Job security, asymmetric information, and wage rigidity*

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Abstract

We adapt the model of Menzio and Moen (2010) to consider, under both symmetric and asymmetric information, a labour market with directed search in which firms can commit to wage contracts but cannot commit *not* to replace incumbent workers with cheaper current hires. Workers are risk averse, so there exists an incentive for firms to smooth wages over time and in the face of shocks to labour productivity. The possibility of worker replacement creates job insecurity. To avoid it (which saves on the ex ante wage bill), firms may choose a wage for new hires that is never below that paid to incumbents and, hence, is equally unresponsive to negative shocks. This leads to a substantial degree of downward rigidity in the wages of new hires and magnifies the response of unemployment and vacancies to negative shocks. Our version of the model allows for the analysis of positive probability shocks in a tractable way. We further show that the interplay with asymmetric information can substantially enhance wage rigidity and increase the responsiveness of unemployment and vacancies to productivity shocks. In an empirical exercise, we argue that downward — but not upward — real wage rigidity for new hires is apparent in Germany, and we find tentative evidence in favour of the model with asymmetric information.

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1 Introduction

The behaviour of real wages over the business cycle is critical to understanding the mechanisms that drive employment and output fluctuations. The procyclicality or otherwise of real wages was the subject of considerable debate following the publication of Keynes's *General Theory*, and it remains controversial.[1] In this paper, we develop a model that has implications for the cyclicality of real wages and for output volatility, but we emphasize *asymmetric* wage responses to different phases of the business cycle. Our model exhibits partial "equal treatment" — the wages of new hires are equal to those of incumbent workers, but only in recessions.[2] Equality arises in order to eliminate job insecurity. The implication is that if there is a reason for ongoing wages to be rigid — here, risk aversion — this will be transmitted to new hires' wages. The latter is important for employment fluctuations, as wages are allocative in our base model.

We first recast insights from Menzio and Moen (2010), henceforth MM, in a more familiar and tractable framework. In their paper, overlapping generations of two-period lived firms interact with infinitely lived workers in the context of a frictional labour market, where employment dynamics are driven by firm entry (each firm employs a fixed number of workers). We develop a model with a fixed number of firms operating subject to decreasing returns to scale. We maintain the essential components of their approach. Firms can commit to current and future wages but not to employment. In particular, they cannot commit *not* to lay off a worker. If the wage of new hires is below that of incumbents, the firm will have an incentive to replace its incumbents if it can find suitable applicants, and depending on matching probabilities, there will be a possibility that an incumbent is replaced. Workers will have a preference for a contract with no job insecurity, that is, one in which wages of future hires are never below their own wages; if this situation holds, then the firm will have no incentive to attempt to replace them. It may then be optimal for firms to offer such contracts because the ex ante costs of hiring are lower by a sufficient amount to offset having to forgo the potential benefit of a lower wage for new hires in some future states. That is, it may be optimal for a firm to use its ability to commit to wages that satisfy a "no-replacement constraint" — that the wage for new hires is never below that of incumbents — as a second-best policy because of its inability to commit to not replacing incumbents.[3]

In adverse future states, assuming that the no-replacement constraint is satisfied, the

[1] See Galí (2013) for a comparison of the cyclicality of real wages in the General Theory and in New Keynesian Models and, e.g., Pissarides (2009) for a discussion of more recent empirical evidence in the context of the "unemployment volatility puzzle" (Costain and Reiter (2008), Shimer (2005)).

[2] In our asymmetric information extension to the model, although equality no longer holds, wages of new hires and incumbents remain linked in upswings due to incentive compatibility constraints.

[3] This type of argument was also made in Snell and Thomas (2010) in the context of a perfectly competitive labour market. MM's model, however, concerns a frictional labour market, and we follow their approach.

firm will trade off a desire to smooth the wages of workers in ongoing employment with the benefits from cutting the wage for new entrants. Treated on their own merit, the latter will receive a lower wage, but to avoid violating the constraint, the low wage will also be paid to incumbents, for whom a constant wage would be optimal from the insurance point of view. Then, the upshot is that there is some compromise and a degree of downward wage rigidity. The opposite is not true, however. In good states, there is no problem in paying a higher wage to new entrants than to incumbents, so the rigidity operates only in a downward direction. Because the wage for new entrants is allocational, the downwardly rigid wage affects hiring and increases the variability of both unemployment and vacancies in response to productivity shocks, a point also made by MM.[4] Our focus on employment fluctuations with a constant number of firms with decreasing returns to scale allows us to develop a transparent demand and supply version of the mechanics in a two-period setting (we also show our results extend to a multi-period setting), which in turn facilitates our asymmetric information extension.

We extend the base model to incorporate asymmetric information about the state of nature (productivity) by assuming that firms are better informed. In this case, we show that wages may be fully rigid downwards (to be precise: wages may fall but the rate of fall will be independent of the severity of negative shocks), thus further amplifying the variability of unemployment and vacancies. We show that it is the *interplay* between equal treatment and asymmetric information that leads to this result; without equal treatment, introducing asymmetric information has no impact on allocations. A rough intuition for this result is as follows: consider a bad state, say x, in which firms would like to cut new hire wages but cannot because they do not want to undercut incumbents — that is, we are in a region where the no replacement constraint binds. Suppose there is also a worse state x' in which wages for both incumbents and new hires are again equal but lower than in x (as may arise in the symmetric information version of the model.) Ex post, the firm may prefer to announce state x' because it would lead to both lower new hire wages *and* lower incumbent wages. Although ex ante, higher incumbent wages are better (as they offer better wage smoothing and hence reduce labor costs), ex post, there are strong incentives to cut these and save on wages for workers already in post. The firm thus benefits on both counts. We show that the only incentive-compatible contract may involve a (large) range of shocks for which wages of incumbents and new hires — the latter are allocational — are equal and do not vary with the severity of the shock.

In an empirical section, we argue that downward, but not upward, real rigidity for new hires is apparent in the German administrative IAB Employment History (BeH) panel dataset and that our model is broadly consistent with these empirical findings. We test

[4] Equal treatment can also lead to amplified unemployment fluctuations in competitive models. See Thomas (2005) and Snell and Thomas (2010). See Gertler and Trigari (2009a) for a somewhat related mechanism within a search-matching model with staggered Nash bargaining rather than optimal contracting, as employed here.

a prediction of the model with asymmetric information that the degree of downward — but not upward — rigidity should not depend on the current shock realization, but on the predicted distribution. We find tentative evidence supporting this prediction.

In Snell et al. (2018), we also examined evidence of downward real rigidity in German data in the context of testing Snell and Thomas (2010). The latter model exhibits downward rigidity, but for a somewhat different reason than the frictional model of this paper. In Snell and Thomas (2010), equal treatment holds in *both* down- and upswings (whereas here, it holds only in downswings). Despite this symmetry, wage responses to productivity shocks are asymmetric; in upswings, the labour market clears and wages respond fully to shocks, whereas in downswings, the response to shocks is smaller — following a similar logic to that of the symmetric information version of our model presented here. In the current paper, we present new evidence of downward rigidity based on sectoral variances, and moreover, we test a prediction of the asymmetric information version of our model, mentioned above, that in downturns, wages should be responsive to the distribution of shocks but not their realization.

1.1 Related Literature

Aside from MM, the most closely related theoretical work is Menzio (2005). Similar to this paper (but in contrast to MM), it considers a situation in which firms are informed about the current state of productivity and workers are not, and it exhibits equal treatment. It uses a bargaining model to show that firms satisfy equal treatment because workers receive, during bargaining, information about deals struck by the firm with other workers; an attempt to offer a higher wage when productivity is high, say, to a new hire, reveals that the firm has a positive shock and is willing to pay more and will cause other workers to re-open negotiations. In equilibrium, the firm offers a wage to all workers that is the lowest wage consistent with the observed history of the firm (there is a permanent idiosyncratic component). There are transitory shocks, but if these are not very persistent, the firm does not respond to them; the cost of responding to a positive shock involves paying all workers extra because of equal treatment, while the benefit in terms of additional hiring and retention is smaller when the shock is not expected to persist for long. By contrast, we develop and test a model of asymmetric wage rigidity using an explicit contracting framework. Nevertheless, a related logic applies across downswing states in that as the economy improves, the cost of rewarding incumbents along with new hires is greater than any benefit and, therefore, (by incentive compatibility) wages are constant across such states.

Other related work in which asymmetric information amplifies fluctuations includes Kennan (2010), who develops a model of procyclical information rents to firms: if a privately observed (to firms) component of match surplus has more dispersion when the aggregate state of the economy is better, and bargaining leads to an outcome in which firms capture the informational rent, wages are again relatively rigid, and procyclical rents to employer mean that employment fluctuations are magnified. Moen and Rosen (2011) analyse a model of moral hazard (unobservable worker effort) and competitive search and show that it introduces a counter-cyclical element to rents accruing to workers relative to a standard search-matching model, enhancing fluctuations in employment over the cycle. However, see also Guerrieri (2007) for a model in which workers have private information about match characteristics but which exhibits little amplification. Bruegemann and Moscarini (2010) derive a bound on extra employment amplification that can arise in frictional labor markets when there is acyclicality in worker *rents* (surplus relative to outside options) rather than wages per se, which is weaker than wage acyclicality when, as usual, outside options are procyclical. They argue that standard asymmetric bargaining models may achieve rent acyclicality but will not exceed their bound.

For the empirical results, we attempt to identify asymmetric responses by real wages to business cycle up- and downswings, in contrast to the empirical literature on wage stickiness, which typically has looked for evidence of downward real (and also nominal) rigidity by comparing empirical wage-change distributions with notional distributions, i.e., an attempt to capture how wage changes will be distributed in the absence of downward rigidities. An example is Dickens et al. (2007), who summarize results from the *International Wage Flexibility Project.* They use data from 16 OECD countries and find evidence of wage changes clustered around the expected inflation rate and fewer than the expected number of changes below that rate. This and similar evidence points to the existence of some real downward rigidity in individual wage changes in ongoing employment relationships. Our approach differs in that we concentrate on the wages of new hires (which are omitted by construction in the usual approach) and look at how wages respond to different phases of the cycle. See also Basu and House (2016) for a recent survey of the literature relevant to downward nominal rigidity, which also considers how real labour costs are impacted by rigidities.

Recent evidence from a study of 15 European Union countries by Galuscak et al. (2012) suggests that new hire wages are intimately related to wage structures that already exist in the firm; moreover, this relationship is stronger in periods of labour market slack, which is a feature of the equilibrium we derive here. Galuscak et al. argue that fairness and incentive issues are important in leading to this linkage, which is consistent with evidence collected by Bewley (1999), who argue that internal equity considerations make it difficult for firms to employ new hires at a wage lower than that paid to incumbents. Gertler and Trigari (2009a) estimate the cyclicality of hiring wages in the U.S. by using Survey of Income and Program Participation data and argue that wages of new hires do not appear to be more procyclical than those of ongoing employees. Likewise, using the same data, Gertler et al. (2015) find that the composition of match quality explains the greater wage flexibility for new hires from unemployment.

The outline of the paper is as follows. In Section 2, we lay out the basic model that underlies the analysis. In Section 3, we introduce asymmetric information and show that it increases downwards rigidity. In Section 4, we test certain predictions of the model using German administrative data. Section 5 contains concluding comments.

2 The Model

We adapt the model of MM and adopt their notation where possible. There are two periods $t = 1, 2$ (we extend this to an arbitrary horizon in the appendix), and a large number of identical firms and workers.[5] Each firm and worker lives for both periods, and the ratio of workers to firms equals S. We identify each firm with the entrepreneur who owns it; entrepreneurs do not supply labour. In each period, each firm operates a decreasing returns technology that produces a perishable good, with production function $f(n; x)$, where n is the current number of workers employed at the firm, which we treat as a continuous variable, $x \in X$ is a productivity shock observable at the start of the period, and derivatives with respect to the first argument are $f' > 0$, $f'' < 0$, with $f(0; x) = 0$. (Hours per worker are not variable.) We assume that $x = x_0$ is fixed at $t = 1$, but at $t = 2$, x is a random variable, common across firms, with finite support. Henceforth, x without a 0 subscript will refer to the second period productivity shock. Each worker has a per-period utility of consumption function $v(c)$, with $v' > 0$ and $v'' < 0$. Workers cannot borrow or save, so they consume all their current income; we assume for simplicity that there is no discounting of the future by workers. Entrepreneurs, on the other hand, are risk-neutral, but they also have a zero discount rate (nothing depends on this, provided that discounting is symmetric). A worker who is unemployed in any period receives an income of b.

A firm has a wage policy $\sigma = \left(w_1, (w_{2,i})_{i=1,2}\right)$ to which it commits, where i is the length of the worker's tenure and $w_{2,i}$ may be random (state contingent); so at $t = 1$, workers are offered a wage contract $(w_1, w_{2,2})$ and period-2 hires are offered $w_{2,1}$. A worker who accepts a contract at $t = 1$ suffers exogenous separation from the firm at the end of the first period, with probability δ. In this case, they will be in the same position as a worker who failed to gain employment in the first period; in the second period, such unattached workers seek work.[6] As in MM, employment is assumed to be "at will", so during the matching stage of the second period (after observing x), the firm can dismiss a worker without compensation; crucially, the firm can dismiss a worker after matching with a worker who can replace him or her. Additionally, a worker can quit without penalty but

[5] Formally, we will treat these as measures.

[6] MM assume that separated workers cannot work in the period immediately following separation.

will remain unemployed in the second period.[7]

At the start of each period (in period 2, after x is observed), search and matching occur (see Figure 1). We assume directed search (see Moen (1997), Acemoglu and Shimer (1999), and Rudanko (2009)). We follow MM in the following. Briefly, an unemployed worker can apply for one job at a single firm in each period. We rule out on-the-job search so that at $t = 2$, a worker cannot apply for a job if he is already employed. We identify the 'type' of a job with the utility V a successful applicant obtains from it. The application succeeds with probability $p(\theta(V))$, where $\theta(V)$, 'the expected queue length for the job,' is the ratio of applicants to jobs of type V, that is, the inverse of labor market tightness.[8] (The determination of $\theta(V)$ is discussed below.) The function $p(\cdot)$ is assumed to be strictly decreasing, differentiable and such that $p(0) = 1$, $p(\infty) = 0$. Correspondingly, the firm fills a job of type V with probability $q(\theta(V))$ where $q(\cdot)$ is strictly increasing, and satisfies $q(\theta) = p(\theta)\theta$, $q(0) = 0$ and $q(\infty) = 1$. Moreover, denoting the elasticity of q with respect to θ by $\epsilon_q(\theta)$, $q(\theta)\epsilon_q(\theta) / (1 - \epsilon_q(\theta))$ is assumed to be a decreasing function of θ.[9] At $t = 2$, unemployed workers can apply for jobs that are already filled; if there is a successful applicant, the firm can, by at-will contracting, choose whether to replace the incumbent or not. If $w_{2,1} \geq w_{2,2}$ firms will have no incentive to do this, but for $w_{2,1} < w_{2,2}$ the incentive exists to replace. In the latter case, then, to the extent that the matching process succeeds in selecting a successful applicant, the incumbent is at risk of losing her position.

Simultaneously to committing to a wage policy at the start of $t = 1$, firms choose how many new jobs $\overline{n}_i$ to create in period $i = 1, 2$, at a cost of $k > 0$ per job; $\overline{n}_2$ depends on the shock x. *There is no cost associated with receiving applicants for filled jobs,* so if $w_{2,1} < w_{2,2}$, then a filled job is as attractive as an unfilled one from the point of view of an applicant, as the new hire wage $w_{2,1}$ is assumed to apply to any new hire.[10] Unfilled jobs from the first period 'die' at the end of the period, along with filled jobs in which exogenous separation occurred (little depends on this assumption). The implication is that employment at the firm in period i will increase by $q(\theta(V))\overline{n}_i$.

Our base model differs from MM in the following principal respects. First, our workers are two-period lived rather than infinitely lived (firms in MM are two-period lived), and we have a two-period horizon. Second, rather than having firms of a fixed size (number of jobs) with constant productivity per filled job and free entry of firms, we suppose that

[7] This situation implies that the only participation constraint that matters for period-1 hires is the period-1 constraint. An alternative assumption that leads to this implication is that a worker who changes jobs incurs a high mobility cost.

[8] Again, we are following MM here. For the moment, we suppress other arguments of $\theta(\cdot)$ corresponding to the economic environment.

[9] MM, who also assume this, point out that many standard matching processes satisfy these assumptions.

[10] To be clear, and following MM, in this case, a filled job will attract the same number of applicants as any newly created unfilled job and will have the same probability of a successful applicant being found and, hence, of the incumbent losing his/her position.

