The Scandinavian
Journal of Economics

Scand. J. of Economics 112(4), 841–863, 2010
DOI: 10.1111/j.1467-9442.2010.01628.x

Downward Wage Rigidity in a Model of Equal Treatment Contracting*

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Abstract

Following insights by Bewley (1999a), this paper analyses a model with downward rigidities in which firms cannot pay discriminately based on year of entry to the firm, and develops an equilibrium model of wages and unemployment. We solve for the dynamics of wages and unemployment under conditions of downward wage rigidity, where forward-looking firms take into account these constraints. We show that there is a frontloading incentive that leads to a simple solution in the case of certainty. Using productivity data from the postwar US economy, we analyse the ability of the model to match certain stylised labour market facts.

Keywords: Labour contracts; business cycle; unemployment; equal treatment; downward rigidity; cross-contract restrictions

JEL classification: *E*32; *J*41

I. Introduction

Truman Bewley has argued that there are two key features constraining wage cuts for new hires in recessions (Bewley, 1999a). First, because wage cuts for incumbents will have a negative impact on morale, firms avoid them under all but extreme circumstances. Second, while new hires may be willing to work at a lower wage than that paid to incumbents, paying them less would disrupt internal equity and so their wages will be set at the same level as incumbents' wages. Bewley explains this as follows:

*We are grateful for suggestions from Nobu Kiyotaki, John Moore, participants at the Centre for Dynamic Macroeconomic Analysis conference in St. Andrews, and to two referees for a number of helpful comments. We are grateful for financial support from the ESRC (RES-062-23-0546). Martins is also a research Fellow of CEG-IST, Lisbon.

> "New employees, in contrast, feel it is inequitable to be paid according to a scale lower than the one that applied to colleagues that were hired earlier. For this reason, downward pay rigidity for new hires exists only because the pay of existing employees is rigid." [Bewley, 1999b]

Bewley's account mainly concentrates on the question of why firms do not cut wages in recession. But it raises the important question of how forward-looking firms take into account the fact that such constraints may arise in the future. For example, a firm, anticipating downward wage rigidity, may temper wage increases in better times. Indeed, one might ask in Bewley's analysis what determines the initial wage that is too high once recession strikes. What are the implications for wages and employment in an equilibrium model with forward-looking firms and workers?

In this paper we attempt to accomplish two things. First, we analyse the dynamic consequences of Bewley's insights, by solving a dynamic equilibrium model in which forward-looking firms take into account downward rigidity constraints. Our main result is a characterisation of equilibrium. Second, we investigate whether the resulting wage and unemployment dynamics have reasonable properties when judged against US postwar experience.

We analyse a model in which the pay of new hires and existing workers is linked within each firm—indeed is identical, given that we assume all workers are perfect substitutes—and in which the pay of incumbents is subject to some downward rigidity. This rigidity is then transmitted to the pay of new hires.[1] Workers and firms anticipate the effects of this, so that, for example, with full downward rigidity, an increase in current wages means that future wages cannot be cut below this level. Despite the extensive literature on downward rigidities, there has been almost no analysis of the forward-looking nature of the decision problem and its labour market implications. The only exception we are aware of is Elsby (2009), who solves a problem involving downward nominal rigidities, but in a very different context.

To do this, we take the equal treatment model of Snell and Thomas (2010) and add an explicit downward rigidity constraint.[2,3] Without the

[1] We stress the point that for there to be significant labour market implications, it is necessary that downward rigidity applies to new hires. If it only applies to incumbent workers so that new hires can be hired at a flexible wage, there is no reason why hiring decisions should not be efficient.

[2] We also extend that model to multiple sectors, an extension that is crucial for addressing recent empirical evidence.

[3] We do not provide explicit foundations for the two constraints. Snell and Thomas (2010) show how equal treatment can be endogenised if the contracting environment is "at will", and a similar argument could be used here. A closely related approach is developed by

downward rigidity restriction, but with equal treatment of workers, firms have to trade off, as we show later, the desire to insure their risk-averse workers against the need to respond to market conditions; this will not only prevent their workers from quitting, but will also take advantage of states of the world where labour is cheap. The insurance motive alone provides a degree of downward rigidity, but it may not be sufficient with plausible parametrisations to generate enough employment variability. Here, we add to this model downward wage restrictions in the spirit of Bewley. A firm will face not only a sequence of participation constraints, but also a constraint each period restricting the degree to which it can cut its wage. A major challenge of the analysis is to solve for an equilibrium in the face of these constraints.

We show that the combination of equal treatment and sufficient labour turnover leads to a frontloading incentive. If a firm is not constrained next period either by the participation constraint or by the downward rigidity constraint, then it will want to increase current wages at the expense of future wages. The reason for this is that a worker employed today puts less weight on future wages than does the firm. Because of (exogenous) turnover, there is a chance that the worker will not receive the future wage, but the firm will still pay—by equal treatment—the same wage to a new hire who replaces the worker. The case with no uncertainty allows us to deduce that the wage will always be up against one of the constraints, and we are able to provide a complete characterisation of equilibrium provided that the downward rigidity constraint is not too loose. Unfortunately, this result does not fully generalise to the case with uncertainty, although we present a result for a two-period model.

The equal treatment assumption prevents firms from cutting wages for new entrants, so that in periods with adverse shocks, the wage may not fall sufficiently to clear the labour market. In the absence of equal treatment, wage rigidity in this model (with fixed hours) would not have employment effects as the terms offered to new entrants would be flexible. We show, however, that under certain conditions firms hire up to the point where the real wage equals the marginal product of labour. To the extent, then, that wages do not correspond to market-clearing levels, hiring will be inefficient. We show that this occurs only in the direction of wages that are too high, leading to inefficiently low employment and an excess supply of labour.

When simulated with sectoral productivity shocks, our model is capable of generating simulation results that are reasonably consistent with empirical unemployment and wage movements over the business cycle. We find

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Menzio and Moen (2008). Similar issues arise in the insider–outsider literature; see Gottfries and Sjöström (2000).

that this model produces more reasonable wage–unemployment dynamics than Snell and Thomas (2010).

An outline of the paper is as follows. We start by looking at whether there is evidence for the equal treatment assumption in Section II. Then we lay out the basic assumptions of our model in Section III. In Section IV, we simulate the model using sectoral TFP data from the postwar US economy to generate predictions of wage and unemployment movements and to see the extent to which these satisfy certain stylised facts. Section V contains concluding comments.

