Supplement to "Efficient Competition through Cheap Talk: The Case of Competing Auctions"

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Appendix A: Finite Markets

In this section, we provide a formal analysis of the finite market model introduced in Section 4.

Strategies. As in other finite market models, we impose anonymity on agents' strategies.[1] In other words, we restrict attention to strategy profiles in which agents' strategies and beliefs depend neither on their own identity nor on others' identities.

Procurers' communication strategies are represented by a function $M : \mathcal{V} \to \mathcal{V}$ where $M(v)$ denotes a message announced by each procurer when his type is v. Notice that symmetry is imposed on procurers' strategies through the requirement that all procurers adopt an identical communication strategy. We do the same for contractors' strategies and beliefs. In other words, all agents' strategies are restricted to be independent of their own identity. Contractors' (search and bidding) strategies are represented by a function $p : \{1, ..., N\} \times \mathcal{R}_+ \times \mathcal{V}^N \to [0, 1]$, where $p(n, b; \vec{v})$ denotes the probability that each contractor selects procurer n and quotes a price below b when the announced value profile is $\vec{v}$. For notational simplicity, we also use $p_n(b; \vec{v})$ to denote $p(n, b; \vec{v})$ and let $p_n(\vec{v}) \equiv \lim_{b \to \infty} p_n(b; \vec{v})$. Contractors' beliefs about procurers' types are represented by a function $\mu : \mathcal{V} \times \{1, ..., N\} \times \mathcal{V}^N \to [0, 1]$, where $\mu(v; n, \vec{v})$ denotes the probability that procurer n's value is strictly less than v given the announced value profile $\vec{v}$.

In addition, we require contractors' strategies and beliefs to be independent of procurers' identities. Formally, if $v_n = v_{n'}$, then $p(n, \cdot; \vec{v}) = p(n', \cdot; \vec{v})$ and $\mu(\cdot; n, \vec{v}) = \mu(\cdot; n', \vec{v})$. In other words, all procurers who announce an identical message are treated identically by contractors within each announced value profile. In addition, if $\vec{v}'$ is a permutation of $\vec{v}$ and $v'_{n'} = v_n$, then

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[1]See Burdett, Shi and Wright (2001) for an excellent discussion on the anonymity assumption.

$p(n, \cdot; \vec{v}) = p(n', \cdot; \vec{v}')$ and $\mu(\cdot; n, \vec{v}) = \mu(\cdot; n', \vec{v}')$. This means that procurers are not discriminated by contractors even across different realizations of announced value profiles. These requirements imply that contractors' strategies and beliefs depend only on the distribution of announced messages, not on procurers' identities and, therefore, can be expressed as functions only of the distribution of announced messages (which corresponds to $Q_{\mathcal{M}}$ in the large market setup), not of the whole message profile ($\vec{v}$). For notational convenience, we continue to denote contractors' strategies and beliefs by $p(n, b; \vec{v})$ and $\mu(v; n, \vec{v})$. However, it should be clear that, with the anonymity restrictions, the strategy spaces for the large market setup and the finite market setup are indeed identical.

Agents' payoffs. We now *derive* agents' payoffs as functions of their strategies and beliefs from the physical environment. Fix contractors' strategies p and their belief system μ. Suppose procurer n has value v and announces message v'. If the other procurers' announcements are $\vec{v}_{-n}$, then the probability that the lowest quote to procurer n is weakly less than b is given by $1 - (1 - p_n(b; v', \vec{v}_{-n}))^M$. Denoting by $f(\vec{v}_{-n})$ the probability that other procurers' announced values are $\vec{v}_{-n}$, procurer n's expected payoff is given by

$$V_n(v', v) = \int \left(\int_{b \leq v} (v - b) d \left(1 - (1 - p_n(b; v', \vec{v}_{-n}))^M \right) \right) df(\vec{v}_{-n}). \qquad (1)$$

Now consider a contractor who selects procurer n and quotes b to the procurer. The contractor wins if no other contractor quotes a lower price than b, he is selected among those who quote the same price b, and the procurer's value is above b. Therefore, given the announced value profile $\vec{v}$, his expected payoff is given by

$$U_n(b; \vec{v}, \mu) = \left(\prod_{k=0}^{M-1} \binom{M-1}{k} (1 - p_n(b; \vec{v}))^{M-1-k} (dp_n(b; \vec{v}))^k \frac{1}{(k+1)!} \right) (1 - \mu(b; n, \vec{v}))(b - c), \quad (2)$$

where $dp_n(b; \vec{v}) \equiv p_n(b; \vec{v}) - \lim_{b' \nearrow b} p_n(b'; \vec{v})$. If there is no atom at b (i.e., $dp_n(b; \vec{v}) = 0$), then the expression simplifies to

$$U_n(b; \vec{v}, \mu) = (1 - p_n(b; \vec{v}))^{M-1} (1 - \mu(b; n, \vec{v}))(b - c).$$

Payoff convergence. Let us formally establish the relationship between large market payoffs used in the main text and finite market payoffs above. Fix a communication strategy by procurers M. As explained above, due to the anonymity restrictions, the outcome of the game depends only on the realized distribution of announced messages. In finite markets, this distribution is random. We denote by $\tilde{Q}_{\mathcal{M}}^i$ the (random) number of procurers who announce message v^i and by $\tilde{Q}_{\mathcal{M}}$ the overall distribution (i.e., $\tilde{Q}_{\mathcal{M}} = (\tilde{Q}_{\mathcal{M}}^1, ..., \tilde{Q}_{\mathcal{M}}^I)$). In addition, let $P^i(b; \tilde{Q}_{\mathcal{M}})$ denote

the probability that each contractor selects message v^i and quotes a price below b.