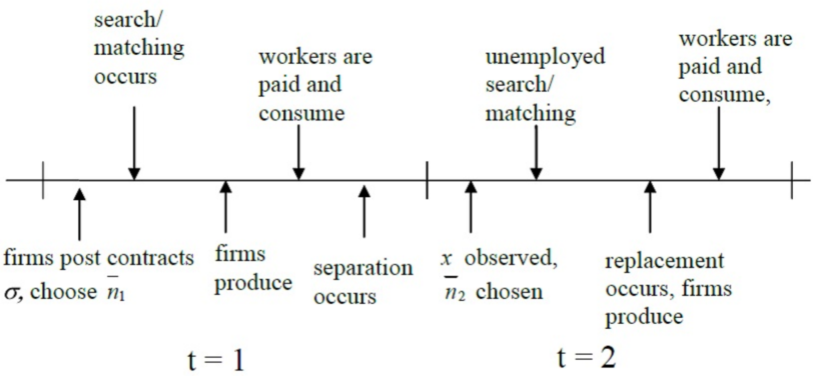

Figure 1: Timeline

there are a fixed number of firms, each with a decreasing returns to scale technology. The supply of jobs then varies not with variations in the number of firms entering the market but with firms' choice about how many jobs (or "vacancies") to create in each period. The fixed cost per job created replaces MM's assumption of a fixed cost incurred per firm that enters.

Let Z_1 be the lifetime utility of a worker at the search stage in period 1 and $Z_2(x)$ be that of a worker in period 2 searching for work in state x. (Z_1 and Z_2 are the endogenous variables determining the economic environment the firm faces.) Define $Z = \left(Z_1, \left(Z_2\left(x\right)\right)_{x\in X}\right)$. The value to a worker at $t = 1$ from being employed by a firm with wage policy σ is then

$$V_1(\sigma; Z) := v\left(w_1\right) + E[\delta Z_2\left(x\right) + (1-\delta)v\left(w_{2,2}\left(x\right)\right)]$$

if the worker faces only a separation risk, where E denotes the expectation. On the other hand, if replacement occurs in some states, that is, if $w_{2,1} < w_{2,2}$, then in such states, the term inside the square brackets must be replaced by

$$\delta Z_2\left(x\right) + (1-\delta)q\left(\theta_2\right)v\left(b\right) + (1-\delta)\left(1-q\left(\theta_2\right)\right)v\left(w_{2,2}\left(x\right)\right),$$

where $\theta_2 = \theta_2(w_{2,1}, Z_2\left(x\right))$ (defined below) is the queue length in that state for a firm offering $w_{2,1}$. This expression reflects the additional risk $q\left(\theta_2\right)$ to a surviving worker of being replaced by a successful applicant.[11]

Let U_1 be the lifetime utility of a worker at $t = 1$ who fails to get a job:

$$U_1\left(Z\right) = v(b) + E\left[Z_2(x)\right],$$

[11] To avoid complicating the exposition, we will ignore the possibility that at the optimal period-2 wage, the firm will prefer to dismiss some of its incumbents. This situation will arise if $w_{22} > f'\left(\left(1-\delta\right)n_1; x\right)$. In our simulations, parameters are chosen so that this scenario does not arise. Without loss of generality, we assume that $w_{22} \geq b$; otherwise, it would be in the worker's interest to quit.

as currently, the worker receives b and is able to search next period. Given U_1 and Z_1, the expected queue length for a job offering V_1 is assumed to satisfy:

$$\theta_1(V_1, Z_1, U_1) = \begin{cases} \theta : p(\theta)V_1 + (1 - p(\theta))U_1 = Z_1, \text{ if } V_1 > Z_1 \\ 0, \text{ if } V_1 \leq Z_1 \end{cases} \tag{1}$$

The idea is that if the value of the job to a successful applicant, V_1, is greater than the value of search, Z_1, the expected queue length is driven up to the point where workers are indifferent between applying for the job and searching somewhere else, and vice versa. The expected queue length for the job will be zero if the value of the job is less than (or equal to) the value of search.

For a worker at $t = 2$, the value from being employed at the wage $w_{2,1}$ is $v(w_{2,1})$, so the expected queue length for period-2 firms and workers for a job with wage $w_{2,1}$ is

$$\theta_2(w_{2,1}, Z_2) = \begin{cases} \theta : p(\theta)v(w_{2,1}) + (1 - p(\theta))v(b) = Z_2, & \text{if } v(w_{2,1}) > Z_2 \\ 0, & \text{if } v(w_{2,1}) \leq Z_2 \end{cases} \tag{2}$$

Assuming that incumbents are not replaced in period 2, that is, if $w_{2,1} \geq w_{2,2}$, all x, a firm's profit is

$$\begin{aligned} F(\sigma; \overline{n}_1, (\overline{n}_2(x))_{x\in X}; Z) = {} & (f(n_1; x_0) - w_1 n_1 - k\overline{n}_1) + \\ & E\left[(f((1-\delta)n_1 + n_2; x) - w_{2,2}(1-\delta)n_1 - w_{2,1}n_2 - k\overline{n}_2)\right] \end{aligned}$$

where n_i is the number of new hires in period i and is given by $n_i = q(\theta_i)\overline{n}_i$, $i = 1, 2$, where θ_i depends on σ, as given by $\theta_1(V_1(\sigma, Z), Z_1, U_1(Z))$ in (1) and $\theta_2(w_{2,1}, Z_2(x))$ in (2) above. Otherwise, in any state where replacement occurs, that is, if $w_{2,1} < w_{2,2}$, the expression for second-period profit is replaced by

$$f((1-\delta)n_1 + n_2; x) - w_{2,2}(1 - q(\theta_2))(1-\delta)n_1 - w_{2,1}(n_2 + q(\theta_2)(1-\delta)n_1) - k\overline{n}_2,$$

where $q(\theta_2)(1-\delta)n_1$ is the number of incumbents who are replaced by new hires, and $n_2 = q(\theta_2)\overline{n}_2$ is the number of new hires *into newly created jobs.*[12]

Competitive Search Equilibrium

We define an equilibrium as follows:

Definition 1 *A symmetric stationary competitive search equilibrium with positive hiring consists of search values* $Z = \left(Z_1, (Z_2(x))_{x\in X}\right)$*, and a wage policy* σ *and job creation plan* $\left(\overline{n}_1, (\overline{n}_2(x))_{x\in X}\right)$ *with the following properties:*

[12] We will assume throughout that positive hiring occurs in equilibrium. Given average annual turnover rates of around 30% in the U.S., for example, this assumption is not restrictive for any reasonable parametrization.

(i) *Profit maximization*: For all $(\sigma'; \overline{n}'_1, (\overline{n}'_2(x))_{x\in X})$,

$$F((\sigma; \overline{n}_1, (\overline{n}_2(x))_{x\in X}); Z) \geq F\left(\sigma'; \overline{n}'_1, (\overline{n}'_2(x))_{x\in X}; Z\right);$$

and

(ii) *Consistency*: $\theta_1(V_1(\sigma, Z), Z_1, U_1) = S/\overline{n}_1$, and, for all x, if $w_{2,1} \geq w_{2,2}$ (no replacement occurs), $\theta_2(w_{2,1}, Z_2(x)) = S_2/\overline{n}_2(x)$ where $S_2 := ((1 - p(S/\overline{n}_1)) + \delta p(S/\overline{n}_1))S$ is the number of workers (per firm) seeking work in period 2, while if $w_{2,1} < w_{2,2}$ (replacement occurs) $\theta_2(w_{2,1}(x), Z_2(x)) = S_2/(\overline{n}_2(x) + (1-\delta) q(S/\overline{n}_1)\overline{n}_1)$.

2.1 No Replacement in State x

We begin by characterizing an optimal policy assuming that in state x, $w_{2,1} \geq w_{2,2}$. We will deal with the issue of whether satisfying this inequality is optimal below, that is, whether a policy with $w_{2,1} < w_{2,2}$ might yield higher profits. We proceed heuristically.[13] In period 2 in any state x, given n_1 and w_1, following MM, it can be shown that the firm must locally maximize profits plus weighted incumbent utility.[14] In particular, given it is assumed optimal *not* to replace, it must maximize

$$\begin{aligned} & f((1-\delta)n_1 + n_2; x) - w_{2,2}(1-\delta)n_1 - w_{2,1}n_2 - k\overline{n}_2 + \\ & \left(1/v'(w_1)\right) n_1 ((1-\delta) v(w_{2,2}) + \delta Z_2(x)), \end{aligned} \tag{3}$$

with respect to $\overline{n}_2, w_{2,1}, w_{2,2}, w_{2,1} \geq w_{2,2}$, where $n_2 = q(\theta_2(w_{2,1}, Z_2(x)))\overline{n}_2 =: \tilde{q}(w_{2,1}, x)\overline{n}_2$. We write $\tilde{q}' \equiv \partial\tilde{q}/\partial w_{2,1}$. Note that the last term in (3) includes the continuation utility of an incumbent, taking into account the separation possibility and multiplied by the number of incumbents. The intuition here is that any change that affects the utility of the firm's old workers can be offset by a change in the first period wage, leaving V_1 unchanged (and, hence, n_1). Multiplying the utility change by the inverse of first-period marginal utility then converts it (for a small change) to the first-period wage savings per worker.

There are two cases to consider:

(A) If the no-replacement constraint, henceforth "no-replacement constraint", $w_{2,1} \geq w_{2,2}$

[13] The necessary conditions that follow in the text are derived formally in the Appendix by considering the two-period problem. Alternatively, it can be directly established that (3) below must hold at a local maximum subject to $w_{21} \geq w_{22}$.

[14] MM introduce a sunspot into their model, which allows the firm to randomize between replacement and no replacement. They can then show that an equivalent of (3) must be maximized across replacement/no-replacement regimes and derive analytical sufficient conditions for no replacement to be optimal. We could follow a similar approach here, but because we are able to compute numerical solutions straightforwardly, the solution can be checked directly. Moreover, the restriction to contracts dependent only on the productivity shock considerably simplifies the analysis.

is not binding, then differentiating (3) with respect to $w_{2,2}$,

$$(1-\delta)n_1 = n_1\left(1/v'(w_1)\right)\left((1-\delta)\,v'(w_{2,2})\right), \tag{4}$$

so that $w_1 = w_{2,2}$. Intuitively, the firm should stabilize the wages of the first period hires if there is no cost of doing this. In this case, also differentiating with respect to $w_{2,1}$, we obtain

$$f'\left((1-\delta)n_1 + n_2; x\right) q'\overline{n}_2 - w_{2,1}q'\overline{n}_2 - q\overline{n}_2 = 0, \tag{5}$$

and simplifying:

$$f'(n)\,\tilde{q}' - w_{2,1}\tilde{q}' - q = 0,$$

where we write $n \equiv (1-\delta)n_1 + n_2$ for total period-2 employment. Finally, differentiating with respect to $\overline{n}_2$,

$$f'(n) = w_{2,1} + k/q. \tag{6}$$

We can combine these latter two to obtain

$$q^2\left(\tilde{q}'\right)^{-1} = k. \tag{7}$$

Intuitively, in order to increase employment by one unit, the firm could open $1/q$ jobs at a cost of k/q. Alternatively a wage increase of $1/\left(\overline{n}_2\tilde{q}'\right)$, holding the number of jobs constant, accomplishes the same result by increasing the queue length and, hence, the probability that each existing job is filled, at a cost of $q\overline{n}_2 \times 1/\left(\overline{n}_2\tilde{q}'\right) = q/\tilde{q}'$. The two must be equal in equilibrium so that (7) follows.

In the proof of Proposition 1 it is shown that (7) can be solved to give a positively sloped locus of values for n_2 and $w_{2,1}$ compatible with equilibrium. This locus defines an upward-sloping "quasi-supply" curve of labor: when equilibrium n_2 is higher, it is more difficult to fill each job because the labor market is tighter (θ_2 is lower, so $k/q(\theta_2)$ is higher); this makes wage increases, as a way to fill jobs, more attractive than creating jobs, and $w_{2,1}$ rises until the two methods cost the same. This locus is independent of the profitability of filling a job. We refer to it as the *commitment quasi-supply curve*. It corresponds to the solution to the first-order conditions in the case where firms can *commit* not to replace incumbent workers and thus ignores the no-replacement constraint $w_{2,1} \geq w_{2,2}$. (The two coincide in the current case because the constraint is not binding by assumption.) Combining this situation with the downward sloping (6), which is a standard labor demand equation, where the unit cost of increasing employment $k/q(\theta_2)$ (itself increasing as n_2 increases)[15] is added to the wage and yields a unique equilibrium for each productivity shock whenever the no-replacement constraint does not bind.[16] As x varies, only the labor

[15] As n_2 increases, we must have $p(\theta)$ increasing from $n_2 = p(\theta)\,S_2$, and hence, θ has fallen as $p' < 0$; thus $q(\theta)$ falls, given that $q' > 0$.

[16] The positions of these two curves depend only on x and n_1, which implies the value of S_2.

demand curve shifts. Denote the solution of (6) and (7) by $\left(w_{2,1}^{C}\left(x, w_1, n_1\right), n_2^{C}\left(x, w_1, n_1\right)\right)$, where the $C-$superscript indicates that this is the solution to the FOCs in the case of commitment.

Since in this case, $w_{2,1} \geq w_{2,2} = w_1$, we conclude that the intersection of (6) and (7) occurs at or above w_1.

(B) If, on the other hand, $w_{2,1} \geq w_{2,2}$ is binding at the optimum (when productivity is sufficiently low), the intersection of (6) and (7) occurs at a wage below w_1, but the wage can be shown to be above $w_{2,1}^{C}\left(x, w_1, n_1\right)$, while employment is below $n_2^{C}\left(x, w_1, n_1\right)$. In the proof, it is shown that $k < q^2/\tilde{q}'$. The unit cost of increasing employment through creating extra jobs, k/q, is lower than that through increasing wages, $q/\tilde{q}'$, so it would be cheaper to cut wages and increase jobs; however, this is not done because the wage cut has a negative externality on incumbents' wage smoothing. More intuitively, if productivity is low enough that the equilibrium hiring wage under commitment $w_{2,1}^{C}$ is below w_1, then the no-replacement constraint will be violated (recall that $w_{2,2}^{C} = w_1$). To satisfy the constraint, $w_{2,2}$ must be cut, which is costly because it reduces wage smoothing, so firms are less willing to let wages fall. Thus, below w_1, the equilibrium lies above the commitment quasi-labor-supply curve.

Consequently, taking w_1 as given, we can plot a *no-commitment quasi-supply* curve in $w_{2,1} - n_2$ space, which coincides with the commitment one above w_1, but below w_1, the curve lies above the commitment curve (it is the locus of points satisfying (25) in the Appendix). Equilibrium occurs at the intersection with the labor demand curve. As x varies, the latter curve shifts. In Figure 2, a situation where the crossing point occurs below w_1 is illustrated.[17] The equilibrium values are at point A, rather than at the commitment solution. If x is sufficiently high such that the intersection occurs above w_1, then the equilibrium will be at the commitment solution, $\left(w_{2,i}^{C}\left(x, w_1, n_1\right), n_2^{C}\left(x, w_1, n_1\right)\right)$. The proposition summarizes the discussion.

Proposition 1 *Suppose that replacement does not occur in state x. Then, (a) if equilibrium hiring wages in period 2 are below period-1 wages, $w_{2,1} < w_1$, we have $w_{2,1} > w_{2,1}^{C}\left(x; w_1, n_1\right)$ and $n_2 < n_2^{C}\left(x; w_1, n_1\right)$: the wage for new hires is higher and employment is lower than they would be if firms were able to commit;*[18] *moreover, $w_{2,2} = w_{2,1} < w_1$. Otherwise, (b) wages and employment are at the commitment levels: $w_{2,1}^{NC}\left(x; w_1, n_1\right) = w_{2,1}^{C}\left(x; w_1, n_1\right)$ and $n_2^{NC}\left(x; w_1, n_1\right) = n_2^{C}\left(x; w_1, n_1\right)$, with $w_{2,2}^{NC}\left(x; w_1, n_1\right) = w_1$. Case (a) occurs when the labor demand curve intersects the commitment quasi-supply curve below*

[17] In simulations, we find that the no-commitment quasi-supply curve will eventually increase as n_2 declines far enough. The intuition is that the number of new hires falls sufficiently low such that the desire to insure incumbents dominates.

[18] If commitment were allowed in such a state, unless the state had a negligible probability, then the equilibrium two-period contract may be different, that is, w_1 and n_1 may differ. The proposition concerns the implied values of w_{21}^{C} and n_2^{C} in a hypothetical equilibrium that has the same period-1 values.

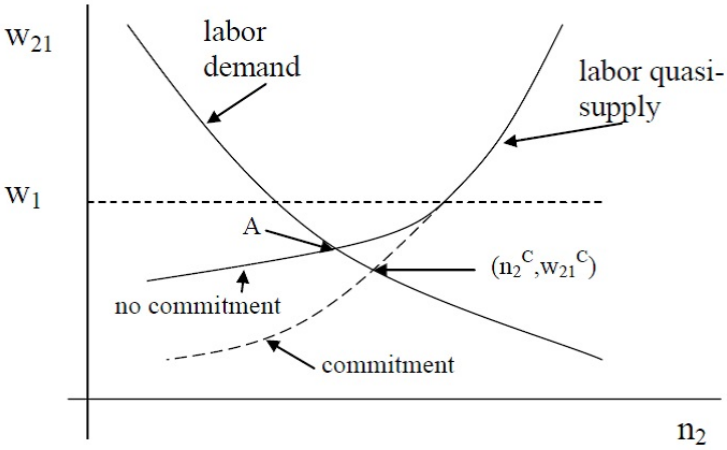

Figure 2: No-commitment quasi-supply

w_1; otherwise, case (b) occurs.

