

# Liquidity, Business Cycles, and Monetary Policy

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Saturday 7 November 2009

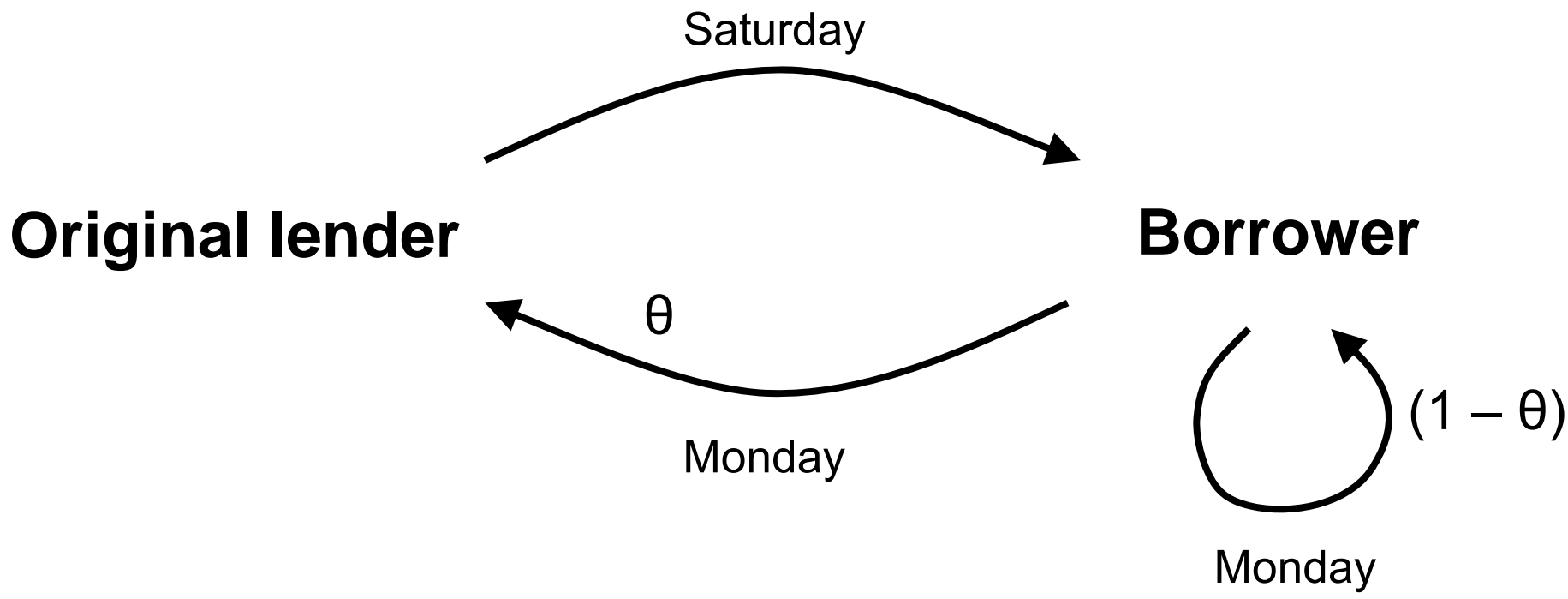
Aims:

Develop model of a “monetary economy”