As N tends to infinity, by the law of large numbers, $\tilde{Q}^i_{\mathcal{M}}/N$ converges to $\sum_{v^j \in M^{-1}(v^i)} f^j$ (the unconditional probability that a procurer announces v^i), where f^i denotes the probability that each procurer draws value v^i. Therefore, for N sufficiently large, $\tilde{Q}^i_{\mathcal{M}}$ is well approximated by $N \cdot \sum_{v^j \in M^{-1}(v^i)} f^j$. This means that if procurer n's announcement is v^i, then the probability that each contractor selects her and quotes a price below b is approximately equal to $P^i(b; \tilde{Q}_{\mathcal{M}})/(N \cdot \sum_{v^j \in M^{-1}(v^i)} f^j)$. It then follows that the probability that the procurer does not receive any price quote below b is approximated by

$$\left(1 - \frac{P^i(b; \tilde{Q}_{\mathcal{M}})}{N \cdot \sum_{v^j \in M^{-1}(v^i)} f^j} \right)^M .$$

As N tends to infinity, M/N converges to β, and $P^i(b; \tilde{Q}_{\mathcal{M}})$ and $\sum_{v^j \in M^{-1}(v^i)} f^j$ correspond to $dP(v^i, b; Q_{\mathcal{M}})$ and $dQ_{\mathcal{M}}(v^i)$, respectively. Therefore, the expression converges to

$$e^{-\frac{\beta dP(v^i, b; Q_{\mathcal{M}})}{dQ_{\mathcal{M}}(v^i)}} = e^{-\lambda(v^i, b; Q_{\mathcal{M}}, P)}.$$

Given this, it is clear that agents' payoffs in (1) and (2) converge to those adopted in Section 2 as N tends to infinity.

Equilibrium definition. A perfect Bayesian equilibrium of this finite market game is a tuple (M, p, μ) such that (i) procurer optimality: $V_n(M(v), v) \geq V_n(v', v)$ for any n and $v' \in \mathcal{V}$, (ii) contractor optimality: for any (n, b) in the support of $p(\cdot, \cdot; \vec{v})$, $U_n(b; \vec{v}, \mu) \geq U_{n'}(b'; \vec{v}, \mu)$ for any (n', b'), and (iii) belief consistency: μ is obtained from M by Bayes' rule whenever possible. The fully revealing equilibrium is a market equilibrium in which $M(v) = v$ for any $v \in \mathcal{V}$. Since contractors' beliefs are degenerate in the fully revealing equilibrium, we suppress μ and use $U_n(b; \vec{v})$ to denote $U_n(b; \vec{v}, \mu)$.

Constrained efficiency benchmark. We now establish the constrained efficiency benchmark for finite markets. We again consider the (utilitarian) social planner who wishes to maximize gross surplus. Respecting the anonymity restrictions, we assume that the social planner is restricted to assign a common search strategy to all contractors. To be formal, denote by $p_n^*(\vec{v})$ the probability that each contractor selects procurer n and by $\vec{p}^*(\vec{v})$ the corresponding probability vector (i.e., $\vec{p}^*(\vec{v}) = (p_1^*(\vec{v}), ..., p_N^*(\vec{v}))$). The social planner's problem is given as follows:

$$\max_{\vec{p}^*(\vec{v})} \sum_{n=1}^{N} (1 - (1 - p_n^*(\vec{v}))^M)(v - c), \text{ subject to } \sum_{n=1}^{N} p_n^*(\vec{v}) = 1. \tag{3}$$

The solution to this problem is reported in the following proposition.

Proposition A1 *Given $\vec{v}$, there exists a unique solution to the social planner's problem: the optimal $p_n^*(\vec{v})$ is characterized by*

$$(1 - p_n^*(\vec{v}))^{M-1}(v - c) \leq \mu(\vec{v}), = \mu(\vec{v}) \ \text{if} \ p_n^*(\vec{v}) > 0,$$

where $\mu(\vec{v})$ is the value that satisfies

$$N - 1 = \sum_{n=1}^{N} \left(\min\left\{ \frac{\mu(\vec{v})}{v_n - c}, 1 \right\} \right)^{\frac{1}{M-1}}.$$

Proof. The objective function is strictly concave, while the constraint is linear in $\vec{p}^*(\vec{v})$. Therefore, letting $M \cdot \mu(\vec{v})$ be the Lagrangian multiplier, $(\vec{p}^*(\vec{v}), \mu(\vec{v}))$ is the solution if and only if

$$(1 - p_n^*(\vec{v}))^{M-1}(v_n - c) \leq \mu(\vec{v}), = \mu(\vec{v}) \ \text{if} \ p_n^*(\vec{v}) > 0, \ \text{and} \ \sum_{n=1}^{N} p_n^*(\vec{v}) = 1.$$

The first condition can be rewritten as

$$1 - p_n^*(\vec{v}) = \left(\min\left\{ \frac{\mu(\vec{v})}{v_n - c}, 1 \right\} \right)^{\frac{1}{M-1}}.$$

Summing this over n and imposing $\sum_{n=1}^{N} p_n^*(\vec{v}) = 1$ result in the condition for $\mu(\vec{v})$ in the proposition. $\square$

Contractors' equilibrium bidding strategies. We begin by presenting a finite-market counterpart to Lemma 1. A proof is omitted because it is essentially identical to that of Lemma 1.

