Clarendon Lectures

Lecture 3

SYSTEMIC RISK

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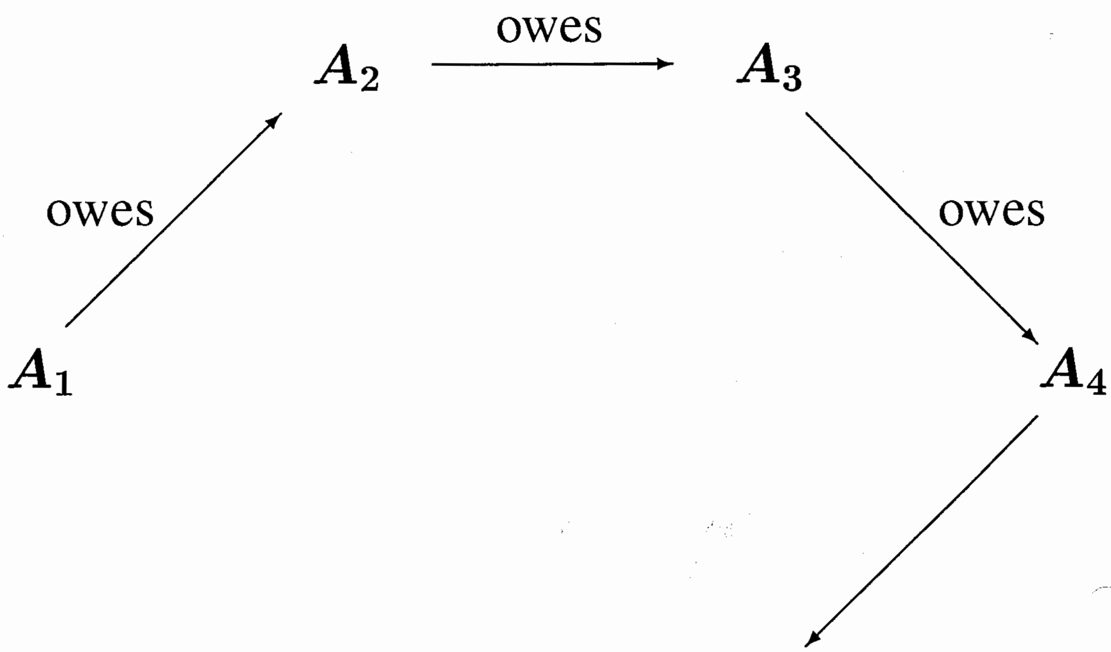
28 November 2001

agents $A_1, A_2, \ldots, A_i, \ldots$



credit chain only if

- each agent holds gross financial positions (simultaneously debtor and creditor)

liquidity shock propagates through chain only if

- agents are liquidity constrained (gross positions cannot be netted out)

propagation causes damage only if

- default creates inefficiency at each link exchange and credit: ordering, delivery and payment do not all happen simultaneously

(time-to-build + time-to-pay)

$\boldsymbol{A_i}$ simultaneously supplies on debit to $\boldsymbol{A_{i-1}}$

and purchases on credit from $\boldsymbol{A_{i+1}}$

$$\boldsymbol{A_{i-1}} \quad \underset{\text{payments}}{\overset{\text{goods}}{\leftrightarrows}} \quad \boldsymbol{A_i} \quad \underset{\text{payments}}{\overset{\text{goods}}{\leftrightarrows}} \quad \boldsymbol{A_{i+1}}$$

$\boldsymbol{A_i}$'s balance sheet:

assets	liabilities
accounts receivable	accounts payable
(from $\boldsymbol{A_{i-1}}$)	(to $\boldsymbol{A_{i+1}}$)

