Leo Hurwicz Lecture

Contagious Illiquidity

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A rudimentary Akerlof example with two types

one seller, many buyers

μ^s = seller's marginal utility of income

μ^b = buyers' marginal utility of income

where $\mu^s > \mu^b$

2 possible types of good: utility H or L

only seller knows type

buyers know good is type H with probability λ

High-price equilibrium

both types of good are traded

price p satisfies

$$\mu^b p = \lambda H + (1-\lambda)L$$

seller with H-type good must want to trade:

$$\mu^s p > H$$

that is, the high-price equilibrium exists iff

$$\frac{H}{\lambda H + (1-\lambda)L} < \frac{\mu^s}{\mu^b}$$

Low-price equilibrium

only type L good is traded

price p satisfies

$$\mu^b p = L$$

seller with H-type good mustn't want to trade:

$$\mu^s p < H$$

that is, the low-price equilibrium exists iff

$$\frac{\mu^s}{\mu^b} < \frac{H}{L}$$

Hence it is possible that there are 2 equilibria

– when

$$\frac{H}{\lambda H + (1-\lambda)L} < \frac{\mu^s}{\mu^b} < \frac{H}{L}$$

↑ condition for high-price equilibrium

↑ condition for low-price equilibrium

When L = 0, the RH inequality is always true: there is always a low-price (zero!) equilibrium, without any trade

Start of a theory of illiquidity/market failure?

If both the inequalities hold, there are two "Walrasian" equilibria (prices are parametric), but only one is "Nash" (agents set prices):

> a buyer could deviate from low price to offer $\varepsilon > 0$ below high price

i.e. if the high-price equilibrium exists (the LH inequality holds) then it is the <u>only</u> Nash inspecting the LH inequality:

$$\frac{H}{\lambda H + (1-\lambda)L} < \frac{\mu^s}{\mu^b}$$

we see that the high-price equilibrium is more likely to exist (market failure is less likely) if there is

- lower variance between H and L
- greater difference between the seller's and the buyers' marginal utilities of income

An Akerlof example with dynamics and assets

discrete time   (“days”)

all agents discount with factor $\beta < 1$

agents alternate their marginal utilities:

today’s seller:	μ^s	μ^b	μ^s	…
	↑	↑	↑	
	today	tomorrow	next day	…
	↓	↓	↓	
today’s buyer:	μ^b	μ^s	μ^b	…

where $\beta\mu^s < \mu^b < \mu^s$

2 types of asset (fraction λ of type H):

type H: pays dividend of 1 dollar, at the start of every day

type L: pays zero dividends

current owner privately receives dividend (1 or 0) thus privately learns the type of asset he holds

to simplify, assume that the trading history of each asset is collectively forgotten

As a preliminary exercise, look at one-shot case:

suppose today is the only opportunity to trade

price p is still dictated by demand competition

given that the L-type asset pays zero dividend, there always exists a zero-price Walrasian equilibrium, in which nothing is traded – i.e. today's asset market breaks down

but this will not be Nash if there exists a high-price equilibrium

One-shot case: high-price equilibrium
(both types of asset are traded)

p satisfies:

$$\mu^b p = \beta\mu^s\lambda + \beta^2\mu^b\lambda + \beta^3\mu^s\lambda + \beta^4\mu^b\lambda + \ldots$$

seller with the H-type asset must want to trade:

$$\mu^s p > \beta\mu^b + \beta^2\mu^s + \beta^3\mu^b + \beta^4\mu^s + \ldots$$

that is, the high-price equilibrium exists iff

$$\left(\frac{\mu^s}{\mu^b} + \beta\right)\lambda > \frac{\mu^b}{\mu^s} + \beta \quad (*)$$

Now consider true dynamic model:

there is a trading opportunity in <u>every</u> period

We will see that the existence condition for a (stationary) equilibrium in which prices are high is now <u>weaker</u>