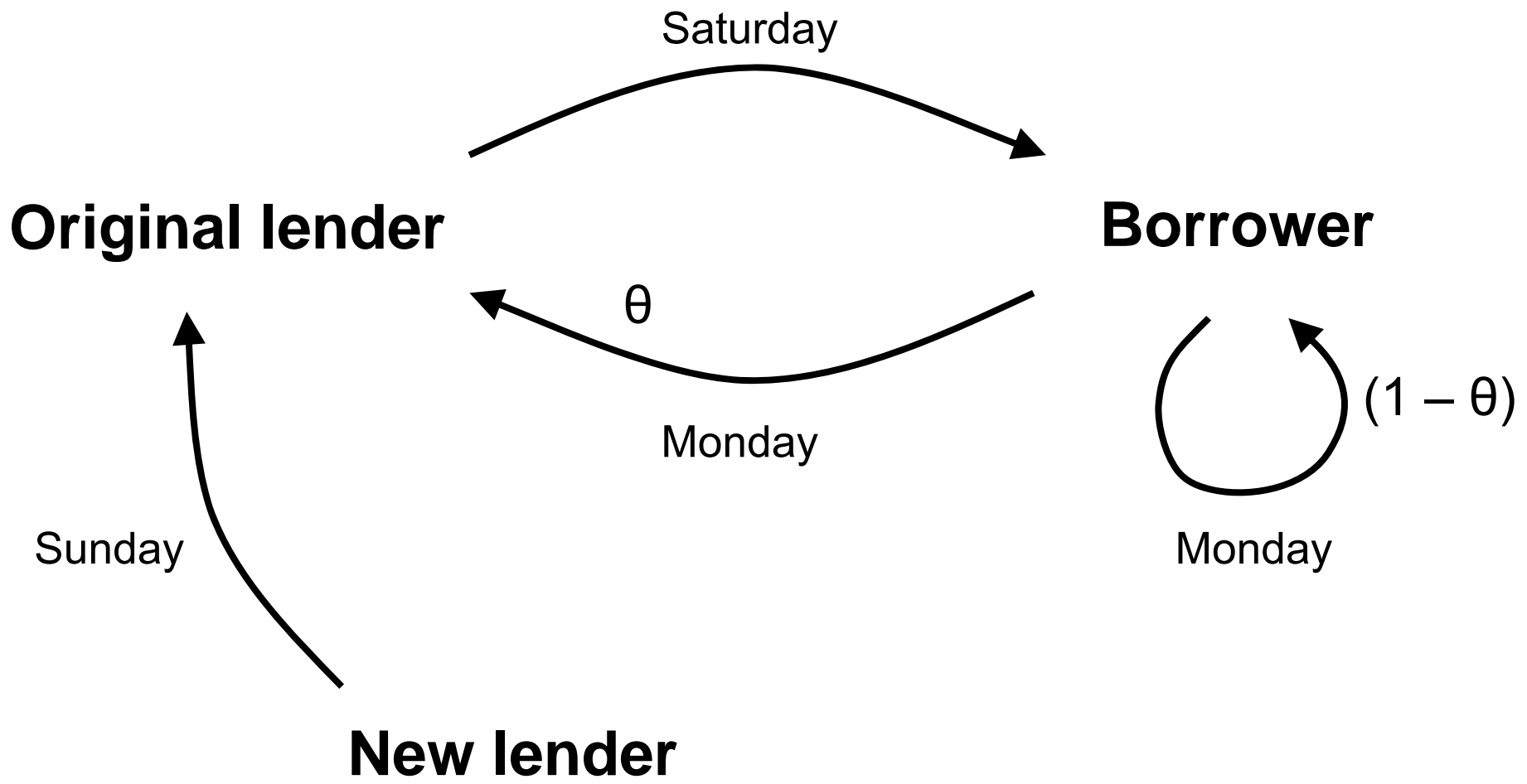
– i.e. where money is essential for the  
smooth running of the economy