Wages are allocational[19] in period 2 so that the flatter quasi-supply in the region where there is downward pressure on wages will also imply more variable employment.[20] The result is unchanged if there is symmetric discounting. If discounting is asymmetric, then we show in Section A.5 of the Appendix that the reference wage in period 2, which determines the regime (and $w_{2,2}$ when no replacement does not bind), differs from w_1.

2.2 Replacement in State x

Next, we characterize outcomes when replacement does occur (although our focus in subsequent sections will be on situations in which no replacement is optimal in equilibrium).

Proposition 2 *Suppose that replacement occurs in state x. Then, for a given w_1 and n_1, the wage for new hires is lower and employment is higher than they would be if firms were able to commit, $w_{2,1} < w^C_{2,1}(x; w_1, n_1) < w_1$; moreover, $w_{2,2} = w_1$.*

[19] i.e., firms hire until the marginal product net of the hiring cost (k/q) is equal to the new hire wage.

[20] e.g., take the matching function $m(u, \nu) = u\nu / \left(u^l + \nu^l\right)^{1/l}$, where u is the number of workers searching and ν is the number of vacancies, where we set $l = 0.5$ (Hagedorn and Manovskii (2008) calibrate $l = 0.407$), with a log production function subject to uniformly distributed multiplicative productivity shocks, CRRA utility and a coefficient of risk aversion of 2, $\delta = 0.1$ (approximate annual separation rate in our German data), $\beta = 0.9$, a replacement rate of 43%, and we calibrate k to yield an average period-2 unemployment rate of 7.5%. The standard deviation of unemployment in the region where the no-replacement constraint is binding is approximately twice that in the commitment model. The effect is smaller, however, under alternative parameterizations. With a Cobb-Douglas matching technology with $p(\theta) = M\theta^{\eta-1}$, $q(\theta) = M\theta^{\eta}$, where $M = 1/10$ and $\eta = 1/2$ (this is the same specification used in MM's example) and $v(c) = c^{0.5}$, and setting $\delta = 0.3$ (appropriate for US annual data), we obtain a much smaller increase of approximately 25%. This result is partly attributable to a lower risk-sharing motive, but the higher separation rate means that in bad states, the incentive to bring in new hires at a lower wage is stronger.

Intuitively, cutting the new hire wage makes a job less attractive, and therefore, given that replacement occurs, the risk of replacement decreases; this positive externality on incumbents makes a wage that is lower than the commitment wage optimal. The firm should stabilize the wages of the first-period hires because there is no cost of doing this — given that the replacement probability is independent of $w_{2,2}$ whenever $w_{2,1} < w_{2,2}$.

Given the tractability of the model, we proceed in our simulations by computing an equilibrium under the assumption that replacement is not optimal in any state. We then check whether replacement can improve profits. If so, we have an equilibrium (but it does not logically rule out the possibility of an equilibrium with replacement existing at the same time).

To understand the logic of the comparison, first consider the limiting case of a competitive labour market, as in Snell and Thomas (2010). In this case, if $w_{2,1} < w_{2,2}$ in some state, all incumbents will be replaced, provided that $w_{2,1}$ is not below the supply price of unemployed workers, as the firm can then hire as many new hires as it wants. Since the supply price of an unemployed worker in period 2 will be at least as great as what a replaced worker would expect to obtain (since we have assumed that the latter will remain unemployed), changing the contract so that $w_{2,2} = w_{2,1}$ clearly does not leave the firm worse off, as it faces the same costs at period 2. Period-1 hires will weakly prefer this contract because they are not replaced. Thus, satisfying the no-replacement constraint is weakly dominant (and strictly so if the supply price of the unemployed exceeds what a replaced worker obtains). However, this logic may not extend to the frictional labour market; suppose that the cost of vacancy creation is sufficiently low such that θ is low in equilibrium, that is, $q(\theta)$ is low. Then, the probability of replacement, $q(\theta)$, may be such that a firm is better off by setting $w_{2,1} < w_{2,2}$ and offering full insurance to an incumbent if he/she remains in the firm, but with a small risk of replacement, and offering a lower wage to new hires. In this case, the no-replacement constraint is (optimally) violated.[21]

2.3 Multi-period Extension

The model extends readily in the obvious way to multiple periods T (with long-lived firms and workers). Proposition 1 also extends to this case in that if no-replacement occurs in equilibrium, incumbents' wages are always no higher than new hire wages and fall only to maintain this relationship, otherwise remaining constant. Moreover, in downturns, wages do not fall as far as firms would like them to in the following sense: if new hire wages fall between periods t and $t+1$, they are above the relevant commitment quasi-supply curve at $t+1$; when new hire wages rise between the two periods, however, then they lie on the

[21] This situation may seem somewhat paradoxical, as a low k implies that search becomes more competitive; but while this is true for new hires, the replacement probability for incumbents will fall to zero, as the ratio of new advertised jobs to existing ones goes to infinity, i.e., the model is not continuous at $k = 0$.

relevant commitment quasi-supply curve at $t+1$.[22]

We now allow for respective discount factors for firms and workers β_f and β_w, $0 < \beta_f, \beta_w < 1$. A firm's wage policy, to which it commits, is $\sigma = \left(w_1, (w_{2,i})_{i=1,2}, ..., (w_{T,i})_{i \leq T}\right)$, where $w_{t,i}$ now denotes the wage paid at t to a worker with i periods tenure (contingent on history up to t) so that $w_{t,1}$ is the wage paid to a new hire at t, etc. See the appendix for further details and proofs.

Proposition 3 *A symmetric stationary competitive search equilibrium with no replacement and positive hiring has the following characterization. (a) (Evolution of incumbent wages) For $i \geq 1$, define $\tilde{w}_{t+1,i+1}(x^t, x)$ to be the solution to*

$$\beta_w v'\left(\tilde{w}_{t+1,i+1}\right) = \beta_f v'\left(w_{t,i}\right)$$

(i.e., when $\beta_w = \beta_f$, $\tilde{w}_{t+1,i+1} = w_{t,i}$). Then

$$w_{t+1,i+1} = Min\{\tilde{w}_{t+1,i+1}, w_{t+1,1}\}.$$

(b) (Evolution of new hire wages) If $\beta_w v'\left(\tilde{w}_{t+1,1}\right) > \beta_f v'\left(w_{t,1}\right)$ (i.e., when $\beta_w = \beta_f$, if new hire wages fall at $t+1$: $w_{t+1,1} < w_{t,1}$), then $w_{t+1,1}$ lies above the commitment quasi-supply curve[23]; if $\beta_w v'\left(\tilde{w}_{t+1,1}\right) \leq \beta_f v'\left(w_{t,1}\right)$ (when $\beta_w = \beta_f$, if new hire wages rise or are constant at $t+1$), $w_{t+1,1} \geq w_{t,1}$, then $w_{t+1,1}$ lies on the commitment quasi-supply curve.

3 Asymmetric Information

So far, we have seen that equal treatment leads to a measure of downward real rigidity. We now consider adding asymmetric information about the period-2 state x, and we argue that for a wide range of adverse shocks, this state may lead to a period-2 wage that is completely rigid for incumbents and, more importantly, for new hires. Moreover, under the assumptions of Proposition 4 below, period-2 wages remain allocational, which leads to enhanced employment variability. We assume that it is always optimal to satisfy the no-replacement constraint.

We will assume that in period 2, ongoing hires in a firm can observe only wages $w_{2,1}$ and $w_{2,2}$ but cannot observe x (nor Z_2 so they cannot infer x). Additionally, they cannot observe the total employment or vacancies at the firm (we relax this later). Equivalently, we assume that such variables are not contractible. The resultant incentive compatibility

[22] The principal qualitative difference is that there may be multiple incumbent wages at each date and that the new hire wage is no longer fully allocative, as future cohorts may be paid more than a newly hired cohort will and may have different associated hiring costs (see Kudlyak (2014)).

[23] In the sense that $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} > k$, where $\tilde{q}\left(w_{t,1}, x^t\right)$ is the probability that a firm that unilaterally varies only the initial wage of a new hire equilibrium contract fills the job.

constraints on the contract imply that the equilibrium contract exhibits a much higher degree of wage rigidity and employment and vacancy fluctuations than induced by equal treatment alone. Throughout this section, we will assume that it is optimal for firms to satisfy the no-replacement constraint $w_{2,1} \geq w_{2,2}$, in every state x, so we will not consider the possibility that replacement is desirable.[24]

In the symmetric information model, when the no-replacement constraint is binding so that $w_{2,1} = w_{2,2}$, as x varies, we pick off points on the quasi-supply curve as in Figure 2. Moving to the asymmetric information model, if wages vary with the state as in the symmetric information solution, then the firm has an incentive to claim that the state corresponding to a lower wage has occurred, as not only is the incumbent wage reduced, which is an unambiguous benefit to the firm, but the new hire wage is also reduced, which is a benefit locally (as $w_{2,1}$ is higher than the committed new hire wage). To satisfy incentive compatibility, then, the wage must be constant across a wide range of states. Because the wage is allocational, this translates into large employment movements.

Our base assumption that firm employment is unobservable to workers contrasts with early models in the asymmetric information implicit contracting literature, such as Chari (1983), Green and Kahn (1983), and Grossman and Hart (1981) — in a single worker model, as is often considered in this literature, observing employment (hours of work) is inevitable of course. In practice, however, the level of employment in a firm can be difficult to define precisely. For example, if the relevant employment level is at the plant, the firm may be able to move production to other companies or plants within the same company, making it difficult to condition on employment (as argued by Stiglitz (1986)). Outside conditions, such as the value of Z_2, may be difficult to contract over. By contrast, in earlier literature, where firms typically had private information about the state of productivity, distortions away from efficient levels of employment occurred to satisfy incentive compatibility. A lower wage-employment combination might be efficient in a low-productivity state, but to prevent the firm from claiming to be in the low-productivity state when it is not – in order to benefit from lower wages – employment in that state might be distorted downward. Our mechanism here is different, and in contrast to that literature, period-2 wages are allocational. However, we also consider an extension below in which we allow contracts to depend on employment levels, and we show that (when shocks are not too far apart) a similar logic applies locally and that wages are essentially constant.

3.1 Incentive Constraints

We will again develop the implications of the model under the assumption that it is not profitable for firms to replace incumbents in period 2, leaving this assumption to be checked

[24]Again, in our simulations reported below, we verify that in the resulting equilibria, this assumption is justified.

numerically. As before, assuming that a firm's profit is

$$F\left(\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X};Z\right)=\left(f\left(n_1\right)-w_1n_1-k\overline{n}_1\right)+E[F^{(x)}]$$

where $F^{(x)}$ is period-2 profits in state x and is given by

$$F^{(x)}\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right):=\\ \left(f\left((1-\delta)n_1+n_2\left(x\right);x\right)-w_{2,2}\left(x\right)(1-\delta)n_1-w_{2,1}\left(x\right)n_2\left(x\right)-k\overline{n}_2\left(x\right)\right)$$

(recall n_i is the number of *new hires* in period i, and is given by $n_i=q\left(\theta_i\right)\overline{n}_i$, $i=1$, 2, where θ_i depends on σ, as given by $\theta_1(V_1\left(\sigma,Z\right),Z_1,U_1\left(Z\right))$ in (1) and $\theta_2(w_{2,1},Z_2\left(x\right))$ in (2) above). We now have the firm's maximization problem as

$(\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X})$ maximizes $F((\sigma;\overline{n}_1,\left(\overline{n}_2\left(x\right)\right)_{x\in X});Z)$ subject to the incentive compatibility constraints for all x,

$$F^{(x)}\left(\sigma;\overline{n}_1,\overline{n}_2\left(x\right);Z\right)=\\ \max_{x',\overline{n}_2'}\left\{\left(f\left((1-\delta)n_1+n_2';x\right)-w_{2,2}(x')(1-\delta)n_1-w_{2,1}(x')n_2'-k\overline{n}_2'\right)\right\}$$

where $n_2'=q\left(\theta_2\right)\overline{n}_2'$ and $\theta_2=\theta_2(w_{2,1}(x'),Z_2\left(x\right))$, and the no-replacement condition is $w_{2,1}\geq w_{2,2}$ for all x. That is, the firm has a menu of wage profiles $\left(w_{21}\left(x\right),w_{22}\left(x\right)\right)$ to choose from and will optimize vacancies, given its choice;[25] incentive compatibility requires that the firm prefers the wage profile associated with the current state to any other.

We now assume that $X\subset R_+$, and that f is differentiable and increasing in x. We can establish the following:

Proposition 4 ***(Asymmetric information)****(i) If commitment is possible, then introducing asymmetric information does not affect the equilibrium. (ii) (wage floor:) Suppose in the no-commitment asymmetric information model with a single period-*2 *productivity state*[26] $\hat{x}$*, that there is an equilibrium with no replacement and the no-replacement constraint binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states,* $\hat{x}-\varepsilon$ *and* $\hat{x}+\varepsilon$ *(i.e., with expected value* $\hat{x}$*), and assuming that there exists* $\overline{\varepsilon}$ *such that for* $\varepsilon\in[0,\overline{\varepsilon})$*, the equilibrium is unique and continuous, period-*2 *wages are* constant *across these states, provided that the perturbation* ε *is sufficiently small.*[27] *Period-2 wages are allocational. (iii) (upward flex-*

[25]These are ex post (after the period-2 state is observed) constraints; for simplicity, we assume that n_1 is contractible. Otherwise, the incentive compatibility constraints should be expressed in terms of an ex ante constraint that requires that should the firm deviate at date 1 and in any period-2 state, it cannot increase its discounted profit. Since in the latter case, the ex post constraints will also hold, the results will be very similar.

[26]Obviously asymmetric information does not bite until we perturb the equilibrium to have multiple states.

[27]For ease of presentation, the proposition considers the case where there is a single period-2 state $\hat{x}$ in

ibility:) In the no-commitment asymmetric information model, at the highest w_{22}, i.e., for $x \in \arg\max_{x'} w_{22}(x')$, $w_{21}(x) \geq w^C_{2,1}(x, w_1, n_1)$ with equality if the no-replacement constraint is not binding; $w_{22}(x) \leq w_1$, all x.

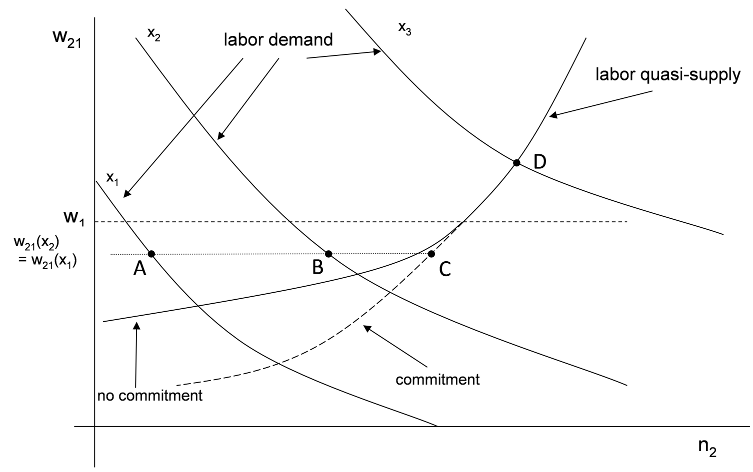

Figure 3: A rigid wage under asymmetric information

Part (i) of Proposition 4 considers the nature of the contract with asymmetric information but with commitment (not to replace) on the part of the firm. The firm will offer a non-contingent period-2 contract wage to period-1 hires (equal to w_1) but will be unrestricted in offering the optimal hiring wage to period-2 workers. Since a stable wage for incumbents is optimal and incentive-compatible, the solution will be identical to the commitment solution considered earlier.

Part (ii) considers what happens in the absence of commitment, where asymmetric information now matters: if there are two states close to each other and the no-replacement constraint is binding, then wages are non-contingent (which as direct implications for hires).