Dynamic case: high-price equilibrium

the (stationary) price p is dictated by a typical buyer's indifference condition:

$$\mu^b p \quad = \quad \beta\mu^s (\lambda + p)$$

↑ he sells tomorrow – no matter which type of asset he learns that he has bought today

seller with the H-type asset must want to trade:

$$\mu^s p \quad > \quad \beta\mu^b \quad + \quad \beta^2\mu^s (1 + p)$$

$$> \quad \beta\mu^b + \beta^2\mu^s + \beta^3\mu^b + \beta^4\mu^s + \ldots$$

that is, in the dynamic case the high-price equilibrium exists iff

$$\frac{(1-\beta^2)\lambda}{\left(\frac{\mu^b}{\mu^s}-\beta\right)} > \frac{\mu^b}{\mu^s} + \beta \qquad (**)$$

this is a <u>weaker</u> existence condition than for the one-shot case:

$$\left(\frac{\mu^s}{\mu^b} + \beta\right)\lambda > \frac{\mu^b}{\mu^s} + \beta \qquad (*)$$

interesting region: (**) holds, but (*) doesn't in this interesting region, we have effectively shown that there are two (stationary) <u>Nash</u> equilibria of the dynamic model:

> one equilibrium in which both types of asset are traded and prices are positive
>
> > – because (**) holds
>
> another equilibrium with market failure (zero prices and no trading)
>
> > – because (*) doesn’t hold

the one-shot model is akin to the dynamic model with market failure from tomorrow onwards

Intuition:

If tomorrow's market is <u>not</u> expected to fail, then today's buyer of an unknown asset will sell the asset tomorrow, whether or not it turns out that he bought a lemon (a type L asset).

Thus, the only downside to buying a lemon today is that he suffers a one-time dividend loss.

In other words, the variance in future utility between the good asset and a lemon is low.

But low variance $\Rightarrow$ market doesn't fail today either

Conversely, if markets in the future are expected to fail, then today's buyer of an unknown asset will be stuck with it for a long time.

Thus, the variance in future utility between the good asset and a lemon is high.

But high variance $\Rightarrow$ market fails today too

Important! This is a Nash equilibrium. Today, no single buyer can profitably deviate by offering a higher price that attracts both types seller

– because (*) doesn't hold

To build a coherent macro model …

Challenge #1: which equilibrium to select?

We'll make use of idea that the high-price equilibrium disappears if

$$\frac{\mu^s}{\mu^b} \quad \text{is too close to 1}$$

– i.e. if the difference between sellers and buyers is not great enough

Challenge #2: endogenize μ^s and μ^b

bare bones of model:

discrete time; single good

agents' utility of consumption path $\{ c_t \}$:

$$\sum_t \beta^t \log c_t$$

agents (firms, banks) invest only periodically

e.g. agents take turns to invest; or
investment opportunities are stochastic

investing agents ("investors") raise funds by borrowing from non-investing agents ("savers")

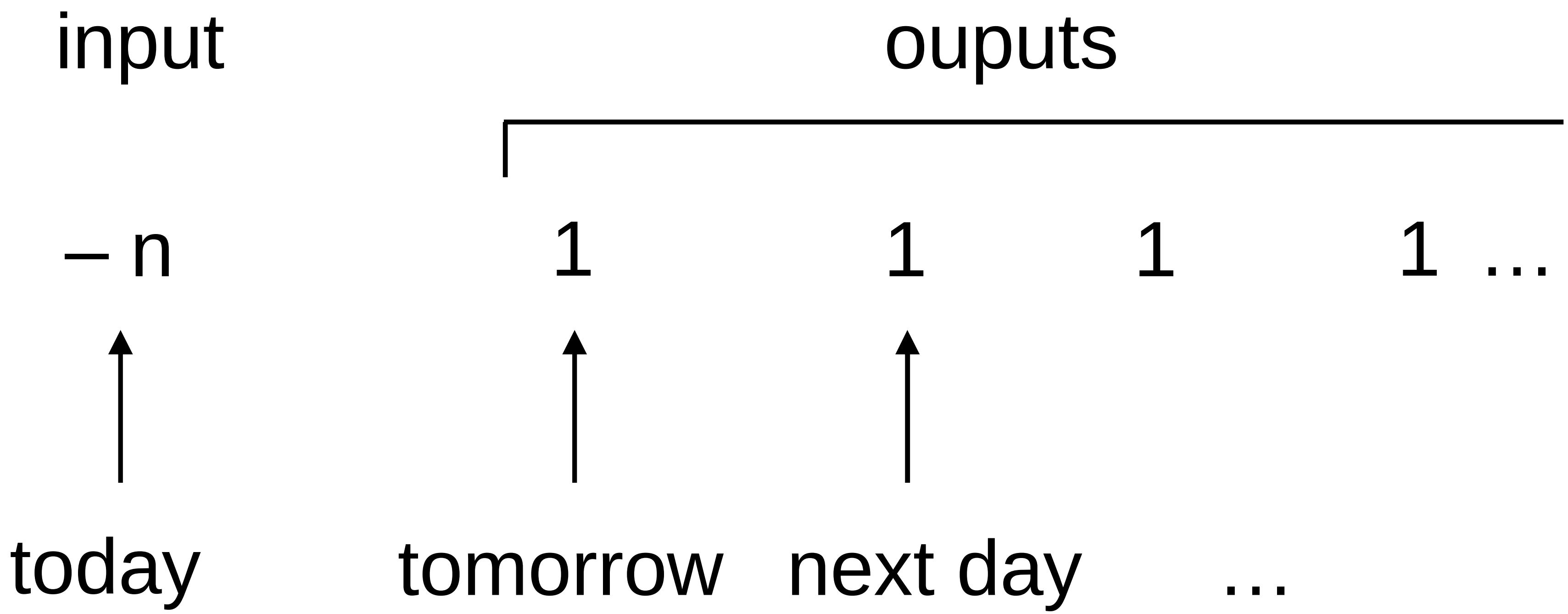
credit markets are imperfect ($\because$ moral hazard)