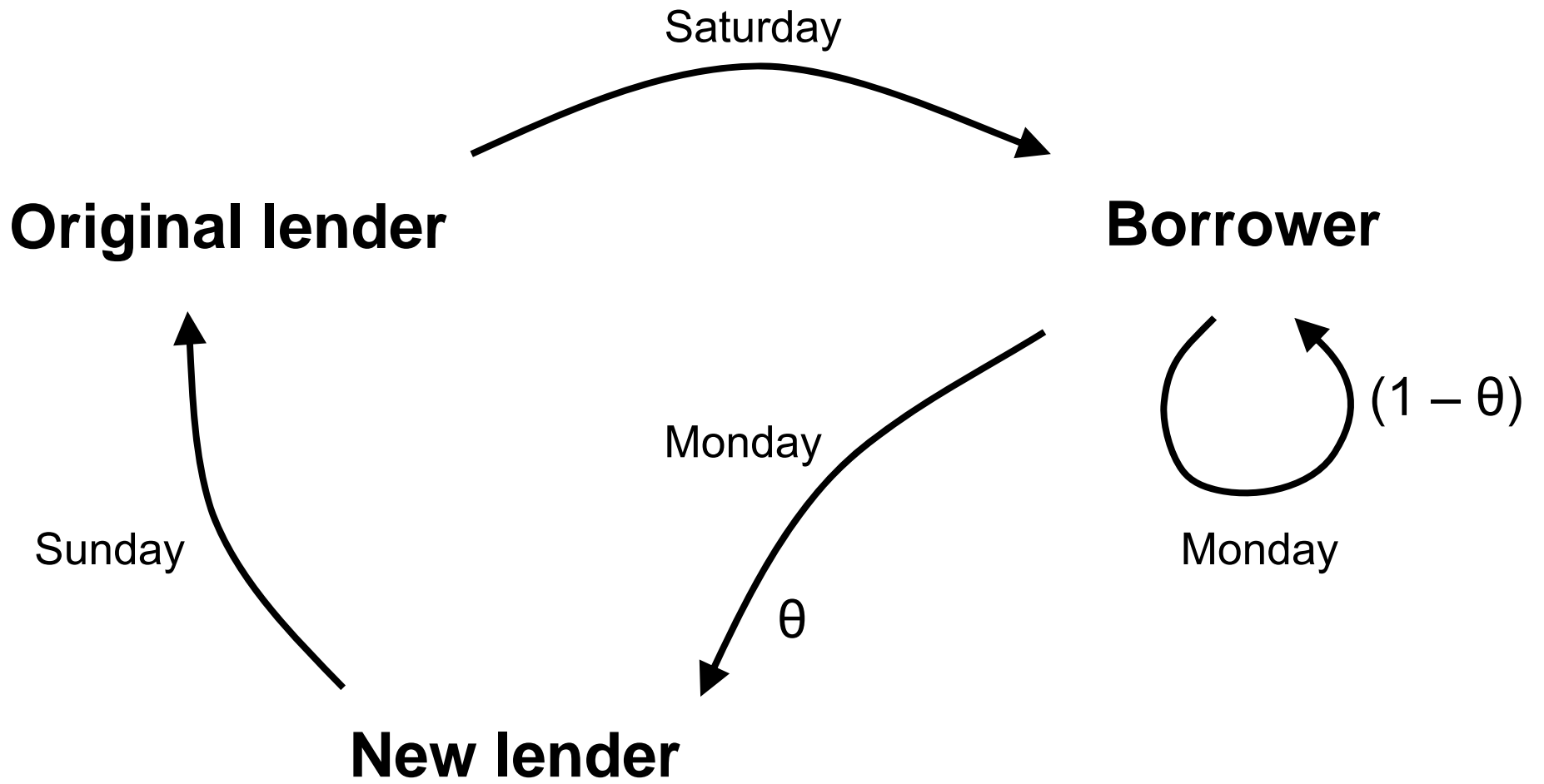
Subject economy to productivity and liquidity  
shocks