While the formal proposition requires the variance of the shocks to be small, simulations suggest that the optimal contract has a fixed period-2 wage for a very wide range of shocks, where there are multiple shocks. To see this intuition, consider the no-commitment solution, and suppose there are two states x_1 and x_2 at $t = 2$ and that we are in the region where the no-replacement constraint is binding in both states, $w_{1,2}(x) = w_{2,2}(x)$, $x = x_1$, x_2. If the wage varies with the state, say if $w_{1,2}(x_1) = w_{2,2}(x_1) < w_{1,2}(x_2) = w_{2,2}(x_2)$, in state x_2, the firm will prefer to "announce" state x_1: it benefits from paying a lower wage to its existing employees. In addition, because the no-replacement constraint is binding, the wage for new hires will optimally be set lower, and the firm will benefit from a lower

the initial situation. If there are other states in which the no-replacement constraint is not binding, the argument can be extended straightforwardly. The argument also extends readily to non-equi-probability perturbations.

wage if it can ignore the no-replacement constraint. Therefore, for both reasons, period-2 profits increase. Consequently, the no-commitment solution will violate incentive compatibility, but a similar logic applies more generally when wages vary across states. This argument works for small wage variations across states; however, for a very wide variation in shocks ,the lower $w_{1,2}$ might be so low — below the optimal level in the other state — that switching to it reduces profits from new hires. These profits are unlikely to outweigh the gains from cutting $w_{2,2}$ though, as these are first-order and large, while around the optimal hiring wage, the change in profits on cutting $w_{2,1}$ will initially be second-order.[28] An incentive compatible contract is illustrated by points A and B in Figure 3, assuming there are only two states, x_1 and x_2.[29]

Part (iii) implies that in the state with the highest w_{22}, if the no-replacement constraint is not binding, new hire wages are at the commitment solution (where labour demand and commitment quasi-supply curves intersect), and if it is binding, wages are at least at this level. Intuitively, continuing the previous discussion, suppose that there is a third state $x_3 > x_1$, x_2, and suppose that this state of nature improves; the new hire (equilibrium) wage that is optimal under commitment, that is, ignoring the no-replacement and incentive compatibility constraints in that state, rises above the constant wage (say $\overline{w}$) for the lower two states. Then, it is incentive compatible to have $w_{2,1}(x_3)$ at the optimal level (see point D in Figure 3) but $w_{2,2}(x_3) = \overline{w}$: announcing a lower state in state x_3 will reduce profits ($w_{2,1}$ will be at a suboptimal level, while w_{22} will be the same). In fact, the firm can do even better: $w_{2,2}$ will be slightly higher than $\overline{w}$.[30] For sufficiently favorable x_3, $w_{2,2}$ can increase all the way to w_1 without violating incentive compatibility, but as shown in general in the proposition, it is never optimal to exceed w_1. Nevertheless, incumbent wages are procyclical — though within the restricted interval of wages $[\overline{w}, w_1]$ — over a wider range of "positive" shocks than in the symmetric information case, something that may accord better with empirical evidence.[31]

[28]For very high rates of turnover (such that incumbents become a very small fraction of the workforce) and for large negative shocks (such that wages are not very close together in the no-commitment solution), the latter *will* satisfy incentive compatibility. However, in our simulations with parameterizations as in Footnote 20, constant wage contracts remain optimal across negative shocks, where the worst shock is up to even 50% below the best shock, even when the turnover rate is as high as 80%. For lower turnover rates, the range of shocks where constant wages are optimal is still higher.

[29]The level of the wage floor will depend on the severity of the distribution where the constrained regime applies, as roughly speaking, the wage floor averages across the wages on the no-commitment supply curve in this region. Below, we proxy for this distribution with a linear function of forecast productivity that is conditional on being in the constrained region.

[30]There will now be a cost of deviating by announcing a lower state, given that the new hire wage will fall below the optimal level, so $w_{2,2}(x_3)$ can increase towards w_1 (recall that $w_{2,2} = w_1$ will improve profits). Hence, $w_{2,2}(x_3)$ will be set to exactly satisfy the incentive compatibility constraint subject to not exceeding w_1. Initially, this scenario is a comparison between a second-order cost and a first-order gain, so the increase in $w_{2,2}$ is itself second-order to avoid violating the incentive constraints.

[31]Using the same calibrations as in Footnote 20, we find that the standard deviation of unemployment in the rigid wage region is increased by approximately 60% relative to the no-commitment model. Once the spot wage rises above the wage floor, incumbent wages are also increasing in the shock, but by a smaller amount, as explained in the text; in the simulations, incumbent wages increase up to the point where the

When there is just one state in which wages exceed a wage floor, the latter logic also implies that the no-commitment quasi-supply curve now coincides with the commitment one for a range of wages below w_1, down to the "wage floor" $\overline{w}$ (in contrast to the symmetric information case).[32] Therefore, the region of "flexibility" for new hire wages extends further (i.e., wages are initially more flexible downwards, but then fully rigid) than in the symmetric information case. Consider point C in Figure 3: if there is a state with demand curve passing through this point, the fact that incentive compatibility lowers the incumbent wage even in such a state implies that the no-replacement constraint first binds only at lower levels of the new hire wage so that $w_{2,1}$ will be set at this level.

3.1.1 Employment-Contingent Contracts

The above concerns the case in which no variables that are observable to both parties can be contracted upon. While in a model such as this, which features a frictional labor market, it is plausible to suppose that it may be difficult to condition contracts on labor market variables such as wages offered by other firms, employment at the firm in which the worker is employed may be a variable that could be conditioned upon. Intuitively, in a low-productivity state, employment could be specified to be inefficiently low to discourage the firm from underreporting productivity in a better state to avail itself of lower wages, given that such inefficiency harms profits more in the better state. We next consider how matters change if employment contingent contracts are possible; for small variations in productivity, in fact, it does not affect the constant wage result.

Proposition 5 ***(Contractible employment levels)*** *In the no-commitment asymmetric information model* where period-2 employment is contractible *and with a single period-*2 *productivity state $\hat{x}$, suppose that for given parameter values, there is a unique equilibrium with no replacement and that the no-replacement constraint binds strictly. Then, in a perturbed version of this model where this state is replaced with two different equal probability states, $x' = \hat{x} - \varepsilon$ and $x'' = \hat{x} + \varepsilon$ (i.e., with expected value $\hat{x}$), and assuming the differentiability of equilibrium values,*[33] *equilibrium period-*2 *wages are approximately*

new hire wage is approximately 10% higher than w_1.

[32] If there are multiple states with wages above the wage floor, we can establish the following result (details available on request). For any equilibrium satisfying monotonicity in the sense that whenever $w_{2,2}(x) > w_{2,2}(x')$, $Z_2(x) \geq Z_2(x')$ and $w_{2,1}(x) \geq w_{2,1}(x')$, and also no replacement binds in x if and only if $w_{2,2}(x)$ is below some critical $w_{2,2}$ (which can be the empty set), then only downward incentive compatibility constraints can bind, and for all states x where no replacement is not binding, $w_{2,1}(x) \leq w^{**}(x)$. That is, new hire wages are no higher than the commitment level. Moreover, if only local downward constraints bind (as is true in our simulations) and $w_{2,2} < w_1$ for higher states (higher by $w_{2,2}$ ranking), it is a strict inequality: $w_{2,1}(x) < w^{**}(x)$. The intuition here is that cutting $w_{2,1}(x)$ a small amount below $w^{**}(x)$ imposes only a second-order cost in state x, but announcing x in a higher state will suffer a first-order cost by this change; this cut would relax the incentive compatibility constraint and permit a higher $w_{2,2}(x')$.

[33] That is, assuming that Z_2 is a differentiable function of ε in a neighbourhood of 0.

constant *across these states, provided that the perturbation* ε *is sufficiently small; formally,* $\lim_{\varepsilon \to 0_+} \left(w_{2,1}\left(x''\right) - w_{2,1}\left(x'\right) \right) / 2\varepsilon = 0$.

A rough intuition for this result is as follows: Given that for a small perturbation in both states x' and x'', the no-replacement constraint continues to bind, and wages for incumbents and new hires are equal. If in the lower-productivity state, wages are lower by more than a second-order amount, there will be, as earlier, a first-order incentive for the firm in x'' to announce x', as there is a benefit both in terms of lower wages for period-1 hires and in terms of reducing the hiring cost for new hires. To prevent this, hiring can be reduced in x', which would be costly in the state x'', but it must be reduced by a large amount, given that hiring is initially (in the unperturbed equilibrium) optimal; this cut in hiring will also impose *first-order* costs in x', swamping any benefit from the lower wages (which are second-order).

4 Testing the Model's Predictions

In this section, we present a variety of tests of the salient features of our model. Our focus is on the the new hire wage because it is allocational.

4.1 The Data

For our empirical exercises, we use the IAB Beschäftigten-Historik (BeH, version 10.01), the Employee History File of the Institute for Employment Research (IAB) of the German Federal Employment Agency. The BeH covers all workers who were at least once employed subject to social security in Germany since 1975.[34] Not covered are self-employed, civil servants (Beamte), family workers assisting in the operation of a family business, and regular students. The BeH includes roughly 80% of the German workforce. To protect data privacy, we are not allowed to work with the universality of the BeH. Therefore, we use a 20% random sample of all workers that worked full-time during at least one year since 1975.[35]

The BeH is organized by employment spells. A *spell* is a continuous period of employment within an establishment in a particular calendar year. Hence, the maximum spell length is 366 days. For each identified full-time worker, the BeH has a record of all existing employment spells — including part-time employment, apprenticeships, etc. For

[34] The BeH also covers marginal part-time workers employed since 1999.

[35] More precisely, we focus on "regular workers" according to the definition used in the Administrative Wage and Labor Market Flow Panel (AWFP) dataset (see Seth and Stüber, 2017): a regular worker is employed full time and belongs to person group 101 (employee s.t. social security without special features), 140 (seamen) or 143 (maritime pilots). Therefore, all (marginal) part-time employees, employees in partial retirement, interns, etc., are not considered regular workers.

our analyses, we restrict our attention to employment spells of full-time workers[36] aged 16 to 65 years from West Germany for the period from 1978 to 2014. We keep employment spells only if the workers are employed on December 31st of the respective year.[37]

We define a newly hired spell as a worker's first spell at the establishment.[38] Hence, a worker's tenure in an establishment that spans more than one calendar year will consist of multiple spells, with the first being classified as a new hire spell.

Our dependent variable is the real average daily wage of a worker over any spell. As the earnings data are right-censored at the contribution assessment ceiling ("Beitragsbemessungsgrenze"), only non-censored wage spells are considered in the analyses.[39] To calculate the average daily real wage and real output per capita in 2010 prices, we use the German Consumer Price Index (CPI, see Table A2).

As a proxy for aggregate productivity, we use West German GDP per capita. GDP data were obtained from the German Federal Statistical Office and the Federal Statistical Offices of the Federal States. In an initial test of downward rigidity, we also make use of the aggregate unemployment rate, which we obtained from the Federal Unemployment Agency (see Table A2).

The final dataset used in our analyses contains over 97.8 million employment spells for nearly 9 million workers working for more than 2.8 million establishments. The BeH contains an establishment identifier, but henceforth, we refer to establishments as "firms" in keeping with the phrasing used in the discussion of the theory.[40]

[36] The BeH documents only total spell earnings, not hours worked in that spell. We therefore consider only full-time workers, as these workers' hours are likely to be acyclical. In earlier work that is available upon request, we analyse the time series properties of an extraneous estimate of the average hours worked in a year by full-time employees in Germany. We find cyclicality — in the sense of having a significant correlation with output — to be relatively weak.

[37] This specification implies that we only ever have a maximum of one spell per worker per year, so when we compute yearly averages over spells, we do not more heavily weight those workers with multiple within-year spells. It also excludes most short-lived spells in the data, particularly temporary summer work.

[38] Re-hires are therefore not identified as new hires. Our decision to treat returning workers as incumbents is because of the relatively short time of absence; 70% of returners returned after an absence of less than one year, and returners' average length of time away is approximately 20 months. This suggests that these spells are for workers who have long-term relationships with the establishment and whose absences were temporary (for reasons such as paternity/maternity leave).

[39] We drop spells with wages ≥ 0.98 * the contribution assessment ceiling. Dropping top-coded spells leads to an under-representation of highly qualified workers, making the results somewhat less generalizable. Because the wages of highly qualified workers are less likely to be covered by a collective bargaining agreement (see, e.g., Düll, 2013) and because uncovered wages are more flexible than covered wages (see, e.g., Devereux and Hart, 2006), we likely slightly underestimate the wage cyclicality. For a quantitative evaluation of the effect of dropping censored spells, see, for example, Appendix A of Stüber and Beissinger (2012).

[40] The main results of this paper hinge on estimates that control for match fixed effects, with the underlying assumption being that matches are with establishments, not firms. However, even if matches are formed at the firm level, then using worker-establishment fixed effects will absorb them in any event; their use in this case may be inefficient but will not bias the estimated year effects.

4.2 Extracting Composition-Bias-Free Estimates of New Hire Wages

We wish to test the model's predictions concerning the cyclical behavior of new hire wages. To do this, one must extract estimates of these wages from the panel data, controlling for composition bias. Following Solon et al. (1994), this can be achieved with a two-step method. In the first stage, year effects are extracted from the panel using year dummies while controlling for worker-firm characteristics. In the second stage, the year effects are treated as composition-controlled estimates of the average new hire wage in each year. In the two-period asymmetric information model, new hires come from unemployment, not from other firms. Hence, the wage year effects that we would like to identify are those for new hires arriving directly from unemployment. We define these hires as workers who were without a job for over four weeks before arriving at the firm.

As noted above, it is important to control for as much worker-firm heterogeneity as possible, and a natural way to do so is to use worker-firm (match) fixed effects (MFE) as well as proxies for returns to tenure and experience. It is widely believed that match quality is procyclical (see the discussion in Gertler et al. (2016)), and failing to control for it may lead to misleading inferences in this respect (Gertler and Trigari, 2009b).

In the first stage, the primary specification to be estimated is the panel regression

$$w_{ijt} = m_{ijt} + \sum_{\tau=1}^{T} \beta_\tau^I I_t^\tau + \sum_{\tau=1}^{T} \beta_\tau^E E_t^\tau + \sum_{\tau=1}^{T} \beta_\tau^U U_t^\tau + \sum_{k=1}^{2} \lambda_k age_{it}^k + \sum_{k=1}^{4} \phi_k ten_{ijt}^k + v_{ijt}, \quad (8)$$

where w_{ijt} is the log of the real average daily wages of worker i in firm j during year t, and v_{ijt} is an error term assumed to be orthogonal to the regressors.

The equation allows for three distinct sets of year effects written in the first three summation terms. The first consists of the dummies I_t^τ ($\tau = 1, \ldots, 37$) with coefficients β_τ^I where I_t^τ equals one if $t = \tau$ and the worker is an incumbent, but is zero otherwise. The β^I coefficients are the incumbents' year effects. The second (third) set of dummies E_t^τ (U_t^τ) take the value of one if the wage is from an ee (ue) new hire and $t = \tau$, but is equal to zero otherwise. The β_t^E (β_t^U) are the corresponding year effects. The variable age_{it} is the worker's age in years, and ten_{ijt} is the worker's firm tenure measured in days at the end of the spell. Finally, m_{ijt} is an MFE. Note that this effect controls for (estimates) the sum of a firm j's effect plus a worker i's effect plus a match quality effect. While the use of MFEs is a general way to absorb heterogeneity in the panel, a drawback is that if new hire wages are excessively sensitive to the state of the cycle at entry and if (part of) this effect remains constant throughout the entire relationship with the firm, then it will be absorbed into the MFE and will not appear as "excess" new hire cyclicality; for example, if new hire effects are procyclical and permanent, they will be observationally equivalent to procyclical match quality effects. This is one of a number of problems that makes a rigorous test of equal treatment difficult and is one reason why we do not execute such tests in this paper.

4.3 Testing the Model

A key prediction common to all versions of our model is that new hire wage growth is relatively rigid in downturns but moves closely with productivity in upturns. To get an initial grasp on whether or not our data support this feature, we look at cross-sector new hire wage growth variance over time.

Suppose the German economy consists of several "sectors" – that is, smaller economies, each with its own distinct and separate labour market. Additionally, suppose that sectoral productivity reacts to aggregate productivity in a heterogenous fashion. In this world, we would expect the cross-sectional (across-sector) variance of new hire wage growth to be higher in upturns than in downturns.[41] We test this simple idea by disaggregating the single new hire (from unemployment) year dummies U_t^τ in (8) into 29 sectoral dummies, according to the classification defined in Appendix A1. Hence, we initially extract 29 time series of new hire wage year effects — one for each sector. We first-difference each sector year effect to obtain a new hire wage growth rate and compute a cross-sector standard deviation for each of the 37 available years, which we denote by σ_{wt}.[42] To proxy — somewhat crudely — cyclical movements in productivity, we use i) the de-meaned aggregate West German unemployment rate ($\widetilde{u}_t$) and ii) the growth rate of West German GDP per capita ($\widetilde{\Delta y_t}$). We refer to years when demeaned unemployment (GDP growth) is below (above) zero as "upswings" and, vice versa, as "downswings". Line 1 in Table 1 shows the results of regressing a) σ_{wt} on $\widetilde{u}_t$, b) σ_{wt} on $\widetilde{u}_t$ when $\widetilde{u}_t < 0$, c) σ_{wt} on $\widetilde{u}_t$ when $\widetilde{u}_t > 0$, and d) σ_{wt} on a dummy that is one in upswings and zero otherwise. Line 2 gives analogous results using GDP growth as a cyclical indicator in place of unemployment (with, of course, upswings/downswings defined here as periods of positive/negative growth).[43]

Regarding unemployment, the table shows that there is a clear-cut negative relationship between aggregate unemployment and cross-sectoral wage growth volatility, although as columns *b*) and *c*) show, this holds in upswings only. The final column shows that volatility is significantly higher in upswings — during these years, it almost doubles (the estimated intercept is around 0.04). Line 2 shows there is less co-movement with GDP

[41] Consider the case where, in the asymmetric information model, all sectors start in the same position. If aggregate productivity falls, then most sectors will be in a downturn and are likely to have wage growth (negative) at the wage floor, whereas if aggregate productivity rises, wage growth in a sector will depend on the realization of the sectoral productivity growth when it is positive.