$\Rightarrow$ investing agents face borrowing constraints

$\Rightarrow$ agents restrict consumption when investing

$$\Rightarrow \quad \frac{\text{marginal utility of income} \mid \text{investing}}{\text{marginal utility of income} \mid \text{saving}} > 1$$

that is, $\mu^s > \mu^b$ (now endogenized) investment project (constant returns to scale):



but each night, from tomorrow night on, there is a small probability that agent (and project) dies

aside from today’s investing agents, death is i.i.d. (population replenished by birth + endowment) nature of borrowing:

assume that a bond is a “stochastic console”:

promise to repay 1 every day from tomorrow on

– unless the issuing agent has died,
  because his projects have died too

– unless the current holder of the paper (the bondholder) has died
  cf. an annuity: no bequest motive

death is a public event,

but a fraction of people privately learn today that someone is going to die tonight

$\Rightarrow$ potential for adverse selection in bond market
(a current bondholder will resell today if he learns that the issuer is going to die tonight)

Are such bonds true Akerlof lemons?

Not really, because all the bonds issued by any given agent are identical, whereas Akerlof's used cars are idiosyncratic

Challenge #3: modelling adverse selection in a ‘homogeneous’ bond market

suppose bonds are retraded in larger markets

in any particular market, bonds initially issued by different agents, all from an identified group, are retraded at a common price: buyers cannot identify the amounts supplied into the market of the various bonds (various issuers and vintages)

$\Rightarrow$ buyers are indifferent about whose bonds they purchase; they rationally assess the average quality in this particular market

Each bond market is identified by the group of agents who initially issued the bonds which are retraded on that market.

It may be

either liquid: trade occurs, at a price $r > 0$

or illiquid: zero price, and no trading

Newly-issued debt is always sold at a price $q \geq r$

– given that investing agents won't die tonight

q = new-issue price of own bond

r = resale price of own bond

q^* = new-issue of others' bonds

r^* = resale price of others' bonds

n = cost of new investment project, per unit

ℓ = liquidation value of project, per unit

= q for investing agent $\because$ he won't die tonight

Consider an agent who today has an investment opportunity and who, as of last night, had:

y units of projects in progress
(“old projects”)

b units of his own debt outstanding

a units of others’ debt accumulated

he has to choose: $\Delta y \geq 0$ (investment)

$\Delta a, \Delta b \geq 0$ (asset sales)

flow-of-funds constraint

$$n\Delta y \quad + \quad b \quad + \quad c$$

investment in new projects — payment due on outstanding debt — consumption

$$\leq \quad y \quad + \quad a \quad + \quad r^*\Delta a \quad + \quad q\Delta b$$

return on old projects — receipts due from others' debt — sales of others' debt — issue of new debt borrowing constraint

just after investment, the face value of liabilities mustn't exceed the market value of assets:

ℓ equals q ↓

$$\frac{1}{1-\beta}(b + \Delta b) \leq \ell\,(y + \Delta y) + r^*(a - \Delta a)$$

face value of own debt (discounted by β)

liquidation value of projects

market value of others' debt

In choosing scale of new (levered) investment, the agent faces a crucial decision:

What to do with his holdings, a, of others' debt?

