Search and Matching - Ph.D. Training Course

Lecture 2: Frictionless Transferable Utility Matching

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Transferable Utility: The Setup

The setup is similar to non-transferable utility:
The "Players":

- Set of Men (or firms): $M = \{m_1, m_2, ..., m_{|M|}\}$
- Set of Women (or workers): $W = \{w_1, w_2, ..., w_{|W|}\}$
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Preferences:

- Say that $u_{m_i}(w_1) > u_{m_i}(w_2)$ if men m_i strictly prefers w_1 to w_2.
- Say that $0 > u_{m_i}(w_2)$ if men m_i strictly prefers being single to w_2.
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New element: Transferability. People value the utility from matching PLUS a monetary transfer (linear utility). So matching yields $u(w_i) + t$ for men and $u(m_j) - t$ for women. Now difference in utility matters.

Definition of a Matching

Matching (μ, t) has two components:

- μ assigns to each men either a women or himself (being single), such that no two men get the same women ($\mu(m_i) \neq \mu(m_j)$ if $m_i \neq m_j$)
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Let $v(w_i)$ be the matched parter for woman w_i, and $\tau(w_i) = t(v(w_i))$ the transfer to her.

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  if $u_{m_i}(w_j) + t' > u_{m_i}(\mu(m_i)) + t(m_i)$ then
  $u_{w_j}(v(w_j)) - \tau(w_j) > u_{w_j}(m_i) - t'$.

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- Equivalent to pairwise stability:

  $$T(m_i) \geq 0 \text{ and } T(m_i) + T(w_j) > V(m_i, w_j)$$

  where $T(m_i) = u_{m_i}(\mu(m_i)) + t(m_i)$ is his total equilibrium payoff
  where $T(w_j) = u_{w_j}(\nu(w_j)) - \tau(w_j)$ is her total equilibrium payoff
  where $V(m_i, w_j) = u_{m_i}(\mu(m_i))+ u_{w_j}(v(w_j))$ is total value created.

Insights and Comments

- Existence: yes, but hard to prove (Shapley+Shubic 71)
- Efficient: yes. Maximizes joint utility (if not, deviation).
- Unique: Matching is generically unique, transfers are not (example)

Assortative Matching

Assortative Matching: finite or continuum of players [continnum on board]
Let m and w be elements of the real numbers
Let $G_M(m)$ be the CDF of men
Let $G_W(w)$ be the CDF of females
Let $V(m, w) > 0$ for illustration.

Positive assortative matching (PAM):
better men are matched to better women,i.e.,

$$\mu(m) = w \text{ iff } G_M(m) = G_W(w)$$

Assortative Matching

Sufficient condition for PAM to optimal (and therefore part of any stable matching):
Increasing differences or "supermodularity":
For all m and m' with $m' > m$, for all w and w' with $w' > w$

$$v(m', w') - v(m', w) > v(m, w') - v(m, w)$$

or equivalently $\frac{\partial^2 v}{\partial m \partial w} > 0$.

Submodularity ($\frac{\partial^2 v}{\partial m \partial w} < 0$) implies negative assortative matching. Discuss economic implications and consequences of transferable utility.

Connection to Competitive Markets

Continuum: competitive market and stable matching coincide (Zame...)
Competitive: m maximizes profits given equilibrium "price" $T(w)$:

$$
\begin{aligned}
& \max_{w} V(m,w) - T(w) \\
\Rightarrow \quad & T'(w) = \frac{\partial V(\mu^{-1}(w), w)}{\partial w} \\
\Rightarrow \quad & T(w) = \int_{\underline{w}}^{w} \frac{\partial V(\mu^{-1}(\tilde{w}), \tilde{w})}{\partial \tilde{w}} d\tilde{w} + T(0)
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Implications for wage dispersion: changes in production function or in types on other side changes the dispersion of wages.

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Applications

- CEO compensation [Tervio...]
- Matching based on intrinsic and extrinsic motivation [Besley, Ghatak...]
- Empirics (are better workers in better jobs? see second part)
- Firm size (how can we embedd "size" better - see next slides)
- Worker Peer Effects (simple ways?)
- Matching under uncertainty of types [Postlewaite, Mailath... ECTR]. Much to do...

Many-to-One Matching (TU)

Setup (Kelso-Crawford ECTR):

- Each firm m can hire multiple workers $\{w_j, w_k, ..\}$ to create value $V(m, w_j, w_k, ...)$
- (One-to-one matching special case where value of multiple workers is the max of the values with each individual worker.)
- Otherwise obvious extension of stability.

- Result: Existence if V fulfills gross-substitutes condition, otherwise existence problems.
- Gross-substitutes: If the firm looses a worker, the marginal loss from loosing any second worker goes up (then existence through ascending multi-unit auctions).
- Similar results in Migrom-Hatfield, Gul-Staccetti, Cole-Prescott...
- Problem: Not much help in characterization of equilibrium.

Simple Notion of Firm Size

Simple many-to-one matching based on Eeckhout-Kircher-WP:

Assume firm value is $V(m, w, l)$ where l is simply the number of workers.
Let $\mu(m)$ still be worker type (no worker type dispersion within firm).
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Market clearing: $\int_A l(m) dG_M = \int_{\mu(A)} dG_W$ for all A.

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But: Missing interactions amongst workers.