[42] In the following, we assume the (small sample) measurement error arising from using estimated rather than actual variance is uncorrelated with the regressors. Estimates of annual new hire wage growth are of course derived from very large samples and raise no such issues.

[43] T-ratios are computed using robust standard errors.

Table 1: The Relationship Between New Hire Wage Growth Volatility and Unemployment and GDP Growth

Regression of σ_{wt} on	*a)*	*b)*	*c)*	*d)*
$\widetilde{u}_t$	-1.27	-2.05	-0.15	0.04
	(4.02)	(2.24)	(0.50)	(2.59)
$\widetilde{\Delta y_t}$	0.52	0.95	0.03	0.02
	(1.83)	(1.76)	(0.09)	(1.22)
Regression of Δw_t^n *on*				
$\Delta \widetilde{u}_t$	-0.87	-1.30	-0.61	.003
	(3.44)	(4.57)	(1.94)	(0.56)
$\widetilde{\Delta y_t}$	0.42	0.55	0.11	0.015
	(4.45)	(2.92)	(.81)	(3.68)

Note: T-ratios in brackets.

growth. There is a positively (borderline) significant relationship with wage growth volatility — a result confirmed by separating upswing and downswing years, as we do in columns *b)* and *c)*. However, column *d)* shows that although the volatility of wage growth is higher in upswings, this increase is not statistically significant. Notwithstanding the latter result, these findings offer some indicative support for downward real wage rigidity.

We may also use the aggregate series for new hire wages extracted from (8), denoted by w_t^n, to test certain aspects of the model. We begin by looking again at the broad question of whether or not new hire wages are relatively rigid in recessions and relatively flexible in booms. We repeat the regressions *a)* to *d)* in Table 1 above, but this time, with Δw_t^n as the regressand and with Δu_t replacing u_t as a regressor.[44] The results — in the lower part of the Table — are broadly indicative of downward real wage rigidity although the findings using GDP growth as a cyclical indicator are more definitive than those using unemployment.

Above, we have used observables to definitively indicate up and downswing years, but it would be interesting to see if the results hold up for a latent variable approach instead. We allow the mean of σ_{wt} (alternatively, Δw_t^n) to take two values according to whether a latent indicator variable is positive or negative, where this indicator variable is, in turn, linear in $\widetilde{u}_t$ (alternatively, $\widetilde{\Delta y_t}$). (Full details of this model are given below for a case that nests the current one). The results for these two cyclical indicators and for the two wage measures are in Table 2.

The table shows that the volatility and mean of wage growth are both higher in the upswing regime, regardless of which of the two cyclical variates is used to determine the

[44] It is more common in this context to regress wages on unemployment in levels rather than first differences. This step does not change the results qualitatively. However, we prefer the current specification over levels because the latter has a highly persistent error term that causes problems for inference.

Table 2: A Switching Regime Model for the Means of σ_{wt} and Δw_t^n.

Volatility Measure (Cyclical Indicator)	$\sigma_{wt}(\widetilde{u_t})$	$\sigma_{wt}(\widetilde{\Delta y_t})$	$\Delta w_t^n(\widetilde{u_t})$	$\Delta w_t^n(\widetilde{\Delta y_t})$
Upswing Mean	0.12	0.17	0.020	0.017
	(6.49)	(22.15)	(8.47)	(6.86)
Downswing Mean	0.03	0.04	−0.00	−0.01
	(8.68)	(8.15)	(1.23)	(2.42)

Note: T-ratios in brackets.

regime. Furthermore, in all cases, a bounds test[45] that these (four) differences are zero is roundly rejected. Finally, the two downswing estimates of wage growth are insignificant and negative, respectively, whilst their upswing counterparts are both significantly positive.

We now turn to a test of the more specific predictions of our theory. To this end, we focus on the predictions of the two-period asymmetric information model; this model has the most interesting and distinct implications for new hire wages. As before, we concentrate on new hire wages because they are allocational in our model. The model has two possible wage "regimes" under no replacement in its second period: a "spot rate" regime and a "constrained" regime. In the spot regime, the new hire wage is determined at the intersection of the labour demand curve and the commitment quasi-supply curve, as depicted, for example, by point D in Figure 3. However, if productivity falls in the second period to the extent that the no-replacement constraint binds, wages fall at a rate that is independent of the current state; under asymmetric information, forecasted productivity, not actual productivity, matters in this regime.[46] Our challenge here is to examine the extent to which these features are present in the time series observations on new hire wages that we obtained from the panel.

We begin with a simple model for the "spot" wage, which, as noted above, lies on the commitment quasi-supply curve in Figure 3. The first-order driving force in our model for any time t endogenous variable — including the spot wage — is the current state of productivity. Lagged productivities obviously also matter, but we take these as being of second-order importance. The theory implies that the spot wage is increasing in productivity. The simplest possible model for the log of the new hire (spot) wage (w_t^s) is therefore the constant elasticity specification

$$w_t^s \approx \alpha + \gamma x_t$$

[45] The variance of the difference between two random variables $a-b$ (say) cannot exceed $var(a)+var(b)+2var(a)var(b)$. We use this upper bound to compute the smallest possible t-test of the difference between up and downswing means.

[46] The constrained regime is also one of equal treatment — as noted already in the text, this prediction is difficult to test rigorously, and we do not address it in this paper.

where x_t is the log of productivity. Clearly, w_t^s is a latent variable; in the spot regime, it is equal to the (log of the) new hire wage w_t^n, but in the constrained regime, it lies below it.

It is not straightforward to use the two-period model with time series data. However, given the previous discussion, a reasonable interpretation of the model's period-two wage might be

$$w_t^n = \alpha + \gamma x_t \quad if \quad \alpha + \gamma x_t > w_{t-1}^n \tag{9}$$

$$\Delta w_t^n = -\theta(w_{t-1}^n - \alpha - \gamma \widehat{x_t}) \qquad if \qquad \alpha + \gamma x_t < w_{t-1}^n \tag{10}$$

where $\widehat{x_t}$ is the forecast of productivity conditional on being in the constrained regime.[47]

In period 1, the model is — by assumption — in the spot regime. If this regime also holds in period 2, then wage growth will be proportional to productivity growth, Δx_t.[48] If, by contrast, wages are constrained in period 2, then wage growth is negative and proportional to $w_{t-1}^n - \alpha - \gamma \widehat{x_t}$.

In general, there is no way of knowing a priori which regime exists in any particular year.[49] Therefore, we use the following latent variable switching model. Let I_t be an indicator defined on the real line that takes positive (negative) values when we are in the spot (constrained) regime. The model is

$$I_t = const + \delta(\gamma x_t - w_{t-1}^n) + u_t \tag{11}$$

$$\Delta w_t^n = const + \beta^u \Delta x_t + \theta^u(\gamma \widehat{x_t} - w_{t-1}^n) + v_t \quad \text{if} \quad I_t > 0 \tag{12}$$

$$\Delta w_t^n = const + \beta^d \Delta x_t + \theta^d(\gamma \widehat{x_t} - w_{t-1}^n) + v_t \quad \text{if} \quad I_t < 0 \tag{13}$$

$$u_t \sim IIDN(0,1), v_t \sim IIDN(0, \sigma_v^2) \tag{14}$$

If the two-period model with asymmetric information is true, then we have $i)$ $\beta^d = 0$,

[47] See Footnote 29.

[48] In a multi-period setting, we may also move from a constrained regime to a spot regime in consecutive observations. If such occurrences are frequent, there will be downward bias in the estimated spot regime productivity elasticity.

[49] Given that when new hire wages rise, we know we must be in the unconstrained regime, it would be tempting to use this fact to split the sample to run separate regressions. However, this approach would be a mistake for two reasons. First, years in which new hire wages fall are not necessarily constrained years (if the fall is relatively modest). Second, using information on a regressand to preselect a regression subsample is well known to lead to biased estimates (upwards in this case). We can, of course, preselect the sample for regression purposes using the value of (presumed exogenous) regressors — an exercise we describe in the text below — but this approach does not achieve an exact split into constrained and unconstrained years.

$ii)\ \beta^u = \gamma, iii)\ 1 > \theta^d > 0,\ \ iv)\ \ \theta^u = 0.$[50]

We do not have a measure of West German productivity (TFP), so in the following, we proxy for it using West German GDP per capita and henceforth refer to x_t as simply "output".

There are problems implementing and interpreting (11) to (14). The assumptions behind the model (particularly the i.i.d. normality of errors) are quite strong. The model requires a proxy for forecasted output $\widehat{x_t}$. Our output and wage series are $I(1)$ and are not cointegrated, so we cannot use their levels on the RHS of (11) or in our estimate of $\widehat{x_t}$. Finally, there are eight parameters (including two intercepts that allow for differential long-run wage growth in upswings and downswings), which "stretches" the information in our 38 data points somewhat.

To help ameliorate the scarcity of data points, we calibrate γ using values from simulations to save having to estimate it. The elasticity of the spot wage with respect to model productivity from these simulations generally lays in the region of 0.6 to 1.0 for a wide array of parameter values and productivity processes. We therefore set γ to 0.8 and subsequently check how sensitive the results are to changing that number by $+/-$ 0.2 (i.e., to $\gamma = 0.6$ and $\gamma = 1$). We should also note at this point that the scarcity of observations is a pervasive problem in business cycle analyses and is not limited to this study.

The lack of cointegration between wages and output is hardly surprising; it is unlikely that there is a single common stochastic trend driving both the wage series we have extracted from a sample of workers and economy-wide output. Nonetheless, it does mean that even if we know the value of γ, we cannot use the levels of wages and output in the model without first rendering them stationary. To obtain stationary measures of the cyclical component of these variables, we follow standard macroeconomic procedures and use HP filtered (log) output and (log) wages.

Whilst we cannot test the i.i.d. assumptions behind the model, we can test for the normality it requires. Applying Shapiro-Wilk tests for the null of normality to wage growth, output growth and actual and forecasted wage pressure gives p-values of 0.84, 0.32, 0.33 and 0.58, respectively.

To obtain an estimate of forecasted output, we use the fitted value from an AR(2) model for (HP filtered) log output.[51] The regression coefficients on the two lags are almost exactly of equal and opposite signs (p-value for this test is 0.85), and imposing this equality as a constraint gives

[50] $1 > \theta^d$, as the slope of the quasi-supply curve is less than that of the commitment curve in this region.

[51] Strictly speaking, we should use only data points from the constrained regime, and obviously, in the current setup, we cannot do this. However, the AR coefficients are stable over sub-periods. Furthermore, in an additional exercise, we used the latent variable model's fitted probabilities to determine the most likely regime in each year and split the sample accordingly. The estimated AR process using only the data points classified as being constrained was practically identical to that from the full sample.

$$x_t^h = const + 0.43 \Delta x_{t-1}^h + \varepsilon_t \tag{15}$$

$$(4.55) \tag{16}$$

where here and henceforth, superscript h denotes an HP filtered quantity.

In addition to estimating the full model, we also estimate a restricted version in which we ignore the impact of forecasted output in downturns by setting the $\theta's$ to zero. This exercise is similar to that conducted in Table 1 in the sense we regress wage growth on a cyclical indicator (this time output growth), with the crucial difference being that the two regimes are endogenously determined via our latent variable model. The models are estimated by ML, and the results for the restricted version are presented in Table 3.

Table 3: ML Estimates of the Model

	β^u	β^d	θ^u	θ^d	t_δ
Coefficient	.50	.17	–	–	6.26
	(8.22)	(2.49)			
Coefficient	.47	.12	.17	.42	4.62
	(6.39)	1.68	(1.21)	(2.66)	

Note: T-ratios in brackets.

The values of t_δ in the table indicate that delta is significant in both restricted and unrestricted models. This, in turn, implies that our cyclical indicator $\gamma\widehat{x_t} - w_{t-1}^n$ is a significant determinant of the current state (i.e., an upswing or a downswing). The restricted model results in the first line of the table, reinforcing the findings above, namely, that in a downswing, wage growth is poorly related to output growth, whilst in an upswing, it is closely related to it.

The lower part of Table 3 presents the results for the model including forecasted output $\widehat{x_t^h}$, offering some support for the model's predictions. In particular, θ^u is insignificant, θ^d is significant, β^u is positive and highly significant (albeit smaller than γ) and β^d is small in magnitude and of borderline significance. However, these inferences must be treated with caution not just because of the restrictive assumptions of the model and the small sample but also because $\widehat{x_t^h}$ is a generated regressor. This issue does not occur when testing the null of $\theta^u = 0$ or $\theta^d = 0$ (see Pagan (1984)) but may appear for inferences about other (non-generated) regressors. Generally, however, the finding in the generated regressor literature is that the latter problem is insubstantial. Nonetheless, we carried out an alternative exercise to check the impact of generated regressors. We re-estimated the unrestricted model while allowing $\widehat{x_t^h}$ and $\widehat{w_t^h}$ to enter with separate parameters. The estimates of these two terms i) were jointly significant (insignificant) in down (upswings),

ii) were "correctly" signed, and iii) passed an LR test of the theory's restriction that their ratio should equal $\gamma\hat{\rho}$, where $\hat{\rho}$ is the estimate of the coefficient in (15). Modulo the degrees of freedom issue, this exercise provides some reassurance that our inferences are not likely to be affected by the generated regressor problem.

A further concern is the low degrees of freedom caused by the endogenous switching process. To partially address this issue, we carried out a further sensitivity analysis; in the same vein as in the early part of this section, we split the sample a priori into constrained/unconstrained data points according to whether or not HP filtered output was below/above zero. The reasoning here is that whilst this split is "noisy", it at least achieves parsimony with respect to degrees of freedom. The estimates (available on request) were very close to their latent variable counterparts, and despite higher standard errors, the test results were the same.

Finally, we note that the results for both restricted and unrestricted models (again available on request) are qualitatively unchanged when we decrease/increase β to 0.6/1.0.[52]

5 Concluding Comments

We have considered a simple frictional model of the labor market, in which a desire to avoid job insecurity through replacement by cheaper new hires leads to a degree of downward wage rigidity for new hires. A key feature of our model is that the new hire wages are allocational. The rigidity arises from worker risk aversion and a desire to limit temporal wage variation for incumbent workers, which also transmits to new hires in downturns. That is, to save on the ex ante wage bill, it may be optimal to avoid worker replacement, which leads to a wage for new hires that is as unresponsive to negative shocks, as is the incumbent wage. This magnifies the response of unemployment and vacancies to negative shocks. We further show that the interplay with asymmetric information can substantially enhance downward wage rigidity and increase the responsiveness of unemployment and vacancies to productivity shocks. We argue that downward, but not upward, real wage rigidity for new hires is apparent in the German BeH panel dataset and that our model is broadly consistent with these empirical findings.

[52]Of course, the model also requires that $\beta^u = \gamma$, so this lack of sensitivity is not all good news for its predictions. However, the maximized likelihood was relatively flat over the relevant ranges, suggesting that β was poorly identified.

References

Acemoglu, D. and Shimer, R. (1999). Efficient unemployment insurance. *Journal of Political Economy*, 107(5):893–928.

Basu, S. and House, C. L. (2016). Allocative and remitted wages: New facts and challenges for keynesian models. In Taylor, J. B. and Uhlig, H., editors, *Handbook of Macroeconomics, Volume 2A*, pages 297–354. Elsevier.

Bewley, T. F. (1999). *Why Wages Don't Fall During a Recession.* Harvard University Press, Harvard.

Bruegemann, B. and Moscarini, G. (2010). Rent Rigidity, Asymmetric Information, and Volatility Bounds in Labor. *Review of Economic Dynamics*, 13(3):575–596.

Chari, V. V. (1983). Involuntary unemployment and implicit contracts. *Quarterly Journal of Economics*, 98:107–122.

Costain, J. S. and Reiter, M. (2008). Business cycles, unemployment insurance, and the calibration of matching models. *Journal of Economic Dynamics and Control*, 32(4):1120–1155.

Devereux, P. J. and Hart, R. A. (2006). Real wage cyclicality of job stayers, within-company job movers, and between-company job-movers. *Industrial and Labor Relations Review*, 60(1):105–119.

Dickens, W. T., Goette, L., Groshen, E. L., Holden, S., Messina, J., Schweitzer, M. E., Turunen, J., and Ward, M. E. (2007). How wages change: Micro evidence from the international wage flexibility project. *Journal of Economic Perspectives*, 21(2):195–214.