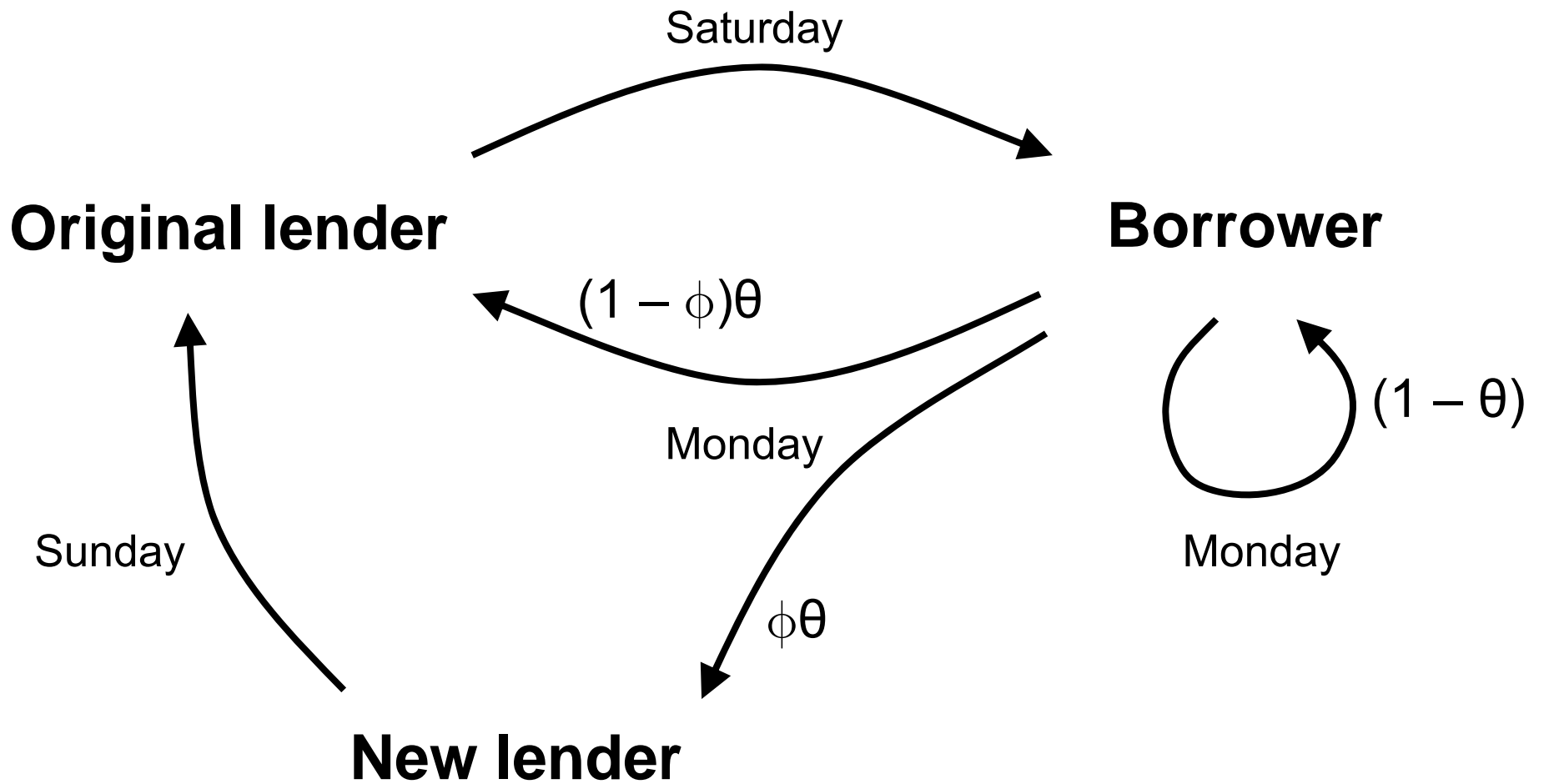
Examine role of (unconventional) monetary  
policy



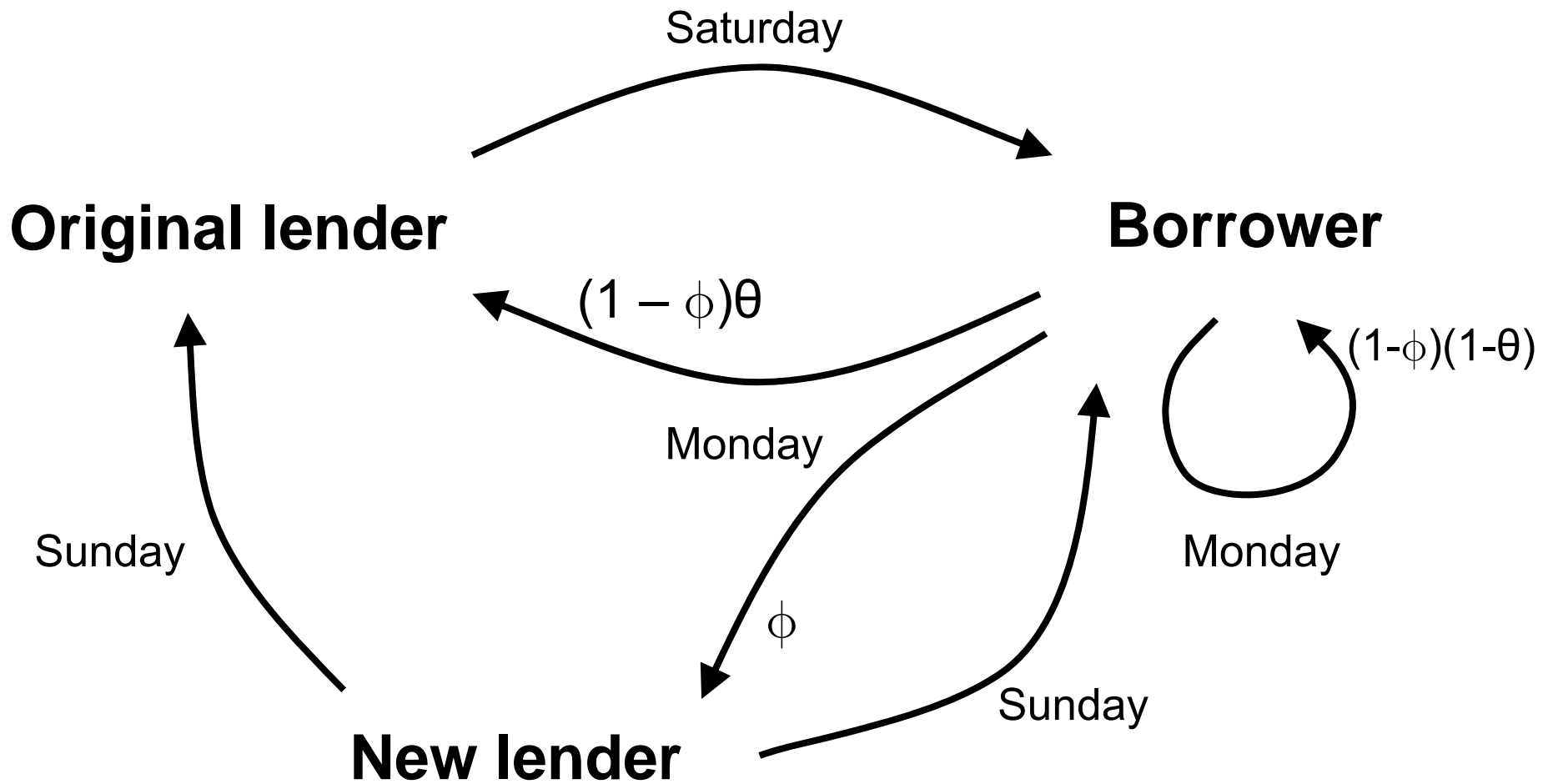
$\theta < 1$  (borrowing constraint)