II. Evidence for Equal Treatment

While we simply impose that all workers within a firm must be paid the same, here we briefly review some empirical evidence on the issue of equal treatment. An early study was Baker *et al.* (1994), who examined the pay of managerial employees in a single firm over time. They found that incumbents' pay tends to move together, but the pay of entrants is significantly more variable, suggesting that the pay of new hires may be more subject to outside conditions than that of incumbents. However, they do not formally control for composition of entry cohorts, so it is difficult to know what causes this extra variability. Wachter and Bender (2008) have recently run a similar analysis on a number of firms in a large German manufacturing sector, and they, too, find evidence of substantial and quite persistent entry cohort effects. However, these seem to be widely distributed across firms at any given date, suggesting, as they note, that they are not driven by cyclical phenomena. A study by Kwon and Milgrom (2005) of Swedish workers finds that if cohort effects for labour market entry and occupation entry are included in addition to firm entry cohort effects, the former two are procyclical in line with expectations, while the latter actually appear *counter*cyclical. In other words, a worker entering the labour market in a downturn will tend to do worse than those already active, but entering a firm in a downturn does not of itself lead to a lower wage than that received by incumbents. Haefke *et al.* (2007) argue that wages of those entering the labour market from non-employment are considerably more variable than wages of those who remain in employment. However, this does not imply that new hires are treated differently within particular firms. In a similar vein, Pissarides (2007) summarises empirical studies that find wages for workers who change jobs are considerably more procyclical than those who remain with the same employer. Gertler and Trigari (2009), however, argue that such studies are not demonstrating that the new hire wage is more procyclical than a stayer's wage *within a particular firm*. The reason for the empirical finding could simply be that match quality varies procyclically.

Finally, as discussed above, survey evidence in Bewley (1999a) suggests that violations of equal treatment are unusual, particularly in the primary sector. Similar findings exist for other countries. Swedish managers responded that "hiring underbidders would violate their internal wage policy" (Agell and Lundborg, 1999, p. 7). In a British survey, Kaufman (1984) reported that almost all managers viewed bringing in similarly qualified workers at lower wage rates as "infeasible". Akerlof and Yellen (1990) argue that personnel management texts treat the need for equitable pay as virtually self-evident.

III. The Model

The model is as follows. Time t runs from 1 to T, and there is a single consumption good in each period. There are M equal-sized sectors. All workers are assumed to be identical, except for the date at which they enter the labour market and the sector with which they are currently associated. We abstract from any tenure or experience effects on productivity. Workers are risk averse with per-period twice-differentiable utility function $u(w), u' > 0, u'' < 0,$ where $w \geq 0$ is the real wage that must be consumed within the period. There is no disutility of work, and hours are fixed so that workers are either employed or unemployed. If workers are not employed in a period, they receive some low consumption level $\underline{c} > 0$. There is a large number of identical risk-neutral firms in each sector. A firm in sector m has a diminishing-returns technology where output is $F^m(N, s_t)$ with $\partial F^m / \partial N > 0, \partial^2 F^m / \partial N^2 < 0,$ where N is the labour input and s_t is the current shock, which specifies current productivity in each sector. It is assumed that a firm must always employ some workers in each period.[4] Workers and firms discount the future with the common factor $\beta \in (0, 1]$.

Workers exogenously separate with probability $1 - \delta$. Of the separated, a proportion ϕ remains in the same sector while $(1 - \phi)$ are distributed evenly across sectors.[5] Likewise, $(1 - \phi)$ of the unemployed move to other sectors. Thus all movements between firms and between sectors are due to exogenous separations and workers cannot choose, for example, which sector to move to. Separation occurs at the end of a period so that separated workers who find a job in the following period do not suffer any unemployment. We assume there are a large number of workers relative to the number of firms, and we normalise the ratio of workers to firms to one

[4] This can be motivated by an assumption that firms cannot produce after a period of zero production.

[5] We want to include multiple sectors in the model for the later simulation exercise. The fact that generally some sectors will have positive unemployment while others have zero unemployment means that the model does not have an aggregate unemployment rate that is often at a minimum bound.

in each sector.[6] We assume that the "spot wage"/full-employment solution ($N = 1$) is always greater than the unemployment consumption level; that is, $\partial F^m/\partial N(1, s_t) > \underline{c}$ for all t.

The shock s_t follows a stochastic process taking a finite number of possible values, and with initial value s_1, which we specify in more detail below. Let $h_t \equiv (s_1, s_2, \ldots, s_t)$ be the history at t. The labour market offers a worker currently looking for work in sector m an expected lifetime utility, $\chi_t^m = \chi_t^m(h_t)$. A firm must offer at least $\chi_t^m(h_t)$ to prevent its workers from quitting, and this is also the minimum utility that must be offered to hire. The firm can hire any number of workers by offering at least χ_t^m.

The timing is as follows. At date 1 each firm in sector m offers a *single* state-contingent wage contract $(w_t^m(h_t))_{t=1}^T$ *to which it is committed*. Workers can then accept contracts, and period 1 production takes place. At the end of period 1, a firm loses a fraction $(1-\delta)$ of its workforce due to exogenous separation, as described above. At the start of each subsequent period $t \geq 2$, firms and workers observe s_t. Workers may quit costlessly at this point and join the pool of those previously separated, the unemployed, and new entrants to the sector; they face the same probability of employment (so receive $\chi_t^m(h_t)$), but they may not switch sectors. Provided the continuation utility offered by the contract at least matches χ_t^m, the firm is able to retain its staff and hire as many new workers as it requires from the pool of those looking for work. Production takes place, wages $w_t^m(h_t)$ are paid, and so on.

The Firm's Problem

At the start of date 1, after s_1 is observed, firms in each sector m commit to contracts $(w_t^m(h_t))_{t=1}^T$, $w_t^m(h_t) \geq 0$, which we assume are not binding on workers. *We assume equal treatment within the firm.* A worker joining subsequently, at τ after history h_τ, is offered a continuation of this same contract: $(w_t^m(h_t))_{t=\tau}^T$. To avoid cluttering the notation, we omit sector superscripts in what follows, unless necessary. Let $V_t(h_t)$ denote the continuation utility from t onwards from the contract, defined recursively by

$$V_t(h_t) = u(w_t(h_t)) + \beta\Bigg[E\delta V_{t+1}(h_{t+1}) + (1-\delta)\phi\chi_{t+1} + \sum_{m' \neq m}(M-1)^{-1}(1-\delta)(1-\phi)\chi_{t+1}^{m'} \mid h_t\Bigg], \quad (1)$$

[6] We keep the labour force fixed, although extending the model to allow exit and entry is straightforward and does not affect the results.

with $V_{T+1}=0$, where E denotes expectation, and the terms involving χ_{t+1} and $\chi_{t+1}^{m'}$ reflect the utility after exogenous separation, if the worker respectively remains in the same sector or moves to another sector. Each firm also has a planned employment path $(N_t(h_t))_{t=1}^T$, where $N_t(h_t)\geq 0$.

Note that it is assumed in (1) that there are no lay-offs, only exogenously determined separations. Our aim is to construct an equilibrium in which lay-offs do not occur, largely because it substantially simplifies the analytics of the solution. Provided exogenous turnover is high enough relative to negative shocks, it can be shown that the firm always wants to hire new workers and does not want to replace any existing ones, so we henceforth assume the parameters are consistent with this.[7]

The problem faced by the firm, which takes the stochastic sequence of outside option values in its sector, $(\chi_t)_{t=1}^T$, as parametric (as well as those in other sectors), is

$$\max_{(w_t(h_t))_{t=1}^T,(N_t(h_t))_{t=1}^T} E\left[\sum_{t=1}^{T}(\beta)^{t-1}(F^m(N_t(h_t),s_t)-N_t(h_t)w_t(h_t))\right], \quad (2)$$

subject to

$$V_t(h_t)\geq \chi_t(h_t) \quad (3)$$

for all positive probability h_t, $T\geq t\geq 1$, and

$$w_t(h_{t-1},s)\geq b(h_{t-1},s)w(h_{t-1}), \quad (4)$$

for all positive probability h_{t-1}, all $s\in S$ with $\pi_{s_{t-1}s}>0$, $T\geq t\geq 2$. Equation (3) is the *participation constraint* that says that at any point the contract must offer at least what a worker can get by quitting, and (4) is the downward constraint that imposes that wages cannot fall at a rate faster than an amount that may depend on the current state, given by $b(h_t)$. For $b=1$ we have downward real wage rigidity, and for $b=0$ we have the problem in which there is no downward constraint on wages. Downward nominal rigidity would be captured by $b(h_t)=p_{t-1}/p_t$, where p_t is the price level at t.