Düll, N. (2013). Collective wage agreement and minimum wage in Germany. mimeo. Ad hoc request of the European Employment Observatory.

Galí, J. (2013). Notes For A New Guide To Keynes (I): Wages, Aggregate Demand, And Employment. *Journal of the European Economic Association*, 11(5):973–1003.

Galuscak, K., Keeney, M., Nicolitsas, D., Smets, F., Strzelecki, P., and Vodopivec, M. (2012). The determination of wages of newly hired employees: Survey evidence on internal versus external factors. *Labour Economics*, 19(5):802 – 812.

Gertler, M., Huckfeldt, C., and Trigari, A. (2015). Unemployment fluctuations, match quality,and the wage cyclicality of new hires. Technical report, Cornell University. mimeo, Cornell University.

Gertler, M., Huckfeldt, C., and Trigari, A. (2016). Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires. NBER Working Papers 22341, National Bureau of Economic Research, Inc.

Gertler, M. and Trigari, A. (2009a). Unemployment fluctuations with staggered nash wage bargaining. *Journal of Political Economy*, 117(1):38–86.

Gertler, M. and Trigari, A. (2009b). Unemployment fluctuations with staggered nash wage bargaining. *The Journal of Political Economy*, 117(1):38–86.

Green, J. and Kahn, C. M. (1983). Wage employment contracts. *Quarterly Journal of Economics*, 98(Supplement):173–187.

Grossman, S. J. and Hart, O. D. (1981). Implicit contracts, moral hazard, and unemployment. *American Economic Review*, 71:301–307.

Guerrieri, V. (2007). Heterogeneity, job creation and unemployment volatility. *The Scandinavian Journal of Economics*, 109(4):667–693.

Hagedorn, M. and Manovskii, I. (2008). The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *American Economic Review*, 98(4):1692–1706.

Kennan, J. (2010). Private Information, Wage Bargaining and Employment Fluctuations. *Review of Economic Studies*, 77(2):633–664.

Kudlyak, M. (2014). The cyclicality of the user cost of labor. *Journal of Monetary Economics*, 68:53–67.

Menzio, G. (2005). High frequency wage rigidity. Manuscript, Univeristy of Pennsylvania.

Menzio, G. and Moen, E. R. (2010). Worker replacement. *Journal of Monetary Economics*, 57(6):623–636.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of Political Economy*, 105(2):385–411.

Moen, E. R. and Rosen, A. (2011). Incentives in competitive search equilibrium. *Review of Economic Studies*, 78(2):733–761.

Pagan, A. (1984). Econometric Issues in the Analysis of Regressions with Generated Regressors. *International Economic Review*, 25(1):221–247.

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, 77(5):1339–1369.

Rudanko, L. (2009). Labor market dynamics under long-term wage contracting. *Journal of Monetary Economics*, 56(2):170–183.

Seth, S. and Stüber, H. (2017). Administrative wage and labor market flow panel (*AWFP*) 1975–2014. *FAU Discussion Papers in Economics*, 01/2017.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Snell, A., Stüber, H., and Thomas, J. P. (2018). Downward real wage rigidity and equal treatment wage contracts: Theory and evidence. *Review of Economic Dynamics*, 30:265 – 284.

Snell, A. and Thomas, J. P. (2010). Labor contracts, equal treatment and wage-unemployment dynamics. *American Economic Journal: Macroeconomics*, 2(3):98–127.

Solon, G., Barsky, R., and Parker, J. A. (1994). Measuring the cyclicality of real wages: How important is composition bias? *The Quarterly Journal of Economics*, 109(1):1–25.

Stiglitz, J. E. (1986). Theories of wage rigidity. In Butkiewicz, J. L., Koford, K. J., and Miller, J. B., editors, *Keynes' economic legacy: Contemporary economic theories*, pages 153–206. Praeger, New York.

Stüber, H. and Beissinger, T. (2012). Does downward nominal wage rigidity dampen wage increases? *European Economic Review*, 56(4):870–887.

Thomas, J. P. (2005). Fair pay and a wage-bill argument for low real wage cyclicality and excessive employment variability. *Economic Journal*, 115(506):833–859.

A Proofs

A.1 Proof of Proposition 1

Proof. We derive the necessary conditions by considering the following Lagrangian, assuming that there is an interior solution and that there is no replacement in state x. We give the appropriate expression if there is no replacement in period 2 in *any* state; otherwise, an analogous argument applies (if there is replacement in some state $x' \neq x$, it modifies the expectation term in (17) and (20), but they cancel one another out).

$$\mathcal{L} = \left(f\left(\tilde{q}_1\left(V_1\right)\overline{n}_1\right) - w_1\tilde{q}_1\left(V_1\right)\overline{n}_1 - k\overline{n}_1\right) + E_{x'}\left[\left(f\left((1-\delta)\tilde{q}_1\left(V_1\right)\overline{n}_1 + \tilde{q}\left(w_{2,1}, x'\right)\overline{n}_2; x'\right) - w_{2,2}(1-\delta)\tilde{q}_1\left(V_1\right)\overline{n}_1\right.\right.$$

where $\tilde{q}_1(V_1)$ is defined analogously to $\tilde{q}(w_{2,1}, x)$, $\lambda_{x'}$ is the multiplier on the $w_{2,1} \geq w_{2,2}$ constraint in state x' and recall $V_1 = v(w_1) + E[\delta Z_2(x') + (1-\delta)v(w_{2,2}(x'))]$. This expression leads to the FOCs:

$$\tilde{q}_1' v'(w_1)\overline{n}_1 (f'(n_1) - w_1 + E_{x'}[f'(n; x')(1-\delta) - w_{2,2}(x')(1-\delta)]) - \tilde{q}_1(V_1)\overline{n}_1 = 0 \quad (17)$$

$$f'(n; x)\tilde{q}(w_{2,1}, x) - w_{2,1}\tilde{q}(w_{2,1}, x) - k = 0 \quad (18)$$

$$f'(n; x)\tilde{q}'\overline{n}_2 - \tilde{q}(w_{2,1}, x)\overline{n}_2 - w_{2,1}\tilde{q}'\overline{n}_2 + \lambda_x = 0 \quad (19)$$

$$\begin{aligned} &\tilde{q}_1' v'(w_{2,2}(x))(1-\delta)\overline{n}_1 (f'(n_1) - w_1 + \\ &E_{x'}[f'(n; x')(1-\delta) - w_{2,2}(x')(1-\delta)]) - \lambda_x - (1-\delta)\tilde{q}_1(V_1)\overline{n}_1 = 0 \end{aligned} \quad (20)$$

together with the complementary slackness conditions. Note that (18) implies (6) in the text.

From (17) and (20),

$$\frac{v'(w_1)}{v'(w_{2,2})}\left(q_1 + \frac{\lambda_x}{\overline{n}_1(1-\delta)}\right) = q_1. \quad (21)$$

Using this to eliminate λ_x in (19):

$$f'(n; x)\tilde{q}'\overline{n}_2 - \tilde{q}(w_{2,1}, x)\overline{n}_2 - w_{2,1}\tilde{q}'\overline{n}_2 + q_1\overline{n}_1(1-\delta)\left(\frac{v'(w_{2,2})}{v'(w_1)} - 1\right) = 0. \quad (22)$$

There are two cases.

A. If $\lambda_x = 0$, then (21) $w_1 = w_{2,2}$, and (22) implies (5) in the text, and hence, we get (7). We characterize points that satisfy (7). For clarity, we let $\tilde{w}_{2,1}$ and $\tilde{\theta}_2$ denote the individual firm's values. Then

$$\tilde{q}' = \frac{dq}{d\theta_2}\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,1}}\,|_{Z_2 \text{ constant}}\,.$$

From (2),

$$\frac{d\tilde{\theta}_2}{d\tilde{w}_{2,1}}\,|_{Z_2 \text{ constant}} = -\frac{pv'(w_{2,1})}{\frac{dp}{d\theta_2}\left(v\left(w_{2,1}\right)-v\left(b\right)\right)},$$

and differentiating $q = p \cdot \theta_2$ to eliminate $\frac{dp}{d\theta_2}$, we obtain

$$\tilde{q}' = -\frac{dq}{d\theta_2}\frac{p\theta_2 v'(w_{2,1})}{\left(\frac{dq}{d\theta_2}-p\right)\left(v\left(w_{2,1}\right)-v\left(b\right)\right)}. \tag{23}$$

After rearrangement,

$$\frac{q^2}{\tilde{q}'} = q^2\frac{\left(1-\frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)}{\theta_2\frac{dq}{d\theta_2}}\frac{v\left(w_{2,1}\right)-v\left(b\right)}{v'\left(w_{2,1}\right)}.$$

From our assumption on q, q^2 is increasing in θ_2, and the second term in the product is also increasing in θ_2 by assumption (it is the inverse of $q\left(\theta\right)\epsilon_q\left(\theta\right)/\left(1-\epsilon_q\left(\theta\right)\right)$) while the final term is increasing in $w_{2,1}$. Thus, the locus of values of θ_2 and $w_{2,1}$ such that (7) holds is negatively sloped. Recall that $n_2 = p\left(\theta_2\right)S_2$, and as $p' < 0$, there is a one-to-one negative relationship between n_2 and θ_2. Therefore, (7) can be solved to give a positively sloped locus of values for n_2 and $w_{2,1}$ that is compatible with equilibrium.

Next, (18) is negatively sloped in n_2–$w_{2,1}$ space by $f'' < 0$ and $q\left(\theta_2\right) = q\left(p^{-1}\left(n_2/S_2\right)\right)$, $q' > 0$, $p' < 0$. Therefore, $\left(w_{2,1}, n_2\right)$ is at the unique intersection point, denoted by $\left(w_{2,1}^C\left(x; w_1, n_1\right), n_2^C\left(x; w_1, n_1\right)\right)$ in the text. Since $w_{2,1} \geq w_1$ implies $\lambda_x = 0$ (see next line), claim (b) is established.

B. If $\lambda_x > 0$, then $w_{2,2} = w_{2,1}$ and from (21) $w_1 > w_{2,2} = w_{2,1}$, and (22) implies

$$(1-\delta)n_1 - \left(f'\left(n;x\right)\tilde{q}'\overline{n}_2 - w_{2,1}\tilde{q}'\overline{n}_2 - q\overline{n}_2\right) = n_1\left(1/v'\left(w_1\right)\right)\left(\left(1-\delta\right)v'\left(w_{2,1}\right)\right). \tag{24}$$

(This equation also follows from differentiating (3) with respect to $w_{2,1}$ after setting $w_{2,1} = w_{2,2}$.) Thus, eliminating f' using (18), and using $n_2 = q\overline{n}_2$,

$$1 + \frac{\left(1-k\tilde{q}'/q^2\right)n_2}{n_1\left(1-\delta\right)} = \frac{v'\left(w_{2,1}\right)}{v'\left(w_1\right)}, \tag{25}$$

so that as $w_{2,1} < w_1$, $k\tilde{q}'/q^2 < 1$, i.e., $k < q^2/\tilde{q}'$. Holding n_2 (and hence θ_2) constant, $q^2/\tilde{q}'$ is increasing in $w_{2,1}$, so the locus of points $\left(n_2, w_{2,1}\right)$ satisfying (25) must lie above — $w_{2,1}$ is higher — that defined by (7). At $w_{2,1} = w_1$ we have $k\tilde{q}'/q^2 = 1$, so the two loci coincide. Thus, the downward sloping (18) must intersect (25) at a higher wage and a lower value for n_2 than it would intersect (7). Thus, claim (a) is established.

Since $\lambda_x > 0$ if and only if $w_{2,1} < w_1$, the final claim of the proposition follows. ■

A.2 Proof of Proposition 2

Proof. If replacement occurs, as in Section 2.1, the firm must locally maximize profits plus weighted incumbent utility:

$$f\left((1-\delta)n_1 + n_2; x\right) - w_{2,2}(1-\delta)(1-q)n_1 - w_{2,1}\left(q(1-\delta)n_1 + n_2\right) - k\overline{n}_2 \\ + n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\left(1-q\right)v\left(w_{2,2}\right) + \delta Z_2 + (1-\delta)\,qv\left(b\right)\right),$$

where $\overline{n}_2$ is again the number of *new* jobs created, and $n_2 = q\left(\theta\left(w_{2,1}, Z_2\left(x\right)\right)\right)\overline{n}_2$. This situation differs from (3) in that the probability of replacement q is accounted for in the composition of period-2 workers and workers' period-1 utility. Then, differentiating with respect to $w_{2,2}$,

$$(1-\delta)(1-q)n_1 = n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)\left(1-q\right)v'\left(w_{2,2}\right)\right),$$

so that $w_1 = w_{2,2}$, as expected. Differentiating with respect to $w_{2,1}$, we obtain

$$f'\left(n; x\right)\tilde{q}'\overline{n} + (1-\delta)n_1(w_{2,2} - w_{2,1})\tilde{q}' - w_{2,1}\tilde{q}'\overline{n}_2 - q\left((1-\delta)n_1 + \overline{n}_2\right) + \\ n_1\left(1/v'\left(w_1\right)\right)(1-\delta)\left(q'\right)\left(v\left(b\right) - v\left(w_{2,2}\right)\right) = 0$$

where the latter term is the extra cost of compensating period-1 hires for their increased likelihood of replacement (defining $\tilde{q}'$ as before). Differentiating with respect to $\overline{n}_2$,

$$f'\left(n; x\right)q = w_{2,1}q + k. \tag{26}$$

Thus, employment is on the labour demand curve, as in (6). We can combine these latter two equations to obtain

$$\left(k/q\right)\tilde{q}'\overline{n}_2 + (1-\delta)n_1\tilde{q}'\left((w_{2,2} - w_{2,1}) + \left(1/v'\left(w_1\right)\right)\left(v\left(b\right) - v\left(w_{2,2}\right)\right)\right) = \\ q\left((1-\delta)n_1 + \overline{n}_2\right)$$

or

$$k\tilde{q}'/q^2 = 1 + (1-\delta)n_1\tilde{q}'\left((w_{2,1} - w_{2,2}) + \left(1/v'\left(w_1\right)\right)\left(v\left(w_{2,2}\right) - v\left(b\right)\right)\right)/q\overline{n}_2 + \\ (1-\delta)n_1/\overline{n}_2 \tag{27}$$

Both the second and third terms on the RHS of (27) are positive, the second as v is concave, $w_{2,2} = w_1$ from the above, $w_{2,2} > w_{2,1}$ (as replacement occurs) and $b \leq w_{2,1}$. Recall from the proof of Proposition 1 that $\tilde{q}'/q^2$ is decreasing in θ and $w_{2,1}$.Thus, in comparison to the commitment quasi-supply given by (7), at fixed θ, or equivalently fixed n_2 given $n_2 = p(\theta_2) S_2$ as in Figure 2, $w_{2,1}$ must be lower to satisfy (27). Thus, the intersection with the downward sloping (6) must occur at a lower wage and higher employment than in the commitment solution.

Finally, $w^C_{2,1}(x; w_1, n_1) < w_1$ because otherwise, the commitment solution could be implemented, which would be superior. ■

A.3 Proof of Proposition 4

Proof. (i) In the commitment model, consider an equilibrium in the absence of asymmetric information. We have that $w_{2,2}$ is independent of the period-2 state x, and $w_{2,1}$ is chosen independently of $w_{2,2}$ to minimize the cost of hiring a new worker in state x. With asymmetric information, the firm has no incentive to misreport since the wage paid to non-separated period 1 hires is constant, while any different $w_{2,1}$ can only increase new hire costs. The result follows.

(ii) Let $x' := \hat{x} - \varepsilon$, $x'' := \hat{x} + \varepsilon$. Consider an arbitrary sequence $\{\varepsilon_s\}_{s=0,1,\ldots}$, $\varepsilon_s > 0$, $\varepsilon_s \to 0$; we show that there is some $\bar{s}$ such that for $s \geq \bar{s}$, wages are equal in both states: $w_{2,2}(x') = w_{2,2}(x'') = w_{2,1}(x') = w_{2,1}(x'')$.[53] By the assumptions of continuity and the binding no-replacement constraint at $\hat{x}$,

$$\lim_{s\to\infty} w_{2,2}\left(x'\right) = \lim_{s\to\infty} w_{2,2}\left(x''\right) = \lim_{s\to\infty} w_{2,1}\left(x'\right) = \lim_{s\to\infty} w_{2,1}\left(x''\right) = \hat{w}_{2,2} = \hat{w}_{2,1}, \tag{28}$$

where the original equilibrium corresponding to $\hat{x}$ is denoted by $\hat{}$. In what follows, we will deal with the case where $w_{2,2}(x') \leq w_{2,2}(x'')$ infinitely often as $s = 0, 1, \ldots$, so we consider below the circumstances in which this is true; the arguments apply equally to the opposite case. To consider this case, we define

$$C\left(w_{2,1}, x''\right) := \left(k/q\left(\theta_2(w_{2,1}, Z_2\left(x''\right))\right) + w_{2,1}\right)$$

and

$$w^{**}\left(x''\right) \in \arg\min_{w_{2,1}} \left(k/q\left(\theta_2(w_{2,1}, Z_2\left(x''\right))\right) + w_{2,1}\right) \tag{29}$$

where $\theta_2(w_{2,1}, Z_2(x''))$ is as defined in (2); $C(w_{2,1}, x'')$ is the cost per period-2 hire in state x'' ($k/q + w$ is the total cost of a new hire), while $w^{**}(x'')$ is the wage that minimizes this cost. It is independent of the number of hires, and the cost is strictly convex in $w_{2,1}$ (hence, $w^{**}(x'')$ is unique).