$\phi < 1$  (resaleability constraint)



$\phi$  = fraction of asset that can be resold at each date

(cf “peeling an onion”)

# Model

discrete time:  $t = 0, 1, 2, \dots$

at each date  $t$ : homogeneous output,  $Y_t$

capital,  $K_t$

fiat money,  $M_t$

agents, measure 1, utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \log c_s \quad \beta < 1$$

of consumption path  $\{c_t, c_{t+1}, c_{t+2}, \dots\}$



all agents use their capital to produce goods:

start of date  $t$

end of date  $t$

$$k_t \text{ capital} \longrightarrow \left\{ \begin{array}{l} r_t k_t \text{ goods} \\ \lambda k_t \text{ capital} \end{array} \right. \quad \lambda < 1$$

individually constant returns, but  
decreasing returns in aggregate:

$$r_t = a_t K_t^{\alpha-1} \quad \alpha < 1$$

$$Y_t = r_t K_t = a_t K_t^\alpha$$

only a fraction  $\pi < 1$  of agents can use goods to produce new capital:

start of date  $t$

end of date  $t$

$i_t$  goods  $\longrightarrow$   $i_t$  new capital

(new capital produces goods only from  $t+1$  on)

these investment opportunities are i.i.d.,  
across agents, through time

agents cannot insure against the arrival of an investment opportunity

capital is specific to the agent who produces it,  
but he can mortgage future returns by issuing  
equity:

one unit of equity issued at date  $t$  promises

$$r_{t+1}, \lambda r_{t+2}, \lambda^2 r_{t+3}, \dots$$

Borrowing constraint:

an investing agent can mortgage at most  $\theta$  of  
future returns from his new capital production

think of unmortgaged capital as “inside equity”

Resaleability constraint:

at each date  $t$ , an agent can resell at most  $\phi_t$  of his (inside or outside) equity holdings

borrowing and resaleability constraints

$\equiv$  “liquidity constraints”

NB there is a subscript  $t$  on both

$a_t$  (productivity shock) and  $\phi_t$  (liquidity shock)

## balance sheet at end of date t

assets	liabilities
capital stock	own equity issued
others' equity purchased	net worth
money $m_{t+1}$	

$$\begin{aligned} \text{let } n_{t+1} &= \text{capital stock} - \text{own equity issued} \\ &\quad + \text{others' equity purchased} \\ &= \text{inside equity} + \text{outside equity} \end{aligned}$$

let  $q_t$  = price of equity

$p_t$  = price of money (upside down!)