[7] For details, see the arguments in Snell and Thomas (2010) that apply in the current setting *mutatis mutandis*. Note that given that the rate of separation is exogenous, it is changes in hiring that drive movements in unemployment in our model. This is consistent with Hall (2005), who argues that the separation rate in the US labour market is roughly constant—see also Pissarides (1986) and Shimer (2005)—and he also argues that although job losses rise during recessions, the increase is usually very small in relation to the normal levels of separation. However, these conclusions have been disputed—see Elsby *et al.* (2009).

Equilibrium

We use an asterisk to denote equilibrium values.[8] We are looking for symmetric solutions; that is, all firms in a sector choose the same contract. To close the model, we impose an equation specifying the equilibrium determination, given by $(w_t^*(h_t))_{t=1}^T, (N_t^*(h_t))_{t=1}^T$, of the outside option in sector m:

$$\chi_t = \frac{N_t^* - \delta N_{t-1}^*}{1 - \delta N_{t-1}^*} V_t^* + \frac{1 - N_t^*}{1 - \delta N_{t-1}^*} U_t, \tag{5}$$

$N_0^* = 0$, where V_t^* is the equilibrium contract offer at t and U_t is the discounted utility of a worker who is unemployed at t—that is, who currently has failed to find a job. The number of workers who remain in a job from $t-1$ is δN_{t-1}^*, which is the survival rate times the number employed in the sector at time $t-1$. Thus, the denominator of the coefficient on V_t^* is the number of workers not retained after $t-1$ (i.e., the number seeking work at t), while the numerator is the number of hires at t. U_t is given by

$$\begin{aligned} U_t(h_t) = u(\underline{c}) + \beta\big(&\phi E[\chi_{t+1} \mid h_t] \\ &+ \sum_{m' \neq m} (M-1)^{-1}(1-\phi) E\big[\chi_{t+1}^{m'} \mid h_t\big]\big); \end{aligned} \tag{6}$$

that is, the utility from the reservation wage plus future utility from not having a job at the beginning of $t+1$. Given the endpoint condition $\chi_{T+1}^{m'} = 0$, for all m', (5), (1), and (6) uniquely determine U_t, V_t^*, and χ_t.

Note that there are two cases. If the labour market in sector m at time t clears, $N_t^*(h_t) = 1$; then from (5), $\chi_t(h_t)$ must offer the utility offered by other firms. In symmetric equilibrium, other firms in the same sector are offering an identical contract, and so the utility associated with this, $V_t^*(h_t)$, must be offered. If, on the other hand, there is an excess supply of labour,[9] $N_t^*(h_t) < 1$, the outside opportunity will depend on the probability of getting a job.

We can summarise as follows.

Definition 1. $((w_t^{*m}(h_t))_{t=1}^T, (N_t^{*m}(h_t))_{t=1}^T)_{m=1}^M$ *constitutes* a symmetric equilibrium *if it solves problem (2) for each m where* $(\chi_t^m)_{t=1}^T$ *is determined recursively from (1), (5), and (6).*

Employment is determined by a standard marginal productivity equation (again suppressing sector superscripts).

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[8] This subsection broadly follows Snell and Thomas (2010), *mutatis mutandis*, including proofs of the lemmata, which we therefore omit.

[9] The case of excess demand for labour cannot arise in equilibrium; see footnote 10 below. However, because of equal treatment, the case of excess supply can arise since workers cannot undercut.

Lemma 1. *In a symmetric equilibrium,* $N_t^*(h_t)$ *satisfies*

$$\partial F\big(N_t^*(h_t), s_t\big)\big/\partial N = w_t^*(h_t). \tag{7}$$

This is a very useful implication of the combination of equal treatment and positive hiring. The firm can effectively "neutralise" an extra hire today by hiring δ fewer workers in the next period (possible by positive hiring), so employment from the next period on is unchanged. Notice that this requires equal treatment—if hires in the next period were brought in on a different contract, this neutralisation would not be possible. The only consideration is thus whether the extra worker makes a profit today; that is, whether the current marginal product exceeds the wage. If it does, the firm should hire more workers, so at an optimum there must be equality.

Suppose that at some t, the participation constraint binds. Then there must be full employment, and the wage is determined by marginal productivity at full employment.

Lemma 2. *In a symmetric equilibrium, the participation constraint binds at* h_t *if and only if* $N_t^*(h_t) = 1$*; moreover, if the constraint binds, then* $w_t^*(h_t) = \partial F(1, s_t)/\partial N$.

The argument for the first assertion of the lemma is simply that if there is full employment, quitting will not change a worker's utility because she can move immediately to another firm with an identical contract. This makes the worker indifferent about leaving; that is, the participation constraint binds. If, on the other hand, there is unemployment, then again by symmetry a worker must be worse off if she quits—because at best she will get the same contract, but now there is a chance she will become unemployed. Consequently, the participation constraint must be slack.

We define $\underline{w}_s^{*m} = \partial F^m(1, s)/\partial N$; in view of the second assertion of Lemma 2, this is the equilibrium wage when the participation constraint binds in state s. This would also be the wage in a spot market.

The above is very useful, as Lemma 1 tells us that if the contract wage is below the spot wage for that state, we get employment above unity, which is infeasible. So this case cannot occur,[10] and the contract wage must always be at or above the spot wage. If wages are above the spot wage, on the other hand, there is unemployment, and so by Lemma 2 the participation constraint cannot bind. This does not depend on the form of the optimal wage contract, but only on equal treatment and on an optimal hiring policy

[10] Intuitively, the reason that excess demand for labour at time t is impossible is the following. A firm could, by an infinitesimal increase in its wage, attract as many workers as it wishes. Even with downward constraints, this will add at most a tiny amount to current and future wage costs, but it allows for a non-negligible increase in current profits.

for a given contract, together with the hypothesis that firms always hire and that the equilibrium is symmetric across firms in the sector.[11]

Specific Functional Forms

To proceed to an explicit solution, and in order to facilitate the empirical analysis, we put more structure on the problem.[12] This will allow us to assert, under certain conditions, that the wage-updating rule in any sector m is of the following simple form: given w_t^*, compute w_{t+1} under the hypothesis that the participation constraint at $t+1$ is not binding; if $w_{t+1} > \underline{w}^*_{s_{t+1}}$, then the hypothesis is confirmed and w_{t+1} *is* the equilibrium wage. Otherwise, the constraint *is* binding and the equilibrium wage will be at $\underline{w}^*_{s_{t+1}}$. The structure will also allow us to demonstrate sufficient conditions for the symmetric hiring equilibrium to exist.