Lemma A1 *Denote by $\underline{b}_n(\vec{v})$ and $\overline{b}_n(\vec{v})$ the minimal element and the maximal element of the support of $p_n(\cdot; \vec{v})$. In the fully revealing equilibrium, for any n such that $p_n(\vec{v}) > 0$, $\underline{b}_n(\vec{v}) = c + (1 - p_n(\vec{v}))^{M-1}(v_n - c)$, $\overline{b}_n(\vec{v}) = v_n$, and $p_n(\cdot; \vec{v})$ is a continuous and strictly increasing function such that for any $b \in [\underline{b}_n(\vec{v}), \overline{b}_n(\vec{v})]$,*

$$(1 - p_n(b; \vec{v}))^{M-1}(b - c) = (1 - p_n(\vec{v}))^{M-1}(v_n - c).$$

Constrained efficiency of the fully revealing equilibrium. We first illustrate the efficiency result. For each $\vec{v} \in \mathcal{V}^N$, let $u(\vec{v}) = \max_{n,b} U_n(b; \vec{v})$. This corresponds to contractors' market utility in large markets. Contractor optimality requires that in the fully revealing equi-

librium, each contractor chooses procurer n with a positive probability only when

$$U_n(\bar{b}_n(\vec{v}); \vec{v}) = (1 - p_n(\vec{v}))^{M-1}(v_n - c) = u(\vec{v}).$$

This can be rewritten as

$$(1 - p_n(\vec{v}))^{M-1}(v_n - c) \leq u(\vec{v}), = u(\vec{v}) \text{ if } p_n(\vec{v}) > 0.$$

Constrained efficiency of the fully revealing equilibrium follows from the fact that this condition coincides with the one in Proposition A1.

Existence of the fully revealing equilibrium. The discussion in the main text suffices as a proof for the result that no procurer has an incentive to deviate upward (i.e., announcing a higher value than his true value) in finite markets. For downward deviations, the argument in Hernando-Veciana (2005) directly applies, because a downward-deviating procurer in our model never faces any price quote above her value, and thus his problem is the same as when he commits to a lower reserve price than her value. Hernando-Veciana (2005) proves that no procurer has an incentive to deviate downward provided that the market size is sufficiently large.

Limit market outcomes. We now show that the fully revealing equilibria in finite markets converge to the fully revealing equilibrium in the corresponding large market as the market size (N) grows infinity.

Denote by $\tilde{p}_N^i$ the probability that each contractor selects a type v^i procurer (i.e., $\tilde{p}_N^i \equiv \sum_{v_n = v^i} p_n(\vec{v})$) and by $\tilde{u}_N$ contractors' market utility in the fully revealing equilibrium. Both $\tilde{p}_N^i$ and $\tilde{u}_N$ depend on the realizations of procurers' types and, therefore, are random variables. In order to characterize the limits of the random variables, suppose that for each N there are Nf^i number of type v^i procurers in the market (note that Nf^i needs not be an integer). Let p_N^i and u_N be the values that satisfy the equilibrium conditions for contractors' search strategies, that is,

$$\left(1 - \frac{p_N^i}{Nf^i}\right)^{\beta N - 1} (v^i - c) \leq u_N, = u_N \text{ if } p_N^i > 0, \text{ and } \sum_i p_N^i = 1.$$

By the same logic as in the proof of Proposition A1, there exists a unique vector $(p_N^1, ..., p_N^I, u_N)$ that satisfies the conditions. As N tends to infinity, the deterministic sequence converges to a vector $(p^1, ..., p^I, u)$ that satisfies

$$e^{-\beta p^i / f^i}(v^i - c) \leq u, = u \text{ if } p^i > 0, \text{ and } \sum_i p^i = 1. \tag{4}$$

In order to show that the sequence of random vectors $(\tilde{p}_N^1, ..., \tilde{p}_N^I, \tilde{u}_N)$ also converges to $(p^1, ..., p^I, u)$,

it suffices to use the fact that, by the law of large numbers, the proportion of type v^i procurers converges to f^i almost surely.

Now observe that (4) is equivalent to

$$\beta p^i = \max\left\{\log\left(\frac{v^i - c}{u}\right), 0\right\} f^i, \text{ and } \sum_i p^i = 1.$$

Combining the conditions yields

$$\beta = \sum_i \max\left\{\log\left(\frac{v^i - c}{u}\right), 0\right\} f^i = \int_{v > c + u} \log\left(\frac{v - c}{u}\right) dF(v).$$

This condition coincides with the one for a large market (see Proposition 1 and Section 3.2.2). In other words, contractors' market utility in the limit market is identical to the one in the large market. The equivalence of all other variables follows from this result.

Appendix B: Inefficiency in Finite Competing Auctions

In this section, we consider a finite competing (procurement) auctions model with reserve price posting and show that the market outcome is not constrained efficient whenever there are at least three procurers and at least two of them are heterogeneous.

Setup. The physical environment is as described in Section 4. Now assume that each procurer publicly announces, and commits to, her reserve price. In addition, without loss of generality (due to revenue equivalence), assume that each procurer runs a second-price procurement auction. Denote by r_n the reserve price announced by procurer n. In addition, let $\vec{r} \equiv (r_1, ..., r_n) \in \mathbb{R}_+^N$ and $\vec{r}_{-n} \equiv (r_1, ..., r_{n-1}, r_{n+1}, ..., r_N) \in \mathbb{R}_+^{N-1}$. Finally, for simplicity, suppose that each procurer's type is publicly known, and normalize contractors' opportunity cost of working c to 0. We denote by p_n the probability that each contractor selects procurer n.

Individual procurer's problem. If at least two contractors select a procurer, then the procurer can hire one of them at $c = 0$. If only one contractor selects him, then her payoff is equal to $v_n - r_n$. Therefore, procurer n faces the following maximization problem:

$$\max_{r_n} \quad (1 - Mp_n(1 - p_n)^{M-1} - (1 - p_n)^M)v_n + Mp_n(1 - p_n)^{M-1}(v_n - r_n)$$
$$= (1 - (1 - p_n)^M)v_n - Mp_n(1 - p_n)^{M-1}r_n.$$

Assuming the interior solution and taking the first-order condition,

$$(M(1 - p_n)^{M-1}(v_n - r_n) + M(M - 1)p_n(1 - p_n)^{M-2}r_n)\frac{dp_n}{dr_n} - Mp_n(1 - p_n)^{M-1} = 0.$$

Arranging the terms,

$$\frac{dp_n}{dr_n} = \frac{p_n(1 - p_n)}{(1 - p_n)(v_n - r_n) + (M - 1)p_n r_n}. \tag{5}$$

This equation represents the optimal trade-off for an individual procurer between the reserve price and the probability that each contractor selects the procurer.