flow-of-funds constraint:

$$c_t + i_t + q_t(n_{t+1} - i_t) + p_t m_{t+1} = (r_t + \lambda q_t)n_t + p_t m_t$$

liquidity constraint:

$$n_{t+1} \geq (1 - \theta)i_t + (1 - \phi_t)\lambda n_t$$

$$m_{t+1} \geq 0$$

government

at date  $t$ , government chooses its :

$G_t$  within-period net spending/transfers

$N_{t+1}^g$  end-of-period holding of private equity

$M_{t+1}$  end-of-period money supply

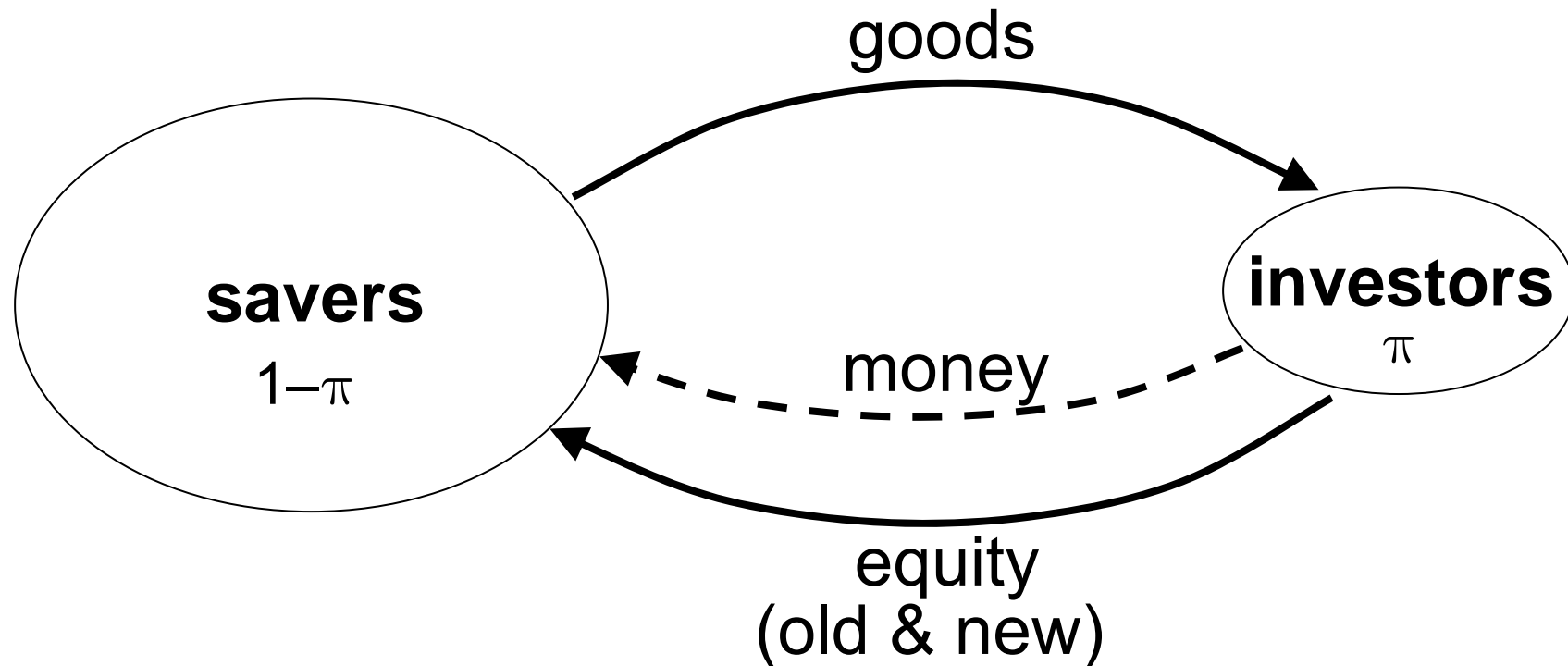
subject to flow-of-funds constraint

$$G_t + q_t(N_{t+1}^g - \lambda N_t^g) = r_t N_t^g + p_t(M_{t+1} - M_t)$$

and resaleability constraint

$$N_{t+1}^g \geq (1 - \phi_t)\lambda N_t^g$$

private economy tries to funnel funds from agents who do not have an investment opportunity (“savers”) into the hands of agents who do (“investors”):