We assume that each firm has technology given by

$$F^m(N, s_t) = a_t^{(m)} N^{1-\alpha}/(1-\alpha), \tag{8}$$

for $\alpha < 1$; for $\alpha = 1$, we specify $F^m(N, s_t) = a_t^{(m)} \log(N)$. The sector-specific shock $a_t^{(m)}$ depends on the aggregate shock s_t. We assume that productivity shocks are not too bad; that is,

$$a_{t+1}/a_t > \delta^\alpha, \tag{9}$$

with probability 1. For example, with a log production function, this requires only that productivity does not fall at a rate equal to turnover. Since the latter is typically estimated in the region of at least 30% on an annual basis, this is a mild restriction. This will ensure that, provided wages are sufficiently downwardly rigid, firms will always need to hire new workers. We also assume henceforth that workers have per-period utility functions of the constant relative risk-aversion family with coefficient $\gamma > 1$ described by $u(c) = c^{1-\gamma}/(1-\gamma)$.[13] Assume $\alpha\gamma > 1$.[14]

The marginal product of labour equals $a_t N_t^{-\alpha}$, so that using (7),

$$N_t = a_t^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}}. \tag{10}$$

Substituting $N_t = 1$, we find that the spot wage is $\underline{w}_t^* = a_t$.

[11] Another useful implication of this is that our assumption that the spot wage is always above the value of not working guarantees that equilibrium wages also always exceed this value. Thus, in our later characterisation, we shall not need to worry about running into the constraint that workers would rather not work than get or keep an employment contract.
[12] We also need the problem faced by the firm to be concave; concave production and utility functions are not sufficient to guarantee this.
[13] For $\gamma = 1$, set $u(c) = \log(c)$; all results go through.
[14] This is needed to make the optimisation problem concave.

Frontloading

First we deal with the case of certainty so that all sectoral productivity sequences are known at date 1 before contracts are entered into.[15] In this case, we show that the wage will always be kept as low as possible subject to its never falling below the spot wage. With downward real rigidity, this implies that the equilibrium satisfies $w(h_t) = \max_{t' \leq t} \underline{w}^*_{s_{t'}}$.

As a first step towards proving this, we demonstrate a frontloading result. Provided the downward constraint is not too weak and unless the participation constraint at the later date binds, wages will fall between any two dates by the maximum allowed by the downward constraint.

The intuition is as follows. If wages in the next period are not up against the downward constraint, then frontloading them by cutting the next period's wage a small amount and simultaneously increasing the current wage to compensate workers does not violate the downward constraint. If, in addition, the next period's participation constraint is not binding, then this too will be satisfied at all dates. This will increase profits, however. The reason is that there is turnover; a number of members of the current workforce will be separated before the next period. To compensate workers, the current wage does not have to be increased too much, as workers discount the future wage by the probability of separation. The firm, however, puts greater weight on wages in the next period because it will have to pay the same wage to new hires as to the surviving incumbents. Thus, the cut in future wages is valued more highly by the firm. The argument works as long as (a) wages are not falling too quickly, because if that were the case, risk-averse workers would need substantial additional compensation now for the steeper wage path, and (b) firms are hiring new workers. Thus the downward constraint must be sufficiently tight ($b(h_t)$ not too small), and also negative productivity shocks should not be so severe that firms do not want to hire new workers in some periods; we had already assumed this condition in order to solve the model.[16]

Lemma 3. *Suppose there is certainty. Then there exists a* $\underline{b} < 1$, *such that if* $b(h_{t+1}) > \underline{b}$ *for all t, the following must hold. If in a solution to problem (2) for sector m the downward rigidity constraint does not bind in sector m between t and* $t+1$, *then the participation constraint binds at* $t+1$.

We can now show that wages that rise by the minimum (given by the downward constraint), constitutes an equilibrium, unless this takes wages below the spot wage—in which case the wage is set to the spot wage. We use an asterisk to denote equilibrium values.

[15] See the Appendix for a discussion of the uncertainty case.
[16] See the Appendix for proofs.

Proposition 1. *Suppose there is no uncertainty and that* $b(h_t) > \underline{b}$ *for all t. Then there is a symmetric equilibrium in which in each sector m,* $w^{*m}_{t+1} = \max\{b(h_{t+1})w^{*m}_t, a^m_{t+1}\}$, *for* $t \geq 2$ *and* $w^{*m}_1 = a^m_1$.

Discussion

Thus, under the conditions of the proposition, the equilibrium wage contract can be computed in a simple recursive fashion, starting at the spot wage in period 1, and then proceeding by reducing wages if possible, due to the frontloading incentive, to the extent permitted by the downward constraint and the need to stay above the spot wage (i.e., to satisfy the participation constraint). If the equilibrium wage were to fall below the spot wage, then given that each firm is on its labour demand function (i.e., the wage is equal to the marginal product of labour) there would be excess demand for labour. Each firm would then have an incentive to slightly raise its wage above that of its competitors in order to attract as many workers as it wants, so this could not be an equilibrium. The equilibrium must occur when all wages are raised to exactly the spot wage. Then a firm would not want to pay less, since its workers would leave (as there is full employment). Nor would it want to pay more. Because of frontloading it will, in the putative equilibrium, be up against the downward constraint in future unemployment states. Hence, paying more now would force it to pay more in the future, raising wage costs for no benefit.

To illustrate the solution, we take the actual productivity series (TFP) from 1955 to 2001 for one of the manufacturing sectors that we use in the empirical exercise (see Section IV for details). Figure 1 displays productivity and simulated wages. The spot wage equals the productivity level (the thicker line), and Proposition 1 says that wages are always at least at this level, but otherwise fall at the rate given by the downward constraint. Whenever the wage lies above productivity, the labour market fails to clear. We show three wage simulations. For much of the time, they are coincident with the spot wage. The horizontal broken lines represent full downward rigidity ($b(h_t) = 1$). The shorter broken line gives a path predicted in Snell and Thomas (2010) and is discussed below. The thin continuous line, only visible before 1960, represents downward nominal rigidity (using the CPI to represent prices). The latter is mostly coincident with productivity because during the period 1955–2001, inflation in most years is greater in absolute value than negative sectoral productivity growth, so the nominal rigidity constraint has little bite. We return to this issue in Section IV.