Contractors' equilibrium search strategies. The expected payoff of a contractor who selects procurer n is equal to $(1 - p_n)^{M-1}r_n$, because he obtains a positive payoff of r_n if and only if no other contractor selects procurer n. Therefore, he selects procurer n with a positive probability only when $(1 - p_n)^{M-1}r_n \geq (1 - p_k)^{M-1}r_k$ for all k. Without loss of generality, assume that $p_k > 0$ for all k: if $p_k = 0$ for some k, then those procurers are irrelevant to the market outcome. Then, the equilibrium conditions for contractors' search strategies are

$$(1 - p_n)^{M-1}r_n = (1 - p_k)^{M-1}r_k, \forall k \text{ and } \sum_{k=1}^{N} p_k = 1.$$

From the former condition,

$$-(M - 1)(1 - p_k)^{M-2}r_k dp_k = -(M - 1)(1 - p_n)^{M-2}r_n dp_n + (1 - p_n)^{M-1}dr_n.$$

Arranging the terms with the fact that $(1 - p_n)^{M-1}r_n = (1 - p_k)^{M-1}r_k$,

$$dp_k = \frac{1 - p_k}{1 - p_n}dp_n - \frac{1 - p_k}{(M - 1)r_n}dr_n.$$

Summing the equations over k (except for n),

$$\sum_{k \neq n} dp_k = \frac{N - 1 - \sum_{k \neq n} p_k}{1 - p_n}dp_n - \frac{N - 1 - \sum_{k \neq n} p_k}{(M - 1)r_n}dr_n.$$

Using the second equilibrium condition $\sum_{k=1}^{N} p_k = 1$, the equation is equivalent to

$$-dp_n = \frac{N - 1 - (1 - p_n)}{1 - p_n}dp_n - \frac{N - 1 - (1 - p_n)}{(M - 1)r_n}dr_n.$$

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Arranging the terms,

$$\frac{dp_n}{dr_n} = \frac{N - 1 - (1 - p_n)}{(M - 1)r_n} \frac{1 - p_n}{N - 1}.$$ (6)

This equation summarizes contractors' responses to a marginal increase in a procurer's reserve price.

Equilibrium reserve prices.   Combining (5) and (6),

$$\frac{p_n(1 - p_n)}{(1 - p_n)(v_n - r_n) + (M - 1)p_n r_n} = \frac{dp_n}{dr_n} = \frac{N - 1 - (1 - p_n)}{(M - 1)r_n} \frac{1 - p_n}{N - 1}.$$

Arranging the terms,

$$(1 - p_n)(v_n - r_n) + (M - 1)p_n r_n = \frac{N - 1}{N - 1 - (1 - p_n)}(M - 1)p_n r_n,$$

which is equivalent to

$$v_n = \left(1 + \frac{(M - 1)p_n}{N - 1 - (1 - p_n)}\right) r_n.$$ (7)

Search (in)efficiency.   From Appendix A, we know that if $r_n = v_n$, then the market outcome is constrained efficient. (7) implies that in equilibrium, $r_n < v_n$, that is, each procurer always sets a reserve price strictly below her true value. Still, there would be no search inefficiency if it were the case that there exists a constant a such that $r_n = a v_n$ for all n: if so, the equilibrium conditions for contractors' search strategies would coincide with the ones for constrained efficiency $((1 - p_n)^{M-1} v_n = (1 - p_{n'})^{M-1} v_{n'}$ for any n and $n')$. This implies that the condition for search efficiency reduces to whether $\frac{(M-1)p_n}{N-1-(1-p_n)}$ is independent of n. There are two special cases where $\frac{(M-1)p_n}{N-1-(1-p_n)}$ is independent of n.

- p_n is independent of n: Obviously, this is the case when procurers are homogeneous (i.e. no heterogeneity).

- $N = 2$: In this case,
$$\frac{(M - 1)p_n}{N - 1 - (1 - p_n)} = M - 1.$$
  This is the case analyzed by Julien et al. (2002).

As soon as there are more than 3 procurers and at least two of them are heterogeneous, $\frac{(M-1)p_n}{N-1-(1-p_n)}$ cannot be independent of n and, therefore, the market outcome fails to achieve constrained efficiency.

Appendix C: Other Equilibria and Equilibrium Selection

In this section, we first construct the set of all *interval partitional* equilibria, which includes the fully revealing equilibrium as a special case. We then show that the fully revealing equilibrium uniquely satisfies *neologism proofness* by Farrell (1993). We also show that in the presence of lying costs, the fully revealing equilibrium is the unique equilibrium that survives iterated deletion of weakly dominated strategies.

Contractors' Equilibrium Bidding Strategies

We first generalize Lemma 1 in order to accommodate multiple procurer types for an identical message. Similarly to the main text, let $\underline{b}(m)$ and $\overline{b}(m)$ denote the minimal element and the maximal element of the support of $\lambda(m, \cdot)$, respectively. We denote by $\Gamma(m)$ the set of procurer types who announce m (i.e., $\Gamma(m) \equiv \{v \in \mathcal{V} : (m, v) \in \text{supp}Q\}$). In addition, we denote by $\underline{v}(m)$ the minimal element of $\Gamma(m)$ and by $\overline{v}(m)$ the maximal element of $\Gamma(m)$.

Lemma A2 *For each m such that $\lambda_{\mathcal{M}}(m) > 0$, there is no atom in the support of $\lambda(m, \cdot)$. In addition, for any $b \in [\underline{b}(m), \overline{b}(m)]$,*

$$U(m, b) = e^{-\lambda(m,b)}(1 - \mu(b; m))(b - c) \leq U(m, \underline{b}(m)) = (1 - \mu(\underline{b}(m); m))(\underline{b}(m) - c),$$

with the equality holding whenever b is in the support of $\lambda(m, \cdot)$.