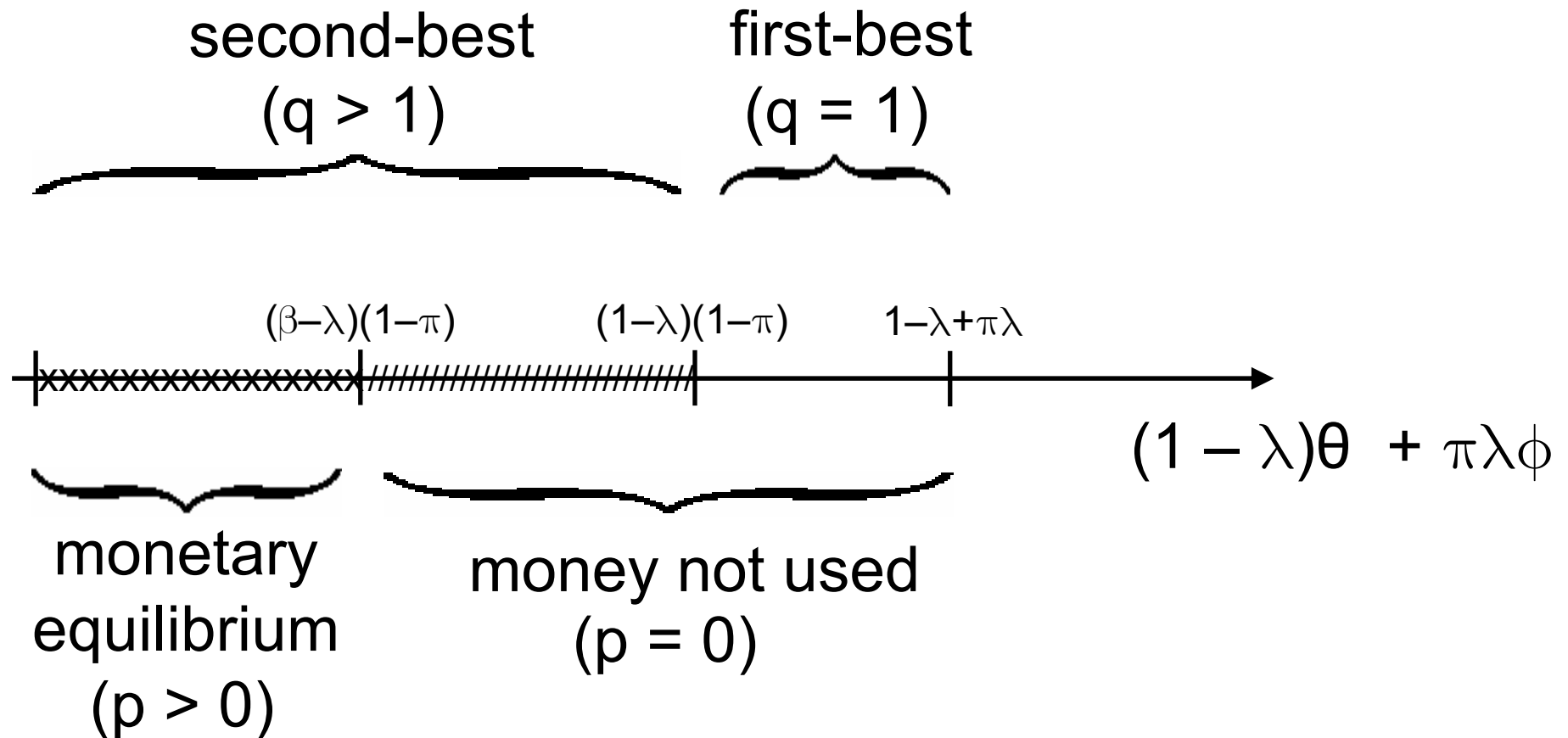
think of this as a “liquidity-in-advance” model



for the moment, ignore government


(set  $G_t \equiv N_t^g \equiv 0$  and  $M_t \equiv M$ )

Proposition (steady-state:  $a_t \equiv a$ ,  $\phi_t \equiv \phi$ )