MODEL

3 dates: t = 0, 1, 2

single good, storable: date t → date t+1

$$z_t \longrightarrow Rz_t$$

identical agents, measure **1**

consume only at date 2; risk neutral

<u>endowments</u> (per capita):

date 0: e

date 1: either $-c$ with probability λ

or d with probability $1-\lambda$

date 2: none

date 1 endowment risk is uninsurable

assume $c > 0$, $d > 0$ and $e > 0$

also $(1-\lambda)d - \lambda c > 0$

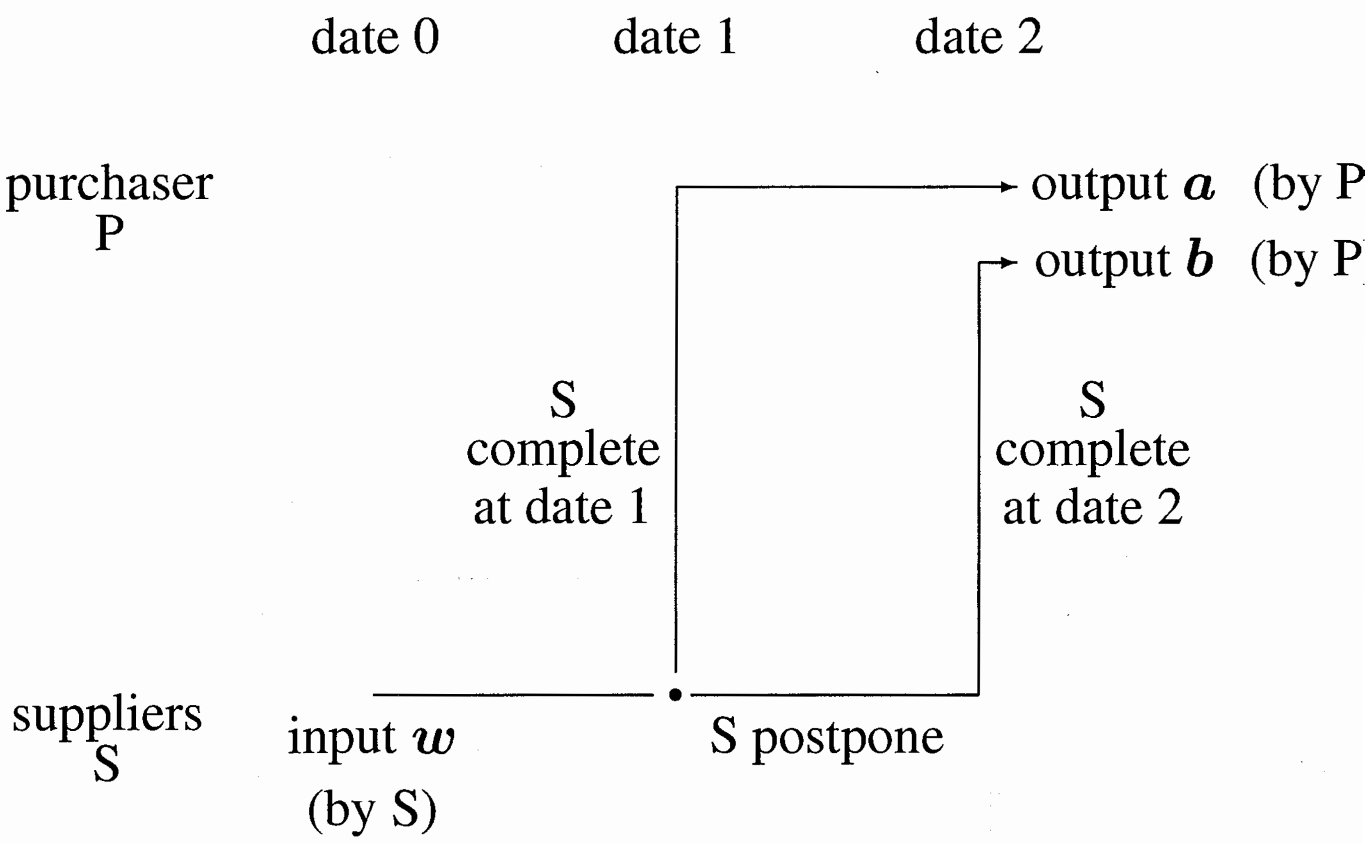
production

production by one agent ("purchaser" P)

requires inputs from measure ε of other agents ("suppliers" S)

inputs are required in equal proportion (Leontief)

constant returns; per composite unit supplied:



assume $a > b > R^2 w$

ex ante competition

at date 0, each agent

chooses level of (composite) purchase, x^p
and is free to purchase from anyone else

chooses level of supply, x^s
and is free to supply to anyone else

except that no-one can supply to their own supplier
($\Rightarrow$ no counter-trade)

ex post lock-in

at dates 1 or 2,

only supplier S can costlessly complete his own job

purchaser P can complete without S if P inputs $\mathbf{1}$,
per composite unit
(backstop technology for P)

assume $b > 1$

incomplete contracts

suppose S can demand payment from P at time of completion,

irrespective of any formal contract

and S has all ex post bargaining power

however, P has an outside option: backstop technology

$\Rightarrow$ ex post, P pays S a price of **1** for completion

ex ante market clears through price, q say, that P pays S

implicit contract (per unit):

date 0:	P places order with S P makes downpayment q to S S inputs w
date 1 or 2:	S completes, and is paid **1** by P

price q will be sensitive to anticipated date of completion symmetric equilibrium $x^p = x^s = x$

date 0 flow-of-funds:

$$qx + wx = e + qx$$

qx	wx	e	qx
downpayment made on purchases	input for supplies	endowment	downpayment received for supplies

guess: no storage $(z_0 = 0)$ at date 0

(to be verified later)

date 1 flow-of-funds:

constrained agents, C (those with negative endowment)

$$\hat{x} = \bar{x} - c$$

$\hat{x}$	$\bar{x}$	c
payment for purchases completed at date 1	average receipts from supplies completed at date 1	endowment

deep pockets, D (those with positive endowment)

$$x + z_1 = \bar{x} + d$$

x	z_1	$\bar{x}$	d
payment for all purchases	storage	average receipts from supplies completed at date 1	endowment

where $\bar{x}$ = average supplies completed at date 1

= average purchases completed at date 1

$= \lambda\hat{x} + (1\text{-}\lambda)x$

credit chains:

$$\hat{x} = \lambda \hat{x} + (1\text{-}\lambda) x - c$$

$$= x - \frac{c}{1 - \lambda}$$

↑

shortage of liquidity
is propagated through
credit chain

$\lambda \sim$ length of chain

example $\lambda = 2/3$

C C D

C C D

C C D

C C D

C C D

C C D

from any C, average length of chain to first D $= 3$

if positions of contrained agents in credit chain weren't random,

then aggregate response would change

examples (keeping $\boldsymbol{\lambda = 2/3}$):

C C
C D
C D
C C
D C
D C
C C
C D
C D

from any C, average length of chain to first D $= \mathbf{2\frac{1}{4}}$

D C
D C
D C
C C
C C
C C
C D
C D
C D

from any C, average length of chain to first D $= \mathbf{1\frac{1}{2}}$

date 2 utilities:

constrained agents C:

$$\widehat{U} = a\widehat{x} + (b-1)(x-\widehat{x}) + (x-\overline{x})$$

output from purchases completed at date 1	output minus payment for purchases completed at date 2	receipts from supplies completed at date 2

deep pockets D:

$$U = ax + Rz_1 + (x-\overline{x})$$

output from purchases	output from storage	receipts from supplies completed at date 2

C's balance sheet at start of date 2

assets		liabilities	
accounts receivable	$x - \overline{x}$	accounts payable	$x - \widehat{x}$
output from purchases completed at date 1	$a\widehat{x}$	net worth (consumption)	$\widehat{U}$
output from purchases to be completed at date 2	$b(x - \widehat{x})$		

ex ante expected utility:

$$EU = \lambda\widehat{U} + (1-\lambda)U$$

$$= \quad a\overline{x} \quad + \quad b(x - \overline{x}) \quad + \quad R(1\text{-}\lambda)z_1$$

output from completion at date 1	output from completion at date 2	output from storage

$$= \Big(ax + R[(1\text{-}\lambda)d - \lambda c]\Big) \quad - \quad (a-b)\frac{\lambda c}{1-\lambda}$$

first best output (no postponement)	welfare loss from postponement

↑

increases with $\boldsymbol{\lambda}$

securitization at date 1

S can securitize a fraction θ of his accounts receivable,
by spending $F(\theta)$ per unit of uncompleted (postponed) supply