It is the interplay of the two constraints that matters for unemployment outcomes. Downward rigidity in the absence of equal treatment, would, in this model, not lead to deviations from full employment. In fact, individual

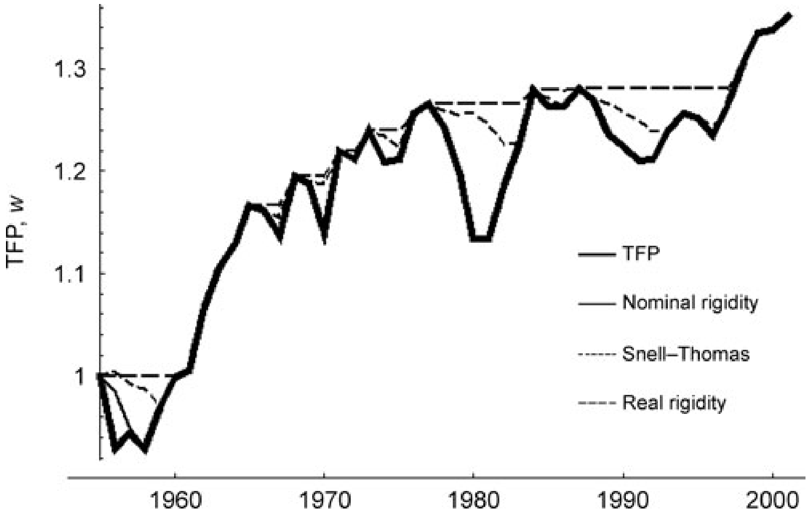

Fig. 1. TFP (thick line) and wage simulations

wages would be the same as those predicted by Beaudry and DiNardo (1991), where wages are downwardly rigid but rise to prevent workers from quitting when wage offers from other firms rise above what was previously offered to a worker.[17] Critically, this downward rigidity only applies to ongoing contracts and would not apply to the wage contracts offered to new hires, and consequently the labour market would always clear. Alternatively, equal treatment *per se* does not lead to deviations from Walrasian outcomes. If workers were risk neutral, for example, then wages that tracked the spot wages would be optimal—firms would be able to hire workers for the minimum possible discounted wages but would still satisfy the participation constraints.

When the two constraints coexist, however, any wage rigidity for incumbents is also transmitted to new hires. We saw in Lemmata 1 and 2 how equal treatment in this model can result in wages above spot wages, leading to unemployment and a non-binding participation constraint. Combined with the other implication of Lemma 2, that wages must be at least at the spot level, we can see that positive productivity shocks—which push spot wages and hence contract wages up—followed by negative ones will lead to wages that are too high to clear the market.

[17] Under the productivity process they assume, this happens when productivity is higher than previously attained during a worker's current tenure. This is the equilibrium outcome in the absence of both constraints, given risk-averse workers, so the downward constraint would play no role.

This answers, at least in this simple model, the question posed at the beginning of the paper: given that adverse shocks may be anticipated by forward-looking firms, what determines the actual level of wages before the adverse shock hits?

However, we conjecture that if labour were not perfectly mobile, the conclusion would change. In a high-productivity state, a firm would face a trade-off between the benefits of raising its wage to retain/attract workers and the future costs of carrying a higher wage forward, which may constrain it in future adverse states. One would therefore expect downward rigidity to constrain upward movements too.

IV. Simulations

In this section, we assess the extent to which the model is consistent with some relevant labour market stylised facts. In particular, we gauge whether the model can generate a plausible degree of unemployment volatility from measured total factor productivity shocks, using US postwar aggregate unemployment and productivity (TFP) data from the Bureau for Labor Statistics (BLS); we also gauge how well wage/unemployment regressions on simulated data correspond to existing stylised facts. In a single sector, unemployment falls to zero whenever the productivity shock is not too bad. In the multi-sector model, however, each sector will be subject to an idiosyncratic productivity shock, so we obtain more realistic unemployment levels. Moreover, when the aggregate productivity shock is positive, there will be more sectors with low unemployment and consequently aggregate employment is likely to be lower.

Given knowledge of the model's parameters, given an initial time period of full employment, and given a TFP series, it is possible to generate the sectoral "real wage" series that would be predicted by versions of our theory. We can only assert that this is an equilibrium if there is no uncertainty, so each sectoral sequence is perfectly anticipated. As noted above, however, we conjecture that this is also an equilibrium with uncertainty, provided shocks are not too negative. It is then possible to derive the corresponding implications for unemployment (rates), and also the relationship between real wages and unemployment.

In accordance with the theory developed earlier, we generate separate predicted wage and unemployment series for each sector, using actual US manufacturing industry multi-factor productivity processes for the 17 manufacturing sectors provided by the BLS for the period 1949–2001.[18] This fixes the variability of shocks and their correlation across

[18] This is the only sectoral TFP series available for such a long time scale and collected on a consistent basis.

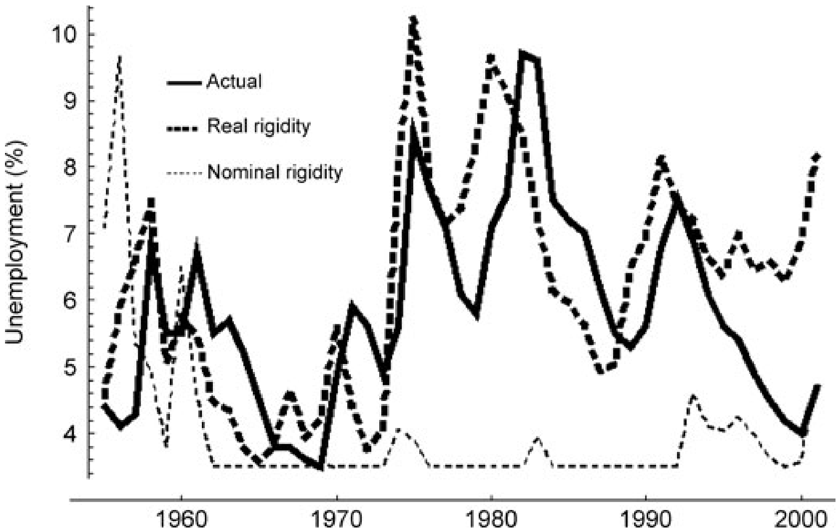

Fig. 2. Unemployment simulations (broken lines) and actual annual US rate (solid line), 1955–2001

sectors, and it also allows us to generate a simulated unemployment series that can be directly compared to the data. None of our theoretical results depended on having sectors that were the same size, so our results readily extend to this asymmetric case; indeed, allowing sector sizes to vary over time would also be a straightforward extension. We then aggregate the model's predicted unemployment for each of these sectors using mid-period unemployment shares as weights[19] (we start simulations at full employment and spot wages in 1949, allowing six years for unemployment to develop in each sector, so we use the period 1955–2001 for our results). Recall that this involves assuming that each sectoral labour market is segmented so that we can compute unemployment in each sector independently and then aggregate.

We do two things. First, we look at the predicted unemployment series and compare this with the actual US experience. Second, we examine the relationship between real wages and unemployment over the business cycle.

To simulate unemployment, we consider in Figure 2 the cases of full downward real rigidity (thick broken line) and downward nominal rigidity (thin broken line). Aside from the level of rigidity, there is only one free parameter, α, which is the curvature of the production function. For

[19] We add 3.5% to represent a constant frictional rate.

downward real rigidity, we use a logarithmic production function ($\alpha = 1$).[20] It should be noted that as manufacturing is only a fraction of the entire economy (and becoming smaller over time), it is inappropriate to compare the predicted unemployment rate for our model manufacturing economy with the general unemployment rate, as we do. Nevertheless, the value of α essentially determines only the extent to which a non-market-clearing wage translates into unemployment; it does not affect the equilibrium wage path but only magnifies the unemployment fluctuations as it falls in size. Thus if the remaining economy were composed of a residual sector with constant unemployment, the similar fluctuations in the economy-wide unemployment rate would result from choosing a lower value for α (which enhances fluctuations). For the downward nominal rigidity, we use a lower value of $\alpha = 0.2$ (the series is even flatter with $\alpha = 1$). Even so, there is far too little variation in unemployment, and as mentioned in Section III, the constraint only has bite at the start and towards the end of our period, when the inflation rate was low.