Proof. The no-atom result follows from the same reasoning as in the proof of Lemma 1: if there is an atom at some b, then a contractor can obtain a strictly higher expected payoff by quoting a price slightly below b. Given this, the second result follows from a contractor's indifference over all prices in the support of $\lambda(m, \cdot)$ and the optimality of those prices. $\qquad\square$

There are two differences from Lemma 1. First, if a contractor quotes a price b above $\underline{v}(m)$, then it may not be accepted even if it is the lowest price to the procurer (hence, $(1 - \mu(b; m))$ in the equation). Second, the support of $\lambda(m, \cdot)$ is not necessarily convex. This arises, in particular, when $\mu(\cdot; m)$ has an atom at some $v \in [\max\{\underline{v}(m), \underline{b}(m)\}, \underline{b}(m)]$ (an important special case is when $\mathcal{V}$ is finite). In this case, the acceptance probability conditional on winning jumps at v. Since a price slightly above v can never be optimal, there may exist a gap in the support of $\lambda(m, \cdot)$.

Lemma A3 *If $\mu(\cdot; m')$ first-order stochastically dominates $\mu(\cdot; m)$, then $\lambda(m', b) \geq \lambda(m, b)$ for any b.*

Proof. Fix b in the support of $\lambda(m, \cdot)$ (not necessarily in the support of $\lambda(m', \cdot)$). In equilibrium,

$$U(m', b) = e^{-\lambda(m',b)}(1 - \mu(b; m'))(b - c) \leq U(m, b) = e^{-\lambda(m,b)}(1 - \mu(b; m))(b - c).$$

Since $\mu(\cdot; m')$ first-order stochastically dominates $\mu(\cdot; m)$, $1 - \mu(b; m') \geq 1 - \mu(b; m)$. Together these imply that $e^{-\lambda(m',b)} \leq e^{-\lambda(m,b)}$, which is equivalent to $\lambda(m', b) \geq \lambda(m, b)$. For b outside of the support of $\lambda(m, \cdot)$ (possibly in the support of $\lambda(m', \cdot)$), $\lambda(m', b)$ is non-decreasing in b, while $\lambda(m, b)$ is necessarily constant. Therefore, $\lambda(m', b) \geq \lambda(m, b)$ even for such b's. $\square$

Interval Partitional Equilibria

Definition A1 *A market equilibrium is interval-partitional if $\Gamma(m)$ is convex for all m.*

In canonical cheap-talk models, all equilibria are necessarily interval-partitional. The result follows from the single crossing property of the sender's preferences. In our model, agents' payoff functions are endogenously derived, rather than exogenously given, and can take a quite complex form. For instance, one must derive agents' expected payoffs for all combinations of procurer types in order to characterize the set of all equilibria (or to exclude non-interval-partitional equilibria). This renders the argument based on the single crossing property infeasible and calls for certain restrictions on equilibrium structure.

The following proposition characterizes the set of all interval partitional equilibria.

Proposition A2 *For any $\tilde{v} \in \mathcal{V}$, there is an equilibrium in which all procurer types strictly below $\tilde{v}$ fully reveal their types, while all procures weakly above $\tilde{v}$ pool altogether (i.e., announce an identical message). There does not exist any other interval partitional equilibrium that yields a different outcome from any of these.*

Proof. We first show that the prescribed strategy profile is indeed an equilibrium, by proving that no procurer type has an incentive to deviate. For notational simplicity, we assume that all procurer types strictly below $\tilde{v}$ simply announce their types, while all procurer types above $\tilde{v}$ announce $\tilde{v}$. As in the main text, we denote by u contractors' market utility in the equilibrium (i.e., $u \equiv \max_{m,b} U(m, b)$). Finally, we let $\underline{b} = c + u$, so that $\underline{b} = \underline{b}(m)$ for any m in the support of $\lambda_{\mathcal{M}}(\cdot)$.

First, consider $v < \tilde{v}$. This procurer's problem is exactly identical to the one in the fully revealing equilibrium: she strictly prefers truth-telling to downward deviations and is indifferent over all upward deviations. Importantly, the latter includes message $\tilde{v}$ (the message sent by procurers above $\tilde{v}$). This follows from the fact that $e^{-\lambda(\tilde{v},b)}(b - c) = u = e^{-\lambda(v,b)}(b - c)$ for any $b \in [\underline{b}, v] \subset [\underline{b}, \tilde{v}]$. Therefore, she does not have an incentive to deviate. Now, consider $v \in [\tilde{v}, \overline{v}]$ and suppose she deviates to $v' < \tilde{v}$. The desired result follows from Lemma A3, because $\mu(\cdot; \tilde{v})$ first-order stochastically dominates $\mu(\cdot; v')$ (i.e., a degenerate distribution on v') and $\overline{b}(v') = v' < \tilde{v} \leq v$.

Next, we prove that there does not exist any other kind of interval partitional equilibrium. To this end, it suffices to show that there cannot exist $m \in \mathcal{M}$ such that $\underline{v}(m) < \overline{v}(m) < \overline{v}$ and

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$\lambda(m) > 0$. The latter condition is due to the possibility that $\underline{v} - c < u$, in which case some lowest procurer types can pool together. Notice that this has no effect on the equilibrium outcome, as those procurers are simply not chosen by contractors, whether they pool or not. To show that this cannot be an equilibrium, let m' be the message sent by type $\overline{v}$ procurers, whether her type is fully revealed or not. In an interval partitional equilibrium, the minimal element of $\Gamma(m')$, $\underline{v}(m')$, is necessarily larger than $\overline{v}(m)$, which ensures the first-order stochastic dominance of $\mu(\cdot; m')$ over $\mu(\cdot; m)$. The desired result (that a type $\overline{v}(m)$ strictly prefers m' to m) then follows from Lemma A3. The strict inequality is guaranteed because $\mu(\overline{v}(m); m) > 0 = \mu(\overline{v}(m); m')$, and thus $\lambda(m, \overline{v}(m)) < \lambda(m', \overline{v}(m))$. $\qquad\square$

Neologism Proofness

Farrell (1993) defines neologism proofness in the context of standard cheap-talk games. We omit economic motivations behind neologism proofness. Interested readers are referred to the original paper by Farrell (1993).

