., 1983. Strategic commitment with R&D: the symmetric case. Bell Journal of Economics 14 (1), 225–235.

Cabon-Dhersin, M.-L., Drouhin, N., 2014. Tacit collusion in a one-shot game of price competition with soft capacity constraints. Journal of Economics & Management Strategy 23 (2), 427–442.

Cournot, A.-A., 1838. In: Hachette, L. (Ed.), Recherches Sur Les Principes Mathématiques de la Théorie Des Richesses. Paris.

Dastidar, K.G., 1995. On the existence of pure strategy Bertrand equilibrium. Economic Theory 5, 19–32.

Dastidar, K.G., 1997. Comparing Cournot and Bertrand in a homogeneous product market. Journal of Economic Theory 75, 205–212.

Davidson, C., Deneckere, R., 1986. Long-run competition in capacity, short-run competition in price, and the Cournot model. The RAND Journal of Economics 17(3), 404–415.

Dixon, H.D., 1992. The competitive outcome as the equilibrium in an Edgeworthian price-quantity model. The Economic Journal 102, 301–309.

Edgeworth, F.Y., 1897. La teoria pura del monopolio. Giornale degli Economisti 15, 13–31.

Holmes, T.J., 1989. The effects of third-degree price discrimination in oligopoly. The American Economic Review 79, 244–250.

Klemperer, P.D., Meyer, M.A., 1989. Supply function equilibria in oligopoly under uncertainty. Econometrica 57 (6), 1243–1277.

Kreps, D.M., Scheinkman, J.A., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. The Bell Journal of Economics 14 (2), 326–337.

Levitan, R., Shubik, M., 1972. Price duopoly and capacity constraints. International Economic Review 13, 111–122.

Stole, L.A., 2007. Price discrimination and competition. In: Handbook of Industrial Organization, vol. 3, pp. 2221–2299. Chapter 34

Tirole, J., 1988. The Theory of Industrial Organization. MIT Press.

Vives, X., 1986. Commitment, flexibility and market outcomes. International Journal of Industrial Organization 4, 217–229.

Vives, X., 1999. Oligopoly Pricing. MIT Press.