Given there is only one free parameter, the simulation with downward real rigidity tracks unemployment surprisingly well until towards the end of the simulation. The actual standard deviation of the unemployment rate was 1.5% over our period; the simulated series has a standard deviation of 1.7%. What seems to happen in the late 1990s is that some sectors experience a sufficiently long trend of poor productivity shocks that unemployment in those sectors builds up with completely downward rigid wages. It is clearly unrealistic to suppose that labour would not move out of these sectors in the long run, so even if downward rigidity were appropriate, we should expect to see lower unemployment towards the end of the period than the simulation suggests. Allowing wages to fall somewhat reduces simulated unemployment, particularly towards the end of the simulation. Keeping to the logarithmic production function, however, we find that unemployment variability falls. For example, if real wages can fall at 1% p.a. ($b(h_t) = 0.99$), we find the standard deviation falls to 1.0%; for a fall of 2%, we get a standard deviation of 0.7%.[21]

For our second exercise, we follow the studies of real wage cyclicality that have looked at how wages respond to contemporaneous unemployment movements. While there is a huge body of literature on this, a very rough

[20] Note that the value of the risk-aversion parameter γ does not affect the solution (it is relevant for the value of $\underline{b}$, however).

[21] Real wage falls in this region do not seem implausible. For example, Elsby (2009) charts the distribution of real wage changes in the PSID over 1983–1992, a relatively low inflation period (so surprise inflation is less likely to lead to unanticipated real wage falls); there is a spike around 2%–4% for real wage falls, and they rarely exceed about 6%. Likewise, Christophides and Stengos (2003) find from Canadian wage contract data in the unionised sector that most real wage reductions in the 1990s were of the order of 1%–2%.

summary would be that wages are roughly acyclical, or mildly procyclical, with panel studies consistently pointing towards the latter. For example, using the PSID for men over the period from 1968–1969 to 1986–1987, Solon *et al.* (1994) found that a reduction of one percentage point in the unemployment rate leads to a rise in the real wage rate of 1.4%. Similar estimates are found in Shin (1994) and Devereux (2001). From our simulations, we can regress real wage changes on changes in unemployment to replicate the typical regression undertaken in the panel studies.

Most studies that use longitudinal data of real wage cyclicality, following Bils (1985), estimate the following:

$$\Delta \ln w_{it} = \beta \Delta U_t + \lambda t + \alpha' X_{it} + \varepsilon_{it}, \tag{11}$$

where $\Delta \ln w_{it}$ is the difference between the natural logarithm of worker i's real wage rate in year t and his log real wage in year $t-1$, ΔU_t is the year-to-year change in the unemployment rate, and X_{it} is a vector containing an intercept and time-varying individual characteristics. The equation also includes a linear time trend (i.e., corresponding to a quadratic in t in levels). We ran the regression equation (11) on our 47 years of simulated data. Since all workers have identical productivity in our model, there are no distinguishing individual characteristics.

Under full downward real rigidity and a logarithmic production function, the estimate of β, the unemployment semi-elasticity of the wage, is -0.25 (with standard error 0.05). That it is negative is perhaps unsurprising—with full downward rigidity, for example, wages only change in a sector when there is full employment, in which case they rise. Aggregating across sectors, a fall in unemployment will tend to be associated with full employment in more sectors and consequently rising wages in more sectors. If we set $b = 0.99$, so the real wage can fall by 1%, we get a β estimate of -0.44 (0.08); for $b = 0.98$, the respective estimate is -0.72 (0.13).[22] While these are correctly signed, their magnitude is on the low side compared with the studies mentioned above. They do improve on the performance of the model in Snell and Thomas (2010), however. Running the same regression gives a coefficient of $+0.51$ (0.06), incorrectly signed.

V. Concluding Comments

This paper has analysed a model with downward rigidities in which firms cannot pay discriminately based on the year of entry to a firm. We solved for the dynamics of wages and unemployment under conditions of downward wage rigidity, where forward-looking firms take these constraints into

[22] If we simultaneously change α to maintain the variability of unemployment, however, the coefficient stays close to the -0.25 estimate.

account. We found that the equilibrium could be solved for under conditions of certainty. Using actual productivity data based on the postwar US economy, we analysed the ability of the model to match certain stylised labour market facts, and we found that the model was able to generate sufficient variability of unemployment and was also able to match to an extent the empirical wage–unemployment relationship.

In Snell and Thomas (2010), rigidity arises due to the desire to smooth wage movements over time. The same frontloading motive exists that was identified in the current model, so firms will want wages to fall over time if productivity is constant and if the participation constraint is slack. Wages that fall too fast, however, impose costs on the firm, because it has to compensate workers for this variability. Balancing these two forces provides a limit on how fast wages will fall. An empirical problem with appealing to risk aversion as the explanation of wage rigidity is that in order to get significant downward rigidity, we must assume that either workers are very risk averse (so wage variability is very costly), that the rate of turnover is implausibly small (so the frontloading incentive is small), or that workers are more patient than firms (which again weakens the frontloading incentive).[23] This suggests that the performance of the model may be improved if other sources of wage rigidity are present.

The current paper demonstrates that it is possible to incorporate wage rigidity directly into the model. Given that we assumed that workers are risk averse, the same forces limiting downward movements of wages are present in the current model, so if the downward rigidity constraint were sufficiently slack, the desire to smooth wages would be the binding constraint.[24]

In this analysis, we have taken the two constraints as given. This might lead one to think that there will be large gains to be made if firms and workers can find a way to avoid them. Here we argue that this is not so obvious. Importantly, workers who are employed benefit from the interplay of the two constraints. Equilibrium wages are never below spot wages and are sometimes above. If the firm could get around equal treatment, then it would benefit from being able to hire new workers in depressed states at lower wages, but this would not benefit incumbent workers. Indeed, to the extent that the firm is unable to commit not to replace incumbents by cheaper new hires, this has the potential for making incumbents worse off (Gottfries and Sjöström, 2000).

[23] Snell and Thomas (2010) argue that extending the model to allow for experience-dependent separation rates can help to resolve this problem.

[24] Some risk aversion is needed in order to make the profit function concave in the proof of Lemma 3. In Figure 1, the simulation with downward nominal rigidity lies below that of the Snell–Thomas model. This implies that under the parameters assumed in the latter, the nominal constraint would not bind.

On the other hand, if the downward rigidity constraint is relaxed, a firm would benefit from a policy that pays higher than spot wages in the beginning (and when the labour market clears) but—following the frontloading argument—lower wages than other firms when there is unemployment. This would allow it to satisfy participation constraints in good states but take advantage of cheap workers in bad states. Whether attempting to move to such a policy seems reasonable depends on why downward rigidity exists in the first place. It may be, for example, that efficiency wage considerations matter, so low wages during the bad states have negative implications for effort. We leave the attempt to marry theories of real wage rigidity to the equal treatment approach of this paper to future research.