We begin by translating the definition of neologism proofness into our context. For each (measurable) subset X of $\mathcal{V}$, let $V(X, v)$ denote the expected payoff a type v procurer obtains if contractors believe that his type is drawn from X according to F (restricted to X) and they can obtain only as much utility as their market utility by selecting such a procurer. Formally, let $\lambda(X, b)$ be the maximal non-negative value such that

$$e^{-\lambda(X,b)} \left(1 - \frac{F_-(b)}{F(X)} \right) (b - c) \leq u,$$

where $F_-(b) = \lim_{b' \nearrow b} F(b')$ and $F(X) = \int_X dF(v)$. In other words, the queue length following an out-of-equilibrium message ("neologism") is determined so that contractors obtain the same utility as their market utility u by selecting the procurer (if this is not possible, then set $\lambda(X, b) = 0$). This restriction on λ amounts to the market utility assumption in canonical competitive search and competing auctions models (see, e.g., Eeckhout and Kircher (2010a) and Peters (2010)). Then, let

$$V(X, v) = \int_{b \leq v} (v - b) d \left(1 - e^{-\lambda(X,b)} \right).$$

Definition A2 *Given the interval partitional equilibrium with cutoff type $\tilde{v}$, a subset X of $\mathcal{V}$ is self-signaling if*

$$K(X|\tilde{v}) \equiv \{v < \tilde{v} : V(v, v) < V(X, v)\} \cup \{v \geq \tilde{v} : V(\tilde{v}, v) < V(X, v)\} = X.$$

An equilibrium is neologism-proof if no subset of $\mathcal{V}$ is self-signaling.

Intuitively, $K(X|\tilde{v})$ is the set of procurer types that would obtain a strictly higher expected

payoff than their equilibrium expected payoff if contractors believe that his type belongs to X. A set X is self-signaling if $K(X|\tilde{v})$ coincides with X itself. Neologism proofness requires that there is no such self-signaling set in a given equilibrium.

We first show that no interval partitional equilibrium in which $\tilde{v} < \overline{v}$ (i.e., non-fully revealing equilibrium) is neologism-proof, provided that the type space $\mathcal{V}$ is finite. We explain the role of finiteness after the proof.

Proposition A3 *Suppose $\mathcal{V}$ is finite. Then, in the interval partitional equilibrium with cutoff procurer type $\tilde{v} < \overline{v}$, the set $X = (\tilde{v}, \overline{v}] \cap \mathcal{V}$ is self-signaling.*

Proof. The result follows from the fact that for any $b \in [\underline{b}, \tilde{v}]$,

$$e^{-\lambda(X,b)}(b-c) = u = e^{-\lambda(\tilde{v},b)}(b-c) \Rightarrow \lambda(X,v) = \lambda(\tilde{v},v),$$

while if $b > \tilde{v}$, then

$$e^{-\lambda(X,b)}\left(1 - \frac{F_-(b)}{F(X)}\right)(b-c) = u = e^{-\lambda(\tilde{v},b)}\left(1 - \frac{F_-(b)}{F([\tilde{v},\overline{v}])}\right)(b-c) \Rightarrow \lambda(X,b) > \lambda(\tilde{v},b),$$

because $F(X) < F([\tilde{v},\overline{v}])$. The first part implies that all procurer types weakly below $\tilde{v}$ do not find the deviation to X strictly profitable, while the second part implies that the deviation to X is strictly profitable to all types strictly above $\tilde{v}$. $\square$

If $F(X) = F((\tilde{v},\overline{v}]) = F([\tilde{v},\overline{v}])$ (i.e., F does not put any mass on $\tilde{v}$), then the strict inequality in the second equation no longer holds, which makes the deviation to X only weakly profitable to the types in $(\tilde{v},\overline{v}]$. The finiteness of $\mathcal{V}$ is a sufficient condition for $F(X) < F([\tilde{v},\overline{v}])$ for any $\tilde{v} \in \mathcal{V}$.

Finally, we show that the fully revealing equilibrium is neologism-proof.

Proposition A4 *There is no self-signaling set in the fully revealing equilibrium.*

Proof. Suppose that X is a self-signaling set in the fully revealing equilibrium. It suffices to show that the deviation payoff to a type $\overline{v}(X)$ procurer (the maximal element of X) cannot be larger than her equilibrium payoff (i.e., $V(X,\overline{v}(X)) < V(\overline{v}(X),\overline{v}(X))$). This result is straightforward from the fact that $\mu(\cdot;\overline{v}(X))$ first-order stochastically dominates $\mu(\cdot;X)$ and $\lambda(X,\overline{v}(X)) < \lambda(\overline{v}(X),\overline{v}(X))$ (see Lemma A3).

$\square$

Iterated Deletion of Weakly Dominated Strategies under Lying Costs

We now introduce the cost of lying into procurers' payoffs, as in Kartik (2009). Specifically, take a continuous function $C : \mathcal{V} \times \mathcal{V} \to \mathbb{R}$ such that for any $v \in \mathcal{V}$, $C(m,v)$ is strictly decreasing in m

if $m < v$ and strictly increasing in m if $m > v$. Then assume that a type v procurer loses utility $C(m, v)$ if she announces m. Since this cost of lying is concerned only with procurers' expected payoffs at the communication stage, no previous characterization, other than the existence of equilibria, is affected by its introduction.