[53]The dependence of values on ε_s will mostly be left implicit to avoid the notation becoming more cluttered.

To see this, as earlier, write $q\left(\theta_2(w_{2,1}, Z_2\left(x''\right))\right) \equiv \tilde{q}\left(w_{2,1}, x''\right)$, so

$$\begin{aligned} \frac{dC\left(w_{2,1}, x''\right)}{dw_{2,1}} &= -\frac{k\tilde{q}'}{\tilde{q}^2} + 1 \qquad (30) \\ &= -\frac{k}{\tilde{q}^2} \frac{\theta_2 \frac{dq}{d\theta_2}}{\left(1 - \frac{\theta_2}{q}\frac{dq}{d\theta_2}\right)} \frac{v'\left(w_{2,1}\right)}{v\left(w_{2,1}\right) - v\left(b\right)} + 1, \end{aligned}$$

using (23). Given that $\tilde{q}' > 0$ (a higher wage increases the job-filling rate), the second term in the product is $q\left(\theta_2\right) \epsilon_q\left(\theta_2\right) / \left(1 - \epsilon_q\left(\theta_2\right)\right)$ and therefore is decreasing in θ_2 (by assumption) and, hence, also decreasing in $w_{2,1}$, while the final term in the product is also decreasing in $w_{2,1}$, we have

$$\frac{d^2C\left(w_{2,1}, x''\right)}{dw_{2,1}^2} > 0. \qquad (31)$$

Additionally, given the assumption that the no-replacement constraint is strictly binding initially, we have $\hat{w}_{2,1} > w^{**} := w^{**}\left(x\right)$ (the value for $w^{**}\left(x''\right)$ when $\varepsilon = 0$, being equal to the optimal hiring wage in the unperturbed model), and therefore, by (28) and the continuity of $w^{**}\left(x'\right)$ and $w^{**}\left(x''\right)$ in ε (by the Theorem of the Maximum, as they are both unique by the strict convexity of C and C is continuous in Z and, hence, in ε),

$$\lim_{s\to\infty} w^{**}\left(x'\right) = \lim_{s\to\infty} w^{**}\left(x''\right) = w^{**} < \hat{w}_{2,1}. \qquad (32)$$

Profits in period 2, in state x'', are

$$\max_{n_2} \left(f\left((1-\delta)n_1 + n_2; x''\right) - w_{2,2}(x'')(1-\delta)n_1 - C\left(w_{2,1}(x''), x''\right) n_2\right).$$

In state x'', the firm can claim that x' occurred and make nonnegative savings in wages paid to incumbents because $w_{2,2}\left(x'\right) \leq w_{2,2}\left(x''\right)$. It follows that we must have

$$C\left(w_{2,1}\left(x''\right), x''\right) \leq C\left(w_{2,1}\left(x'\right), x''\right) \qquad (33)$$

since otherwise, by announcing x', hiring costs are reduced as well.

There are three possibilities to consider, and at least one of which must occur infinitely often along the sequence $s = 0, 1, \ldots$. First, $w_{2,1}\left(x'\right) < w_{2,1}\left(x''\right)$. From (33), $w_{2,1}\left(x'\right) < w^{**}\left(x''\right)$ by (31). But as $s \to \infty$, a contradiction occurs in view of $\lim_{s\to\infty} w_{2,1}\left(x'\right) = \hat{w}_{2,1}$ and (32).

On the other hand, if $w_{2,1}\left(x'\right) > w_{2,1}\left(x''\right)$, then by (33) and (31), $w_{2,1}\left(x'\right) > w^{**}\left(x''\right)$. However, we have

$$w_{2,1}\left(x'\right) > w_{2,1}\left(x''\right) \geq w_{2,2}\left(x''\right) \geq w_{2,2}\left(x'\right),$$

where the second inequality follows from no replacement and the final inequality by hypothesis. However, consider a change where $w_{2,1}(x')$ is cut to $w_{2,1}(x'')$ and $w_{2,2}(x')$ is increased to $w_{2,2}(x'')$ if it is initially below this value. This changed contract satisfies no replacement and (trivially) incentive compatibility. The decrease in $w_{2,1}(x')$ reduces hiring costs by (28) and (32), which imply $w_{2,1}(x') > w^{**}(x')$ for a large s. Additionally, for s large enough, $w_{2,2}(x'') < w_1(\varepsilon_s)$ by the binding no-replacement constraint in Problem A (from Proposition 1, this implies $\hat{w}_{22} < \hat{w}_1$), (28) and, by assumption, $\lim_{s\to\infty} w_1(\varepsilon_s) = \hat{w}_1$ using an obvious notation. Then, $v'' < 0$ implies that a small reduction in w_1 to leave V_1 constant will reduce expected wages while leaving hiring constant. Therefore, for a large enough s, the contract is not optimal, contrary to the assumption. The final possibility has $w_{2,1}(x') = w_{2,1}(x'')$. By no replacement, then,

$$w_{2,1}\left(x'\right) = w_{2,1}\left(x''\right) > w_{2,2}\left(x''\right) = w_{2,2}\left(x'\right),$$

where the final equality follows by incentive compatibility (otherwise, x' would be announced because incumbent wages would be lower), and the inequality is strict by the assumption that it not a constant wage contract. Similar to the previous case, both $w_{2,2}(x'')$ and $w_{2,2}(x')$ can be increased by the same small amount without violating incentive compatibility or no replacement, which is compensated by a small reduction in $w_1(\varepsilon_s)$, reducing expected wages paid to period-1 hires. Thus, again, the equilibrium contract is not optimal, contrary to assumption.

(iii) Period-2 profits from the contract for state x in state x' can be written as

$$\pi\left(x, x'\right) := \max_{n_2}\{f\left((1-\delta)n_1 + n_2; x'\right) - w_{2,2}(x)(1-\delta)n_1 - C\left(w_{2,1}(x), x'\right)n_2\}.$$

We proceed in a number of steps. (a) Suppose that there is a binding incentive compatibility constraint between states x' and x'' such that $\pi(x', x') = \pi(x'', x')$ and $C(w_{2,1}(x'), x') > C(w_{2,1}(x''), x')$, so the firm benefits from announcing x'' in state x' from the point of view of new hire costs. Incentive compatibility implies $w_{2,2}(x') < w_{2,2}(x'')$. Then, consider replacing the x' contract by that at x'' (holding n_1 constant). This must trivially satisfy incentive compatibility and no replacement and leave ex post profits unchanged. However, since $w_{2,2}$ is increased in state x', ex ante utility V_1 rises, which reduces period-1 hiring costs; hence, profits increase, contrary to optimality. We conclude that $\pi(x', x') = \pi(x'', x')$ implies $C(w_{2,1}(x'), x') \leq C(w_{2,1}(x''), x')$, and hence, by incentive compatibility, $w_{2,2}(x') \geq w_{2,2}(x'')$ (and if the first inequality is strict or an equality, so is the second, and vice versa).

(b) Let $X' \subseteq X$ be such that for $x \in X'$, $w_{2,2}(x) > w_1$. We show that $X' = \varnothing$. For $x' \in X'' := X \backslash X'$, $x \in X'$, we cannot have $\pi(x', x') = \pi(x, x')$, since $w_{2,2}(x') < w_{2,2}(x)$, contradicting (a). Hence, $\pi(x', x') > \pi(x, x')$ (incentive compatibility is slack). Hence, we can find (by X finite) an $\eta > 0$ such that $\pi(x', x') \geq \pi(x, x') + \eta$ for all $x' \in X''$, $x \in X'$.

Next, cut $w_{2,2}(x)$ by $\varepsilon < \eta\left((1-\delta)n_1\right)^{-1}$ for all $x \in X'$; this does not affect incentive compatibility between $x, x'' \in X'$ as profits change by the same amount in each state, and by construction of ε, $\pi(x', x') > \pi(x, x')$, $x' \in X''$, $x \in X'$. As $\pi(x,x)$ is increased for each $x \in X'$ by $\varepsilon(1-\delta)n_1$, $\pi(x,x) > \pi(x', x)$, $x' \in X''$, as the RHS is unchanged and a weak inequality held before the change. Thus, (global) IC is satisfied. No replacement is satisfied because only w_{22} is cut. If $X' \neq \emptyset$, for a small enough ε, this uniform cut in $w_{2,2}$ in all states where $w_{2,2} > w_1$ and a corresponding increase in w_1 to leave V_1 unchanged increases profits by standard consumption smoothing arguments (hold n_1 constant), i.e., a profitable deviation that is contrary to the assumption. We conclude that $X' = \emptyset$, i.e., $w_{2,2}(x') \leq w_1$ all $x' \in X$.

(c) Let $\hat{X} := \arg\max_{\hat{x}} w_{22}(\hat{x})$. If this is a singleton, $\{x\}$, then by part (a), there is no other state x' with $\pi(x', x') = \pi(x, x')$. It follows that provided that the no replacement constraint is slack in state x, $w_{2,1}(x) = w^{**}(x)$ and, hence, $w_{21}(x) = w^C_{2,1}(x, w_1, n_1)$, as otherwise, if $w_{2,1}(x) \neq w^{**}(x)$ a small enough change in w_{21} towards w^{**} increases profits in state x (by the strict convexity of $C(\cdot, x)$), satisfies no replacement, violates no $\pi(x', x') \geq \pi(x, x')$ constraint for all $x' \neq x$, and relaxes $\pi(x,x) \geq \pi(x', x)$ for $x' \neq x$. If no replacement binds in state x, this argument implies $w_{2,1}(x) \geq w^{**}(x)$, as $w_{2,1}$ can be increased if $w_{21} < w^{**}$ and, hence, $w_{21}(x) \geq w^C_{2,1}(x, w_1, n_1)$.

If $\hat{X}$ is not a singleton, by a similar argument, consider $x \in \hat{X}$ such that $w_{2,1}(x) \neq w^{**}(x)$. If no replacement is not binding at state x, change $w_{2,1}(x)$ towards $w^{**}(x)$ by an amount ε such that $C(w_{2,1}(x), x)$ falls. Again, by part (a) for all $x' \notin \hat{X}$, we have $\pi(x', x') > \pi(x, x')$, and provided that ε is small enough, these incentive compatibility and no replacement constraints are not violated. If any incentive compatibility constraint for $x'' \in \hat{X}$ is violated, replace $w_{21}(x'')$ by the new value of $w_{21}(x)$; this increases ex post profits in x'' and does not affect period 1, as w_{22} is unchanged. Profits are increased by this change, contrary to the assumption. Hence, $w_{2,1}(x) = w^{**}(x)$ for all $x \in \hat{X}$. If no replacement binds at the lowest $w_{2,1}(x)$, $x \in \hat{X}$, again, $w_{2,1}(x) \geq w^{**}(x)$. ■

A.4 Proof of Proposition 5

Proof. Incentive compatibility in state x'' requires that

$$\begin{aligned}(f((1-\delta)n_1 + n_2(x''); x'') - w_{2,2}(x'')(1-\delta)n_1 - C(w_{2,1}(x''), x'')n_2(x'')) \geq \\ (f((1-\delta)n_1 + n_2(x'); x'') - w_{2,2}(x')(1-\delta)n_1 - C(w_{2,1}(x'), x'')n_2(x')),\end{aligned} \tag{34}$$

where $C(\cdot,\cdot)$ is the total cost of a new period 2 hire as defined as in the proof of Proposition 4, and hiring in state x' is denoted $n_2(x')$, etc. We will write $w_{2,1}(x')$ as $w'_{2,1}$ etc. to simplify notation below.

We start by assuming that the optimal contract is differentiable (from the right) at

$\varepsilon = 0$. Consider ε small and take a first-order approximation for (34) around the intial equilibrium[54] at $\hat{x}$, where (34) trivially holds with equality (and where as in the proof of Proposition 4 we use a $\hat{}$ to denote the corresponding initial equilibrium contract) and defining deviations as $\Delta w'_{2,2} := w'_{2,2} - \hat{w}_{2,2}$ etc., and where $\Delta x''(= -\Delta x') := x'' - \hat{x} = \varepsilon$: $f'\left(\Delta n''_2 - \Delta n'_2\right) - (1-\delta)n_1\left(\Delta w''_{2,2} - \Delta w'_{2,2}\right) - \frac{\partial C}{\partial w}n_2\left(\Delta w''_{2,1} - \Delta w'_{2,1}\right) - C\left(\Delta n''_2 - \Delta n'_2\right) \geq 0$, with the reverse inequality implied by incentive compatibility in state x', so given that $f' = C$ in the initial equilibrium ($\hat{n}_2$ is chosen efficiently given $\hat{w}_{2,1}$ in the absence of incentive compatibility constraints), we get

$$-(1-\delta)n_1\left(\Delta w''_{2,2} - \Delta w'_{2,2}\right) - \frac{\partial C}{\partial w}n_2\left(\Delta w''_{2,1} - \Delta w'_{2,1}\right) = 0. \tag{35}$$

Suppose that $\Delta w''_{2,2} < \Delta w'_{2,2}$; we will establish a contradiction. Since $\frac{\partial C}{\partial w} > 0$ (at the initial equilibrium), (35) implies $sgn\left(\Delta w''_{2,2} - \Delta w'_{2,2}\right) = -sgn\left(\Delta w''_{2,1} - \Delta w'_{2,1}\right)$. Hence $\Delta w''_{2,1} > \Delta w'_{2,1}$; thus $w''_{2,2} < w'_{2,2}$ and $w''_{2,1} > w'_{2,1}$ and

$$w''_{2,2} < w'_{2,2} \leq w'_{2,1} < w''_{2,1},$$

where the weak inequality follows by no replacement in state x'.

Consider the following change to the contract (use a $\tilde{}$ to denote this new contract): set wages in x'' to equal those in x': increase $w''_{2,2}$ to $\tilde{w}''_{2,2} := w'_{2,2}$ and reduce $w''_{2,1}$ to $\tilde{w}''_{2,1} = w'_{2,1}$; hold n_1 constant, set n_2 in each state to maximize period 2 profits given $w'_{2,1}$ and $\tilde{w}''_{2,1}$, and change w_1 to $\tilde{w}_1$ to keep V_1 constant. The cut in $w''_{2,1}$ reduces hiring costs by, for ε small enough, $w''_{2,1} > w^{**}\left(x''\right)$ (the latter being the new hire cost minimizing wage in state x'', using notation and the argument in the proof of Proposition 4 above) and as $\tilde{n}''_2$ is chosen optimally, profits on new hires in x'' must rise. Likewise as $\tilde{n}'_2$ is chosen optimally profits in x' cannot fall. Incentive compatibility is satisfied trivially. From V_1 constant (which implies constant vacancy creation and hence constant period 1 vacancy costs)

$$v\left(\tilde{w}_1\right) - v\left(w_1\right) + 0.5\beta\left(1-\delta\right)\left(v\left(w'_{2,2}\right) - v\left(w''_{2,2}\right)\right) = 0. \tag{36}$$

By $w^*_{2,2} < w^*_1$, $w'_{2,2} < w_1$; also $w'_{2,2} < \tilde{w}_1$ for ε small enough, so

$$w_1 > \tilde{w}_1 > w'_{2,2} > w''_{2,2}.$$

It follows from (36) and by $v'' < 0$ that

$$w_1 - \tilde{w}_1 > 0.5\left(1-\delta\right)\left(w'_{2,2} - w''_{2,2}\right);$$

[54]That is, we omit terms of order smaller than ε in the expressions that follow. We assumed that the equilibrium of the model is differentiable in ε on an interval $[0, \bar{\varepsilon})$ (from the right at 0), so that in particular C is also differentiable in x. In the approximation $\partial C/\partial x$ cancels.

thus the change in costs of period 1 hires is

$$n_1 \left(\tilde{w}_1 - w_1 + 0.5 \left(1-\delta\right) \left(w'_{2,2} - w''_{2,2}\right)\right) < 0.$$

Thus the new contract is more profitable than the putative equilibrium one, a contradiction. This establishes that $\Delta w''_{2,2} < \Delta w'_{2,2}$ is not possible. Similarly $\Delta w''_{2,2} > \Delta w'_{2,2}$ yields a contradiction. Thus $\Delta w''_{2,2} = \Delta w'_{2,2}$ and so by (35) $\Delta w''_{2,1} = \Delta w'_{2,1}$. It follows that $\left(\Delta w''_{2,1} - \Delta w'_{2,1}\right) / \left(2\varepsilon\right) = 0$, which establishes the claim.