Appendix A. Proofs

Proof of Lemma 3

We use time subscripts rather than history-dependent functions as there is no uncertainty, and we suppress sector m superscripts. Suppose to the contrary of the claim that $w_t > b_t w_{t-1}$ and the participation constraint does not bind at $t+1$. Starting from the optimal contract, consider reshuffling wages between t and $t+1$ as follows. Decrease the wage at $t+1$ by a small amount $\Delta w_{t+1} > 0$ so that worker utility falls by $\Delta > 0$, and increase the wage at t by Δw_t so that utility rises by x and so that the worker remains indifferent; do not change the contract otherwise. This implies that

$$-\delta\beta\Delta + x = 0, \tag{A1}$$

where $u'(w_{t+1})\Delta w_{t+1} \simeq \Delta$ and $u'(w_t)\Delta w_t \simeq x$. This frontloading satisfies all participation constraints. Worker utility falls at $t+1$, but the constraint was initially slack by hypothesis, and so from this point on constraints are satisfied. Similarly, participation constraints are also satisfied both at t and earlier because utility is held constant over the two periods. A sufficiently small change also satisfies the downward rigidity constraint because at $t+1$ it was slack, while at t, $w_t + \Delta w_t > w_t \geq b_t w_{t-1}$, and at $t+2$, $w_{t+1} - \Delta w_{t+1} < w_{t+1} \leq w_{t+2}/b_{t+2}$. We write $\Pi(u_t; a_t)$ as the static profit function at productivity level a_t (see below for the explicit function) when workers receive a current-period utility of u_t $(= u(w_t))$, and $N(u_t; a_t)$ for the corresponding optimal labour demand. The optimal contract must generate profits of $\Pi(u_t; a_t)$ at t (the choice of N_t does not affect the other constraints, so N_t must be chosen to maximise current profits at the contract wage). The change in profits (viewed from h_t) resulting from the frontloading is

$$\Delta P \simeq \beta\Pi'(u_{t+1}; a_{t+1})\Delta - \Pi'(u_t; a_t)x. \tag{A2}$$

Define $\varepsilon := \min_{2 \leq t \leq T}[\delta^{-\alpha} a_{t+1}/a_t - 1]$, where, by (9), $\varepsilon > 0$, so that

$$\delta^{-\alpha} a_{t+1}/a_t \geq 1 + \varepsilon. \tag{A3}$$

From Hotelling's lemma (converting wages to utilities), $\Pi'(u;a) = -N(u;a)/u'(w)$. Thus,

$$\frac{\Pi'(u_t;a_{t+1})}{\Pi'(u_t;a_t)} = \frac{N(u_t;a_{t+1})}{N(u_t;a_t)} = \frac{a_{t+1}^{\frac{1}{\alpha}}}{a_t^{\frac{1}{\alpha}}} \geq \delta(1+\varepsilon)^{1/\alpha}, \tag{A4}$$

where the second equality follows from optimal labour demand $N = a^{\frac{1}{\alpha}} w^{-\frac{1}{\alpha}}$ (given that u_t and hence wages are constant in the ratio), and the inequality follows from (A3).

Next, $\Pi(\cdot;a)$ is a concave function. Consider the static problem of maximising profits given that workers receive utility u, so that $w = ((1-\gamma)u)^{1/(1-\gamma)}$. Substituting from the condition that the marginal product of labour equals the wage,

$$N = a^{\frac{1}{\alpha}} w^{-\frac{1}{\alpha}}, \tag{A5}$$

yields profits of

$$\Pi(u;a_t) \equiv \frac{a_t^{\frac{1}{\alpha}} \alpha((1-\gamma)u)^{-\frac{1-\alpha}{\alpha(1-\gamma)}}}{1-\alpha}. \tag{A6}$$

As $\alpha\gamma > 1$, this is a strictly concave function of u.

Given that wages rise at a gross rate greater than b, then as $\gamma > 1$ (to satisfy $\alpha\gamma > 1$),

$$\frac{u_{t+1}}{u_t} = \left(\frac{w_{t+1}}{w_t}\right)^{1-\gamma} < b^{1-\gamma}. \tag{A7}$$

Then from (A6),

$$\frac{\Pi'(u_{t+1};a_{t+1})}{\Pi'(u_t;a_{t+1})} = \left(\frac{u_{t+1}}{u_t}\right)^{\left(\frac{\alpha\gamma-1}{\alpha(1-\gamma)}\right)} > b^{\left(\frac{\alpha\gamma-1}{\alpha}\right)}, \tag{A8}$$

where the inequality follows from (A7). Substituting (A1) into (A2) yields

$$\begin{aligned}
\Delta P &\simeq \beta\Pi'(u_{t+1};a_{t+1})\Delta - \Pi'(u_t;a_t)x \\
&= \beta\Delta\Pi'(u_t;a_t)\left(\frac{\Pi'(u_{t+1};a_{t+1})}{\Pi'(u_t;a_t)} - \delta\right) \\
&> \beta\Delta\Pi'(u_t;a_t)\left(\delta(1+\varepsilon)^{1/\alpha} b^{\left(\frac{\alpha\gamma-1}{\alpha}\right)} - \delta\right),
\end{aligned}$$

where the inequality follows from (A4) and (A8). Thus, provided $b \geq (1+\varepsilon)^{-\frac{1}{\alpha\gamma-1}}$, $\Delta P > 0$. As the initial contract was assumed optimal, this is a contradiction. Given $\alpha\gamma > 1$, $(1+\varepsilon)^{-\frac{1}{\alpha\gamma-1}} < 1$. Hence setting $\underline{b} = (1+\varepsilon)^{-\frac{1}{\alpha\gamma-1}}$, the assertion of the lemma follows. ■

Proof of Proposition 1

Suppose all other firms follow the putative equilibrium strategy and hire so that they are on their labour demand curves—that is, the marginal product of labour is equal to the wage (this defines $(\chi_t)_{t=1}^T$ from (5))—and consider the optimal strategy of a potential deviant firm in sector m. Again we drop sector superscripts and write b_t for $b(h_t)$.

(i) If at T, the final wage in the deviant strategy $w_T < w_T^*$, then if $w_T^* = a_T$, there is full employment and so to satisfy the participation constraint, a wage $w_T \geq w_T^*$ must be paid (see the remark following Lemma 2), so the participation constraint would be violated by the deviation strategy and it would be infeasible. On the other hand, if $w_T^* < a_T$, then at $t-1$, $w_{T-1}^* = w_T^*/b_T$ by definition of the equilibrium strategy, and $w_T < w_T^*$ plus downward rigidity implies $w_{T-1} \leq w_T/b_T < w_T^*/b_T$, so the deviation contract offers less discounted utility at $T-1$. Again, if the participation constraint binds at $T-1$ for the equilibrium contract, the participation constraint would be violated for the deviant, and if it does not bind we can extend the argument back to $T-2$, etc. As soon as the participation constraint binds for the equilibrium contract (it must bind at $t=1$ by $w_1^* = a_1$, as this implies full employment in the sector and hence by (5) a binding constraint), we will get a contradiction. (ii) If at T, $w_T > w_T^*$, then the participation constraint is slack for the deviant contract and so by Lemma 3 (recall we are looking at an optimal deviation contract, i.e., a solution to problem (2)), $w_{T-1} = w_T/b_T$ and as $w_T^* \geq b_T w_{T-1}^*$, $w_{T-1} > w_{T-1}^*$. If at $T-1$ the participation constraint binds for the deviant contract, it would be violated for the equilibrium contract, which is impossible. Thus, it cannot bind at $T-1$, and we can work backwards to the point where it last binds for the deviant (it must bind at least at $t=1$ as otherwise cutting w_1 would improve profits without violating any constraint), at which point again the deviant strategy offers higher discounted utility, which is a contradiction. We conclude that $w_T = w_T^*$. By similar arguments, we can show $w_{T-1} = w_{T-1}^*$ and then work backwards to establish equality of the two contracts. Thus, deviation is not profitable. ■

Appendix B. Uncertainty

The above arguments do not generalise to the case of uncertainty. Lemma 3 may fail as the frontloading of the wage contract between periods t and $t+1$ in a particular state may now affect future wages in other possible states at $t+1$—the wage at t is increased; if the downward constraint binds in some other state at $t+1$, this will imply the wage increases in that state, which may be costly. However, we can show that if $T=2$, this problem cannot arise. For simplicity, assume $b(h_2)=1$ for all h_2 (downward real rigidity).