Choice:

> retain a, but use these outside debt holdings as collateral for additional borrowing, to fund more inside investment

or

> dis-invest a, selling at price r*, and use the funds to finance maximal inside investment

flow-of-funds constraint:

$$n\Delta y \quad + \quad b \quad + \quad c$$

$$\leq \quad y \quad + \quad a \quad + \quad r^*\Delta a \quad + \quad q\Delta b$$

borrowing constraint:

$$\frac{1}{1-\beta}(b + \Delta b) \quad \leq \quad \ell\,(y + \Delta y) \quad + \quad r^*(a - \Delta a)$$

↑

ℓ equals q

every additional unit of outside debt sold ($\Delta a \uparrow 1$)

<u>loses</u> 1 console of an outside asset

but

<u>gains</u> $$\frac{[r^* - (1-\beta)qr^*]}{[n - (1-\beta)q\ell]}$$ consoles of an inside asset

levered price of outside (dis-)investment

levered price of inside investment

ℓ equals q

Crux of the argument about contagious illiquidity:

suppose, for some reason, this agent's own bond market turns illiquid

– the anticipated future resale prices { r } of his debt drop to zero –

then this <u>lowers</u> today's new-issue price, q, of his debt – given that buyers' future marginal utilities vary ($\mu^s > \mu^b$);

but can show: $q \downarrow \;\Rightarrow\; \dfrac{[r^* - (1-\beta)qr^*]}{[n - (1-\beta)q^2]} \downarrow$

continuing with this logic:

suppose this agent's bond market turns illiquid

$\Rightarrow$ his gains from selling outside assets $\downarrow$

(also: future collateral value of inside asset $\downarrow$)

$\Rightarrow$ his sales of others' debt $\downarrow$

remember this agent is trading for liquidity motives, not because of private information

$\Rightarrow$ others' bond markets turn illiquid too

Conclusion: illiquidity is contagious

Moreover, recall our earlier finding:

> dynamic asset markets may have two stationary equilibria
>
> > – one with trade and positive prices; another with zero prices and no trade

thus a <u>temporary</u> shock – large enough to kick the economy into system-wide illiquidity – may leave the economy in that state, even after the shock has disappeared

End of first half of my lecture …

second half of my lecture:

Contagious illiquidity leads to <u>financial fragility</u>:

bond markets turn illiquid

$\Rightarrow$ investing agents issue debt

backed by their holdings of others' debt

i.e. rather than reselling the financial assets

they bought when they were savers,

they borrow against those assets

$\Rightarrow$ agents hold gross financial positions

i.e. they have financial contracts on both sides of their balance sheets

$\Rightarrow$ in the economy as a whole,
debt builds up, backed by other debt

i.e. there are chains of credit

$\Rightarrow$ chains of default?

The danger time is the day <u>after</u> investment:

the agent is owed $(a - \Delta a)$, but owes $(b + \Delta b)$.

In the absence of any default by his debtors, he has more than enough ‘cash’ with which to pay:

$$(b + \Delta b) < (y + \Delta y) + (a - \Delta a).$$

(True! – because of earlier borrowing constraint

$$\frac{1}{1-\beta}(b + \Delta b) \leq \ell\,(y + \Delta y) + r^*(a - \Delta a)$$

and $\ell, r^* < 1/(1-\beta)$)

Crisis arises only if shortfall in receipts due from others' debt is enough to reverse the critical post-investment cash-flow inequality; i.e. if

$$(b + \Delta b) \quad > \quad (y + \Delta y) \quad + \quad (a - \Delta a) \quad - \quad \text{shortfall}$$

$\Rightarrow$ this agent defaults

$\Rightarrow$ other agents default too

$\Rightarrow$ real losses: from reduced new investment,
plus from delays and inefficient
project reallocation in bankruptcy

Proximate cause of these losses: a large enough (temporary) shock to one or more agents' cash flow causes a snowballing of default

Prior cause: in times of liquidity need, agents have chosen to build up gross financial positions, rather than reselling paper assets

Deep cause: illiquidity in one bond market led to illiquidity in the other bond markets

END