As in several other studies (e.g., Chen, Kartik and Sobel (2008), Kartik (2009), and Chen (2011)), we also impose monotonicity on agents' strategies. Specifically, we restrict attention to strategy profiles in which a higher message increases contractors' beliefs about the procurer's type in the sense of first-order stochastic dominance (i.e., if $m' > m$, then $\mu(\cdot; m')$ first-order stochastically dominates $\mu(\cdot; m)$).

Proposition A5 *Assume that $\mathcal{V}$ is finite. In the presence of lying costs, the fully revealing equilibrium is the unique monotone equilibrium that survives iterated deletion of weakly dominated procurer strategies.*

Proof. Suppose $\mathcal{V} = \{v^1, ..., v^I\}$, where $v^i < v^{i+1}$ for all $i = 1, ..., I - 1$. We prove the result by mathematical induction.

(i) Consider a type v^1 procurer. As in the proof of Proposition A4, this type gains nothing from increasing contractors' beliefs (i.e., pooling with the other types). On the other hand, truth-telling minimizes her cost of lying. Therefore, truth-telling is a weakly dominant strategy for her.

(ii) Suppose each type v^j procurer truthfully reveals her type for all $j < i$. If a type v^i contractor truthfully reveals her type, then she can induce at least as optimistic beliefs as the degenerate belief on v^i. In the absence of lying costs, this makes her strictly prefer truth-telling to downward deviations (announcing a message below v^i) and indifferent over all upward deviations. Combining this with the fact that truth-telling minimizes her cost of lying, it follows that truth-telling is a weakly dominant strategy for a type v^i procurer as well. $\qquad\square$

Appendix D: Risk Aversion

In this section, we show that the fully revealing equilibrium exists even if agents are risk averse.

Suppose that if a contractor is hired by a type v procurer at price p, then his utility is given by $\phi(p - c)$ (i.e., $\phi(\cdot)$ is the Bernoulli utility function for contractors), where $\phi(\cdot)$ is a strictly increasing and concave function. We do not introduce a new notation for procurers, because, as shown shortly, procurers' risk aversion does not affect the argument. The utility of a contractor who fails to match is assumed to be $\phi(0) = 0$. We maintain the assumption that each agent maximizes his or her expected utility. Since the translation is straightforward, we omit a detailed specification of agents' payoffs in the market game.

Regarding Lemma 1, the only necessary modification is that now for each $b \in [\underline{b}(v), \overline{b}(v)]$,

$$e^{-\lambda(v,b)}\phi(b - c) = e^{-\lambda_{\mathcal{M}}(v)}\phi(v - c) = \phi(\underline{b}(v) - c). \tag{8}$$

To prove that no procurer has an incentive to deviate from truthful revelation, consider the probability that a procurer with message v does not receive any price quote below b. Similarly to the one in the main text, it is given by

$$e^{-\lambda(v,b)} = \frac{\phi(\underline{b} - c)}{\phi(b - c)}.$$

The equality is due to (8) and the fact that $\underline{b}(v)$ is independent of v, provided that $\lambda(v) > 0$. The independence of this probability with respect to v can be used, as in the main text, to show that each procurer strictly prefers truth-telling to downward deviations and is indifferent over all upward deviations.

Appendix E: Contractor Heterogeneity

In this section, we demonstrate that our main result continues to hold even when contractors are heterogeneous. We consider the two most common specifications where contractors differ either in terms of their opportunity cost of working (see, e.g., Peters, 1997b; Mortensen and Wright, 2002) or in terms of observable skills that add the same value across firms (see, e.g., Shi, 2002; Shimer, 2005).

Heterogeneous Opportunity Costs of Working

Setup. We consider the same basic setup as in Section 2, but with one adjustment. Now, at the beginning of the game each contractor independently and identically draws his opportunity cost of working from the set $\mathcal{C} \equiv [\underline{c}, \overline{c}]$ according to the distribution function G with continuous and everywhere-positive density g. In addition, to ease the exposition and notation, assume that the set of procurers' types $\mathcal{V}$ is also convex (i.e., $\mathcal{V} \equiv [\underline{v}, \overline{v}]$) and the distribution function F also admits a continuous and everywhere-positive density f. To avoid triviality, also assume that $\underline{c} < \underline{v}$ and $\overline{c} < \overline{v}$.

Fully revealing equilibrium. In order to deliver the main result most efficiently, we directly present the candidate (fully revealing) strategy profile and prove that the strategy profile is indeed an equilibrium as well as constrained efficient. We consider the following strategy profile:

- Procurers: each procurer truthfully reveals her type, that is, supp$Q = \{(m, v) \in \mathcal{V} \times \mathcal{V} : m = v\}$.

- Contractors: there exists a continuous and strictly increasing function $w : \mathcal{C} \rightarrow \mathcal{R}_+$ such that all type c procurers quote an identical price $w(c)$ and their quotes are randomly distributed to all procurers above $w(c)$.

The latter property implies that the queue length of type c contractors for a type v procurer is given by

$$\lambda(v, c) = \frac{\beta g(c) \frac{f(v)}{1 - F(w(c))}}{f(v)} = \frac{\beta g(c)}{1 - F(w(c))} \text{ if } v > w(c), \tag{9}$$

and $\lambda(v, c) = 0$, otherwise. For notational convenience, define $\Lambda(v, c) \equiv \int_{\underline{c}}^{c} \lambda(v, x) dx$, which represents the queue length of all contractors below c for a type v procurer.

Truthful revelation. We first prove that no procurer has an incentive to deviate from truthful revelation. As in the main text, consider the probability that the lowest quote to a type v procurer is less than or equal to $w(c)$. Without loss of generality, we consider only the case where $w(c) \leq v$. Given that $w(\cdot)$ is strictly increasing, the probability is given by $1 - e^{-\Lambda(v,c)}$. Since $\lambda(v, c)$ is independent of v conditional on $v > w(c)$ (see (9)), this probability is also independent of v. As in the main text, combining this with the fact that the upper bound of the support of price quotes for a type v procurer is equal to v yields the desired result: each procurer strictly prefers truth-telling to downward deviations and is indifferent over all upward deviations.