Proposition 2. *If* $T=2$, *and* $b(h_2)=1$ *for all* h_2, *there is a symmetric equilibrium in which* $w_t^* = \max_{t' \leq t} \underline{w}_{t'}^*$.

Proof: To establish this, we convert the firm's choice variable (contract) from wages $(w_t(h_t))_{t=1}^2$ to utilities $(u_t(h_t))_{t=1}^2$. We can formulate problem (2) faced by the firm as

$$\max_{(u_t(h_t))_{t=1}^2} E\left[\sum_{t=1}^{2}(\beta)^{t-1}\Pi(u_t(h_t), a_t)\right] \qquad \text{(Problem A}^R\text{)}$$

subject to

$$\tilde{V}_t(h_t) \geq \chi(h_t) \qquad \text{(A9)}$$

for all positive probability h_t, $2 \geq t \geq 1$, and

$$u_t(h_{t-1}, s) \geq u(h_{t-1}), \tag{A10}$$

where $\tilde{V}_t(h_t)$ is defined recursively as before by

$$\tilde{V}_t(h_t) = u(w_t(h_t)) + \beta E[\delta V_{t+1}(h_{t+1}) + (1-\delta)\chi_{t+1} \mid h_t], \tag{A11}$$

with $\tilde{V}_3 = 0$. The maximand is strictly concave (see the proof of Lemma 3) and the constraints are linear. The Slater condition is satisfied by $u_t(h_t) = u(w^*(h_t) + \varepsilon)$, for all h_t and for $\varepsilon > 0$. Moreover, it is straightforward but tedious to show that the Kuhn–Tucker conditions are satisfied at the putative equilibrium contract, hence the putative solution solves problem (2). ■

For $T \geq 3$, we can construct counterexamples to the putative equilibrium, but only if there are shocks sufficiently bad that productivity falls in a range close to δ. We do not have an analytical result, however.[25]

References

Agell, J., and Lundborg, P. (1999), Survey Evidence on Wage Rigidity and Unemployment: Sweden in the 1990s, Department of Economics, Working paper, Uppsala University.

Akerlof, G. A., and Yellen, J. L. (1990), The Fair Wage, *Quarterly Journal of Economics 55*, 255–283.

Baker, G., Gibbs, M., and Holmström, B. (1994), The Wage Policy of a Firm, *Quarterly Journal of Economics 109*, 921–955.

Beaudry, P., and DiNardo, J. (1991), The Effect of Implicit Contracts on the Movement of Wages over the Business-cycle—Evidence from Micro Data, *Journal of Political Economy 99*, 665–688.

Bewley, T. F. (1999a), *Why Wages Don't Fall During a Recession*, Harvard University Press, Cambridge, MA.

Bewley, T. F. (1999b), Work Motivation, *Federal Reserve Bank of St. Louis Review 81* (3), 35–50.

Bils, M. J. (1985), Real Wages over the Business Cycle: Evidence from Panel Data, *Journal of Political Economy 93*, 666–689.

Christophides, L. N., and Stengos, T. (2003), Wage Rigidity in Canadian Collective Bargaining Agreements, *Industrial and Labor Relations Review 56*, 429–448.

Devereux, P. J. (2001), The Cyclicality of Real Wages within Employer–Employee Matches, *Industrial Labor Relations Review 54*, 835–850.

Elsby, M. W. L. (2009), Evaluating the Economic Significance of Downward Nominal Wage Rigidity, *Journal of Monetary Economics 56*, 154–169.

Elsby, M. W., Michaels, R., and Solon, G. (2009), The Ins and Outs of Cyclical Unemployment, *American Economic Journal: Macroeconomics 1*, 84–110.

Gertler, M., and Trigari, A. (2009), Unemployment Fluctuations with Staggered Nash Wage Bargaining, *Journal of Political Economy 117*, 38–86.

Gottfries, N., and Sjöström, T. (2000), Insider Bargaining Power, Starting Wages, and Involuntary Unemployment, *Scandinavian Journal of Economics 102*, 669–688.

[25] We simulated models with i.i.d. two-state multiplicative productivity processes up to $T = 13$. For example, for $\delta = 0.8$, we confirmed the putative equilibrium for a wide range of other parameter values, provided a_{t+1}/a_t was at least approximately 0.82 with probability 1.

Haefke, C., Sonntag, M., and van Rens, T. (2007), Wage Rigidity and Job Creation, Universitat Pompeu Fabra, Economics Working Paper no. 1047.

Hall, R. E. (2005), Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years, in M. Gertler and K. Rogoff (eds.), *NBER Macroeconomics Annual*, MIT Press, Cambridge, MA, 101–137.

Kaufman, R. T. (1984), On Wage Stickiness in Britain's Competitive Sector, *British Journal of Industrial Relations 22*, 101–112.

Kwon, I., and Milgrom, E. M. M. (2005), Cohort Effects and Catch-ups in Wages and Promotions, manuscript, University of Michigan.

Menzio, G., and Moen, E. (2008), Worker Replacement, PIER Working Paper no. 08-040.

Pissarides, C. (1986), Unemployment and Vacancies in Britain, *Economic Policy 1*, 499–599.

Pissarides, C. A. (2007), The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?, CEP Discussion Paper no. dp0839.

Shimer, R. (2005), Reassessing the Ins and Outs of Unemployment, manuscript, University of Chicago.

Shin, D. (1994), Cyclicality of Real Wages among Young Men, *Economics Letters 46*, 137–142.

Snell, A., and Thomas, J. P. (2010), Labor Contracts, Equal Treatment and Wage–Unemployment Dynamics, *American Economic Journal: Macroeconomics 2,* 98–127.

Solon, G., Barsky, R., and Parker, J. A. (1994), Measuring the Cyclicality of Real Wages: How Important is Composition Bias?, *Quarterly Journal of Economics 109*, 1–26.

Wachter, T. von, and Bender, S. (2008), Do Initial Conditions Persist between Firms? An Analysis of Firm-entry Cohort Effects and Job Losers using Matched Employer–Employee Data, in S. Bender, J. Lane, K. Shaw, F. Andersson, and T. von Wachter (eds.), *The Analysis of Firms and Employees: Quantitative and Qualitative Approaches*, University of Chicago Press, Chicago, 135–162.