$F(\cdot)$ increasing and strictly convex

deep pockets D won't securitize their accounts receivable

constrained agents C will choose θ to maximise

$$\widehat{U} = a\widehat{x} + (b-1)(x-\widehat{x}) + (1-\theta)(x-\overline{x})$$

non-securitized receipts from supplies completed at date 2

subject to

$$\widehat{x} = \overline{x} + [\frac{\theta}{R} - F(\theta)](x-\overline{x}) - c$$

↑ sales of securitized accounts receivable

↑ accounts receivable

assume an interior choice of θ, for which $\hat{x} < x$

first-order condition for θ:

$$F'(\theta) = \frac{1}{R} - \frac{1}{a - b + 1} \qquad \text{F.O.C.}$$

date 1 value of date 2 goods to D	date 1 value of date 2 goods to C

securitization shortens the credit chain:

$$\hat{x} = x - \frac{c}{1 - \lambda + \lambda\left(\frac{\theta}{R} - F(\theta)\right)}$$

individual choices in the date 0 market

prices:

q_1 = downpayment anticipating completion at date 1

q_2 = downpayment anticipating completion at date 2

supplies:

x_1^s = units supplied anticipating completion at date 1

x_2^s = units supplied anticipating completion at date 2

purchases:

x^p = units ordered

τ = expected proportion of purchases x^p completed at date 2

$$= \lambda \left(1 - \frac{\widehat{x}^p}{x^p}\right)$$ (see below for $\widehat{x}^p$)

date 0 flow of funds:

$$[(1-\tau)q_1 + \tau q_2]x^p \quad + \quad w(x_1^s + x_2^s) \quad + \quad z_0$$

downpayment made on purchases	input for supplies	storage

$$= \quad e \quad + \quad q_1 x_1^s \quad + \quad q_2 x_2^s$$

endowment	downpayment received for supplies completed at date 1	downpayment received for supplies completed at date 2

date 1 flow of funds:

with negative endowment shock:

$$\widehat{x}^p = x_1^s + \left(\frac{\theta}{R} - F(\theta)\right) x_2^s + R z_0 - c$$

↑
θ given
by F.O.C.

with positive endowment shock:

$$x^p + z_1 + \frac{1}{R}\frac{\lambda}{1\text{-}\lambda}\theta\, x_2^s = x_1^s + R z_0 + d$$

purchases of
securitized
accounts
receivable

date 2 utilities:

with negative endowment shock at date 1

$$\hat{U} = a\hat{x}^p + (b-1)(x^p - \hat{x}^p) + (1-\theta)x_2^s$$

with positive endowment shock at date 1

$$U = ax^p + x_2^s + Rz_1 + \frac{\lambda}{1\text{-}\lambda}\theta\, x_2^s$$

maximise expected utility $EU = \lambda\hat{U} + (1\text{-}\lambda)U$

subject to date 0 flow of funds constraint indifference conditions:

$$\frac{EU}{e} = \frac{\lambda(a-b+1)+(1\text{-}\lambda)R}{w-(1\text{-}\lambda)q_1-\lambda q_2} \qquad (x_1^s$$

$$= \frac{\lambda(a-b+1)\left(\frac{\theta}{R}-F(\theta)\right)+1-\lambda\theta}{w-q_2-\lambda(q_2-q_1)\left(\frac{\theta}{R}-F(\theta)\right)} \qquad (x_2^s$$

$$= \frac{\lambda(b-1)+(1\text{-}\lambda)(a-R)}{(1\text{-}\lambda)q_1+\lambda q_2} \qquad (x^p$$

$$> \frac{\lambda(a-b+1)+(1\text{-}\lambda)R}{\frac{1}{R}-\lambda(q_2-q_1)} \qquad (z_0$$

government

- has endowment at date 0
- cannot produce
- consumes U^g at date 2

social welfare $= \lambda \hat{U} + (1-\lambda)U + U^g$

PROPOSITION:

If, at date 1, the government purchases securitized accounts receivable to raise their price above $1/R$, then social welfare will increase.