$q_t$  must lie below  $1/\theta$   $\therefore$  for an INVESTOR (he):


$$\text{investment } i_t = \frac{\text{available funds} - \text{consumption}}{(1 - \theta q_t)}$$


 required downpayment  
per unit of investment

$$\text{available funds} = [r_t + q_t \phi_t \lambda] n_t + p_t m_t$$

$$\text{consumption } c_t = (1 - \beta) \times \text{his net worth}$$

$$[r_t + q_t \phi_t \lambda + s_t (1 - \phi_t) \lambda] n_t + p_t m_t$$


  
 his shadow price  
of illiquid equity =  $\frac{1 - \theta q_t}{1 - \theta} < 1$

SAVER (she): consumption  $c_t = (1 - \beta)$ net worth

$$\text{her net worth} = [r_t + q_t \phi_t \lambda + q_t (1 - \phi_t) \lambda] n_t + p_t m_t$$

$\uparrow$   
 her shadow price of illiquid equity = market price of equity

her portfolio choice:

$$\begin{aligned}
 & (1 - \pi) E_t \left\{ \frac{r_{t+1} + q_{t+1} \phi_{t+1} \lambda + q_{t+1} (1 - \phi_{t+1}) \lambda}{q_t} - \frac{p_{t+1}}{p_t} \right\} \\
 & \qquad \qquad \qquad \frac{c_{t+1} \mid \text{she is still a saver at } t+1}{\qquad \qquad \qquad} \\
 = & \pi E_t \left\{ \frac{p_{t+1}}{p_t} - \frac{r_{t+1} + q_{t+1} \phi_{t+1} \lambda + s_{t+1} (1 - \phi_{t+1}) \lambda}{q_t} \right\} \\
 & \qquad \qquad \qquad \frac{c_{t+1} \mid \text{she becomes an investor at } t+1}{\qquad \qquad \qquad}
 \end{aligned}$$

these date  $t$  behavioural equations are all linear  
in  $n_t$  and  $m_t$

⇒ aggregation is easy; we do not need to keep  
track of the evolution of the distribution of  
agents' individual asset holdings

⇒ relevant state variables:  $K_t$  (endogenous)  
 $a_t, \phi_t$  (exogenous)

IS equation:

$$Y_t = C_t^{\text{savers}} + C_t^{\text{investors}} + I_t + G_t$$

$(1 - \beta) \times \text{Net Worths}$

$\frac{\pi \times \text{Available Funds} - C_t^{\text{investors}}}{(1 - \theta q_t)}$

The diagram illustrates the IS equation  $Y_t = C_t^{\text{savers}} + C_t^{\text{investors}} + I_t + G_t$ . Below the equation, the term  $(1 - \beta) \times \text{Net Worths}$  has two arrows pointing to  $C_t^{\text{savers}}$  and  $C_t^{\text{investors}}$ . Another arrow points from the fraction  $\frac{\pi \times \text{Available Funds} - C_t^{\text{investors}}}{(1 - \theta q_t)}$  to  $I_t$ .

LM equation:

$$\begin{aligned}
 & (1 - \pi) E_t \left\{ \frac{r_{t+1} + q_{t+1} \phi_{t+1} \lambda + q_{t+1} (1 - \phi_{t+1}) \lambda}{q_t} - \frac{p_{t+1}}{p_t} \right\} \\
 & \qquad \qquad \qquad C_{t+1}^{\text{savers}} \\
 = & \pi E_t \left\{ \frac{p_{t+1}}{p_t} - \frac{r_{t+1} + q_{t+1} \phi_{t+1} \lambda + s_{t+1} (1 - \phi_{t+1}) \lambda}{q_t} \right\} \\
 & \qquad \qquad \qquad C_{t+1}^{\text{investors}}
 \end{aligned}$$

$$E_t r_{t+1} + \lambda > 1/\beta \quad (K_t \text{ less than first-best})$$

return on capital      time preference

$$> E_t \frac{r_{t+1} + q_{t+1}\phi_{t+1}\lambda + q_{t+1}(1 - \phi_{t+1})\lambda}{q_t}$$

return on equity if still a saver at t+1

$$> E_t \frac{p_{t+1}}{p_t}$$

return on money

$$> E_t \frac{r_{t+1} + q_{t+1}\phi_{t+1}\lambda + s_{t+1}(1 - \phi_{t+1})\lambda}{q_t}$$

return on equity if become an investor at t+1