Equilibrium prices. Next, we pin down the function $w(\cdot)$ by exploiting the revenue equivalence between first- and second-price auctions. Suppose each procurer runs a second-price auction with reserve price equal to her own value v, but contractors select procurers as prescribed in the above strategy profile. Then, the expected payoff of a type c contractor is given by

$$u(c) = e^{-\Lambda(v,c)}(v - c) - \int_c^{\overline{c}} (v - c') d \left(1 - e^{-\Lambda(v,c')} \right). \tag{10}$$

The first term reflects the fact that a type c contractor is hired by a type v procurer if and only if there is no other contractor whose type is strictly below c. The second term reflects the fact that the price paid to a contractor in case he wins the auction is determined by the second lowest contractor type. Notice that the expression is indeed independent of v (conditional on $v > w(c)$) for the same reason as above.

Given $u(c)$, we specify $w(c)$ so that a type c contractor's expected payoff from the first-price auction is identical to that from the second-price auction. Formally, let $w(c)$ be the value that satisfies

$$u(c) = e^{-\Lambda(v,c)}(w(c) - c).$$

Since $\Lambda(v, c)$ is independent of c, $w(c)$ is well-defined. By construction, $w(\cdot)$ is also continuous

and strictly increasing. Finally, the incentive compatibility for contractors' bidding also follows from that of the second-price auction: no contractor has an incentive to deviate from quoting $w(c)$.

Constrained efficiency.   It remains to show that the resulting market outcome is constrained efficient. Since the formal argument is not much different from the one in the main text, we only provide an intuition behind the efficiency result. The result is, in fact, straightforward from (10). It shows the expected payoff of a type c contractor $u(c)$, but it also coincides with the contractor's marginal social contribution. To see this, notice that the first term can be (re)interpreted as the amount of expected social surplus a type c procurer can generate by selecting a type v procurer: note that the social planner will assign each project to the lowest cost contractor. The second term can be (re)interpreted as the negative externality imposed by the contractor on the contractors above his type: if a type c contractor did not select the procurer, then the procurer could have hired a contractor whose type is above c. It follows that each contractor's optimal action is also socially optimal, and thus the resulting market outcome is constrained efficient. See the previous working paper version of this paper for a more formal argument.

Heterogeneous Skills

Setup.   We again consider the same basic environment as in Section 2, but now allow for contractor heterogeneity in terms of their observable skills. For simplicity, we normalize c to 0. Each contractor's skill, denoted by s, is independently and identically drawn from the set $\mathcal{S} \equiv [\underline{s}, \overline{s}]$ according to the distribution function G. Assume, again, that the two distribution functions F and G admit continuous and everywhere-positive densities f and g, respectively. If a type s contractor is hired by a type v procurer at p, then the contractor receives utility p, while the procurer obtains utility $v + s - p$.

Fully revealing equilibrium.   Since the analysis is similar to that of the previous one, we illustrate only the necessary changes for this specification. Now the candidate strategy profile is given as follows:

- Procurers: each procurer truthfully reveals her type, that is, supp$Q = \{(m, v) \in \mathcal{V} \times \mathcal{V} : m = v\}$.

- Contractors: there exists a continuous and strictly increasing function $w : \mathcal{S} \to \mathcal{R}_+$ such that $s - w(s)$ is strictly increasing (i.e., the slope of $w(\cdot)$ is strictly smaller than 1 everywhere), all type s contractors quote an identical price $w(s)$, and their quotes are randomly distributed over all procurer types above $w(s) - s$.

The restriction on the slope of $w(\cdot)$ guarantees that although a more skilled contractor quotes a higher price, each procurer is willing to hire the most skilled contractor. In this strategy profile, as before, the queue length of type s contractors for a type v procurer is given by

$$\lambda(v,s) = \frac{\beta g(s)\frac{f(v)}{1-F(w(s)-s)}}{f(v)} = \frac{\beta g(s)}{1-F(w(s)-s)}.$$

In addition, let $\Lambda(v,s) \equiv \int_s^{\overline{s}} \lambda(v,s)ds$ denote the queue length of contractors above s for a type v procurer.

Truthful revelation. Consider the probability that the best quote to a type v procurer is the one made by a type s procurer. Since the procurer's surplus $v + s - w(s)$ is strictly increasing in s, the probability is given by $1 - e^{-\Lambda(v,s)}$. The fact that this probability is independent of v conditional on $v + s \geq w(s)$, once again, can be used to verify procurers' incentives at the communication stage.

Equilibrium prices. As in the previous case, suppose each procurer runs a second-price auction. In the current specification, it is necessary that a procurer's reserve price depends on a contractor's type. Specifically, assume that a type v procurer's reserve price for a type s contractor is given by $v + s$. Then, the expected payoff of a type c contractor who selects a type $v(\geq w(s) - s)$ procurer is given by

$$u(s) = e^{-\Lambda(v,s)}(v+s) - \int_{\underline{s}}^s (v+s')d\left(1 - e^{-\Lambda(v,s')}\right). \tag{11}$$

Given $u(\cdot)$, the function $w(\cdot)$ can be derived from

$$u(s) = e^{-\Lambda(v,s)}w(s).$$

Constrained efficiency. The argument for constrained efficiency is identical to that of the previous case. Each contractor's expected payoff $u(s)$ in (11) coincides with his marginal social contribution: now the second term represents the negative externality imposed by a type s contractor on those contractors whose types are *below* s (who would not be hired because of the type s contractor). Therefore, the resulting market outcome is constrained efficient. See the previous working paper version of this paper for a more formal argument.

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