Now we allow for the contract to be non-differentiable in ε (from the right) at $\varepsilon = 0$. It must be (right) continuous at $\varepsilon = 0$, as otherwise profits would also be discontinuous, while a simple constant wage contract would be continuous so would do better.[55] Consider a sequence for $\varepsilon \equiv \left(x'' - x'\right)/2$: $\{\varepsilon_\nu\}$, $\varepsilon_\nu \to 0$ as $\nu \to \infty$. Assume that the no replacement constraint binds (so that $w_{22} = w_{21} =: w_2$ say) in both states along the sequence (cf. proof of Proposition 4) and that only the downward incentive constraint binds (i.e., (34)). Then by standard arguments $w''_2 \geq w'_2$ and n''_2 is at the optimal level given w''_2.[56] The other possibilities can be dealt with in an analogous manner. We again suppress the explicit dependence of the optimal contract on ε_ν for notational simplicity. We suppose, contrary to hypothesis, that

$$0 < \limsup_{\nu\to\infty} \left|w''_2 - w_2{}'\right| / \varepsilon_\nu. \tag{37}$$

Rearranging (34):

$$f\left((1-\delta)n_1 + n''_2; x''\right) - f\left((1-\delta)n_1 + n'_2; x''\right) - C\left(w_{2,1}\left(x''\right), x''\right) n_2\left(x''\right) + \\ C\left(w_{2,1}\left(x'\right), x''\right) n_2\left(x'\right) - (1-\delta)n_1\left(w''_2 - w'_2\right) \geq 0. \tag{38}$$

By (37) we can take a subsequence such that $\lim_{\nu\to\infty} \left(w''_2 - w_2{}'\right)/\varepsilon = a$ where $|a| > 0$, and where n_1 converges to say $\tilde{n}_1$, we get after dividing (38) by ε_ν and taking the limit:

$$\liminf_{\nu\to\infty} [(f\left((1-\delta)n_1 + n''_2; x''\right) - f\left((1-\delta)n_1 + n'_2; x''\right) - C\left(w''_2, x''\right) n''_2 + \\ C\left(w'_2, x''\right) n'_2)/\varepsilon_\nu] \geq (1-\delta)\tilde{n}_1 a. \tag{39}$$

By $w''_2 - w'_2 \geq 0$, $a > 0$. In other words, assuming for small ε we have lower wages in state x' than in x'' by a first-order amount, implies that the R.H.S. of (39) is positive, that is, there is a (first-order) incentive in state x'' to underreport x to benefit from lower wage costs; to offset this (i.e., to preserve incentive compatibility) the level of new hires in state x' needs to be sufficiently different (below in this case) that in x'' to lead to a fall in profits

[55] Profits are bounded above by a contract which ignores the incentive constraint, which would be continuous, so any discontinuity must imply profits jump down for $\varepsilon > 0$. Holding wages constant across states and setting period 2 employment efficiently at those wages as in the construction in the proof of Proposition 4 would satisfy incentive constraints and lead to profits varying continuously; hence this would be a profitable deviation.

[56] I.e., it maximizes $f\left((1-\delta)n_1 + n''_2; x''\right) - C\left(w''_2, x''\right) n''_2$.

from new hires that is also first-order. We show that such a difference in hires would also imply, contrary to optimality, that a deviation contract is profitable which avoids the costs of distorted employment, where wages are constant and employment in state x' is set at an efficient level given wages.

Consider then the following possible deviation contract. In state x' set $w_{2,1} = w_{2,2} = w_2''$, and set n_2 at the profit maximizing level in state x' for w_2'', say $\tilde{n}_2'$. Change w_1 to leave V_1 unchanged (and leave hiring in period 1 the same). In period 2 this contract differs only in state x', satisfies no replacement, and is incentive compatible as wages are the same across states and n_2 is chosen optimally in each state. Considering only incumbents the wage increase from w_2' to w_2'' must increase profits once the reduction in w_1 is taken into account ($w_2' < w_1$ implies that more smoothing reduces wage costs). As overall profits cannot be improved by any deviation, the change in profits in state x' ignoring incumbents must be nonpositive, i.e.,

$$
\begin{aligned}
0 \geq & \qquad (40)\\
& \left(f\left((1-\delta)n_1 + \tilde{n}_2'; x'\right) - C\left(w_2'', x'\right)\tilde{n}_2'\right) - \left(f\left((1-\delta)n_1 + n_2'; x'\right) - C\left(w_2', x'\right)n_2'\right) \geq \\
& \left(f\left((1-\delta)n_1 + n_2''; x'\right) - C\left(w_2'', x'\right)n_2''\right) - \left(f\left((1-\delta)n_1 + n_2'; x'\right) - C\left(w_2', x'\right)n_2'\right),
\end{aligned}
$$

where the second inequality follows by definition of $\tilde{n}_2'$ yielding at least as much profit as n_2'' at w_2''. Dividing the R.H.S. of (40) by ε_ν, note that this differs from the term in square brackets in (39) only by the argument in x, so that given differentiability of f and C in x the two expressions differ by a term of order less than ε_ν.[57] So taking the limit as $\nu \to \infty$, we get the same value, which is a contradiction as from (39) it is at least $(1-\delta)\tilde{n}_1 a > 0$, whereas from (40) it is nonpositive. ■

A.5 Multi-period Extension to Base Model

We extend the model in a straightforward way to $T > 2$ periods. At the start of each period t a productivity shock $x_t \in X$ is drawn, according to a Markov process with transition probabilities $\Pi = [\pi_{xx'}]_{x,x' \in X}$, where again $x = x_0$ is fixed at $t = 1$. Per-period production and utility functions are as before, but we allow for respective discount factors for firms and workers β_f and β_w, $0 < \beta_f, \beta_w < 1$.

Firms also choose how many new jobs $\overline{n}_t$ to create in period t at a cost of $k > 0$ per job. Each $w_{t,i}$ and $\overline{n}_t$ is a function of the history of shocks $x^t := (x_1, \ldots, x_t)$. As before, an employed worker suffers exogenous separation from the firm at the end of a period with probability δ. Such workers join the existing unemployed in searching for work from the

[57] I.e., by a term $h(\varepsilon) = o(\varepsilon)$ so that $h(\varepsilon)/\varepsilon \to 0$ as $\varepsilon \to 0$. This follows as the derivative of the R.H.S. of (40) with respect to x at the limit contract, i.e., the initial ($\varepsilon = 0$) contract, equals zero. (Recall that by continuity n_2', n_2'', converge to the same value, etc.)

start of the next period. A worker who is unemployed in any period receives an income of b.

Search and matching occurs in a similar manner to the earlier model. A firm will *not* have an incentive to replace a particular cohort having tenure $i > 1$ at date t if the discounted continuation costs, taking into account separation, do not exceed those for a new hire[58]:

$$E[\sum_{\tau=t}^{T}(\beta_f(1-\delta))^{\tau-t}w_{\tau,\tau-t+i} \mid x^t] \leq E[\sum_{\tau=t}^{T}(\beta_f(1-\delta))^{\tau-t}w_{\tau,\tau-t+1} \mid x^t]. \tag{41}$$

For simplicity we will assume that it is optimal for a firm to satisfy the no-replacement constraints, that is we require that (41) holds for all t, $1 < t \leq T$, all i, $1 < i \leq t$, and all x^t.

Let $Z_t(x^t)$ be the lifetime utility of a worker searching in period t. Define $Z := (Z_1, Z_2, \ldots, Z_T)$ (suppressing dependence on x^t where no ambiguity arises). The value to a worker at t with tenure i from being employed by a firm with wage policy σ is defined recursively by

$$V_{t,i}(\sigma; Z, x^t) := v\left(w_{t,i}\left(x^t\right)\right) + \beta_w E[\delta Z_{t+1} + (1-\delta)V_{t+1,i+1}(\sigma; Z) \mid x^t], \tag{42}$$

for $t = 1, \ldots, T$, $i \leq t$, with $Z_{T+1} = V_{T+1,i}(\sigma; Z) = 0$, $i \leq T+1$. Likewise let U_t be the lifetime utility of an unemployed worker at t who fails to find a job:

$$U_t\left(Z, x^t\right) = v(b) + \beta_w E\left[Z_{t+1} \mid x^t\right].$$

Given U_t and Z_t, the expected queue length for a job offering $V_{t,1}$ to a new hire at t is assumed to satisfy:

$$\theta(V_{t,1}, Z_t, U_t) = \begin{cases} \theta : p(\theta)V_{t,1} + (1-p(\theta))U_t = Z_t, \text{ if } V_{t,1} > Z_t \\ 0, \text{ if } V_{t,1} \leq Z_t \end{cases} \tag{43}$$

A firm's profit at t is:

$$F_t\left(\sigma; (\overline{n}_1, \overline{n}_2, \ldots, \overline{n}_T); Z\right) = f\left(\sum_{i=1}^{t} n_{t,i}; x_t\right) - \sum_{i=1}^{t} w_{t,i}n_{t,i} - k\overline{n}_t$$

where $n_{t,i}$ is the number of workers in the firm in period t with tenure i, and is given by $n_{t,i} = (1-\delta)^{i-1} q\left(\theta_{t-i+1}\right)\overline{n}_{t-i+1}$, $i = 1, \ldots, t$, where, from (43), $\theta_{t-i+1} = \theta(V_{t-i+1,1}(\sigma; Z)$, $Z_{t-i+1}, U_{t-i+1}(Z))$.

[58]This inequality is relevant provided that the firm will not try to replace either cohort in the future. Since we will look for contracts that satisfy no-replacement constraints at all dates, this will hold.

We define an equilibrium analogously with the two-period case:

Definition 2 *A symmetric stationary competitive search equilibrium with no replacement and positive hiring consists of search values $Z = (Z_1, Z_2, \ldots, Z_T)$, and a wage policy σ and job creation plan $(\overline{n}_1,\overline{n}_2, \ldots, \overline{n}_T)$, $\overline{n}_t > 0$ all $t = 1, \ldots, T$, with the following properties:*

(i) *Profit maximization*:[59] For all $(\sigma'; (\overline{n}'_1, \ldots, \overline{n}'_T))$,

$$E\sum_{t=1}^{T} \beta_f^{t-1} F_t\left(\sigma; (\overline{n}_1,\overline{n}_2, \ldots, \overline{n}_T); Z\right) \geq E\sum_{t=1}^{T} \beta_f^{t-1} F_t\left(\sigma'; (\overline{n}'_1, \ldots, \overline{n}'_T); Z\right); \tag{44}$$

(ii) *Consistency*: $\theta(V_{t,1}(\sigma, Z), Z_t, U_t) = S_t/\overline{n}_t$, where $S_t := (1 - p(S_{t-1}/\overline{n}_{t-1}))(1-\delta) S_{t-1} + \delta S$ is the number of workers (per firm) seeking work in period t, and

(iii) *No replacement*: (41) holds for all $t, i < t$ and x^t.

Proceeding as in the two-period model, we define a firm's job filling probability as a function of the current wage holding its future wages constant: $\tilde{q}\left(w_{t,1}, x^t\right) := q\left(\theta(V_{t,1}(\sigma, Z), Z_t\left(x^t\right), U_t\left(x^t, Z\right))\right)$, where the dependence on $w_{t,1}$ is via $V_{t,1}$ from (42) holding $V_{t+1,i+1}$ constant. We write $\tilde{q}' \equiv \partial\tilde{q}/\partial w_{t,1}$.

As in the proof of Proposition 1 if the firm can set $w_{t,1}$ without constraint it must satisfy $q^2\left(\tilde{q}'\right)^{-1} = k$; the equilibrium wage and employment must then be at an intersection of the locus of values for n_t and $w_{t,1}$ (the *commitment quasi-supply curve*) which satisfy this equation, and the labor demand curve.[60]

Proof of Proposition 3.

Proof. (a) 1. We use a variational argument. Starting from the optimal contract, consider frontloading wages between t and $t+1$ in some state $x_{t+1} = x \in X$ (we hold x^t and x fixed throughout the proof). Reduce the wage for some cohort with tenure $i+1$ at $t+1$ after state x by a small amount Δ, and increase the wage for this cohort at t by η so as to leave the worker indifferent; do not change the contract, or vacancy creation,

[59] We have not explicitly defined profits for contracts which violate no replacement, but they are defined analogously to the two-period case, and deviations of this type are understood to be included in (41).

[60] The "labor demand" curve now includes a forward looking element which takes into account the reduction in future hiring costs due to an extra worker taken on today, and any difference in future wage costs between a hire made today and one made next period (i.e., until the no replacement constraint induces equality between the two):

$$w_{t,1} = f'_t - k/q_t + E\left(\beta_f(1-\delta)k/q_{t+1} + \sum_{\tau=t+1}^{T} \beta_f^{\tau-t}(1-\delta)^{\tau-t}\left(w_{\tau,\tau-t} - w_{\tau,\tau-t+1}\right)\right),$$

where $q_t \equiv q(\theta_t)$.

otherwise. This implies that

$$-\pi_{x_t x}\left(1-\delta\right)\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right)\Delta + v'\left(w_{t,i}(x^t)\right)\eta \simeq 0. \tag{45}$$

This frontloading satisfies all constraints: worker utility falls at $t+1$, and so from this point on the no replacement constraints are satisfied; similarly, the no replacement constraint is also satisfied for the cohort both at x^t and earlier because utility is held constant over the two periods; likewise the initial utility offered to this cohort is unchanged so hiring remains constant.

The change in profits (viewed from x^t) per worker in this cohort is $\Delta P = \pi_{x_t x}\beta_f \left(1-\delta\right)\Delta - \eta$. Using (45) to eliminate η gives the change in profits as

$$\Delta P \simeq \pi_{x_t x}\beta_f\left(1-\delta\right)\Delta - \frac{\pi_{x_t x}\left(1-\delta\right)\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right)\Delta}{v'\left(w_{t,i}(x^t)\right)}. \tag{46}$$

The change in profits cannot be positive by optimality of the original contract, i.e., $\Delta P \leq 0$, so using (46) (by considering Δ sufficiently small the approximation in (46) can be made arbitrarily precise) we get

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) \geq \beta_f v'\left(w_{t,i}(x^t)\right). \tag{47}$$

2. Next, consider backloading wages, i.e., repeat the above arguments but for an increase in the wage at $t+1$ of Δ, offset by a decrease of η at t. Note that in this case the $t+1$ no replacement constraint *will* be violated if it is binding initially. By an analogous argument to the above, backloading is profitable if

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) > \beta_f v'\left(w_{t,i}(x^t)\right). \tag{48}$$

In this case the $t+1$ no replacement constraint must bind, as otherwise a small backloading would increase profits, and would violate no other constraints by a similar logic to that given above. Thus if the $t+1$ no replacement constraint is slack, from (47) and the negation of (48),

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) = \beta_f v'\left(w_{t,i}(x^t)\right). \tag{49}$$

3. Suppose that

$$\beta_w v'\left(w_{t+1,1}(x^t,x)\right) > \beta_f v'\left(w_{t,i}(x^t)\right), \tag{50}$$

or equivalently, $\tilde{w}_{t+1,i+1} > w_{t+1,1}$. Then there are two possibilities.

A. The no-replacement constraint is binding for this cohort at $t+1$ (i.e., at $t+1$, the cohort with tenure $i+1$):

$$E[\sum_{\tau=t+1}^{T}(\beta_f(1-\delta))^{\tau-t-1}w_{\tau,\tau-t+i}\mid(x^t,x)]=E[\sum_{\tau=t+1}^{T}(\beta_f(1-\delta))^{\tau-t-1}w_{\tau,\tau-t}\mid(x^t,x)].$$

The continuation utilities offered by the two contracts at $t+1$ must be the same: If

$$V_{t+1,i+1}(\sigma;Z,x^{t+1})<V_{t+1,1}(\sigma;Z,x^{t+1}),$$

then it would be optimal for the firm to replace the incumbent continuation by the new hire one since this cannot violate any constraint (both satisfy all no replacement constraints from $t+2$), and would allow the firm to adjust $w_{t+1,i+1}(x^t,x)$ downwards to equalize continuation utilities, slackening the $t+1$ constraint which is therefore satisfied, and saving costs. If on the other hand

$$V_{t+1,i+1}(\sigma;Z,x^{t+1})>V_{t+1,1}(\sigma;Z,x^{t+1}),$$

then replacing the new hire by the incumbent continuation will not violate any constraints, which are satisfied by the incumbent continuation. The higher utility offered to new hires allows the firm to reduce vacancies while hiring the same number, reducing costs, again a contradiction. So

$$V_{t+1,i+1}(\sigma;Z,x^{t+1})=V_{t+1,1}(\sigma;Z,x^{t+1}).$$

It follows that $w_{t+1,i+1}(x^t,x)=w_{t+1,1}(x^t,x)$. To see this, suppose not and consider taking a convex combination of the two continuation contracts,

$$w^c_\tau(x^\tau):=0.5w_{\tau,\tau-t+i}(x^\tau)+0.5w_{\tau,\tau-t}(x^\tau),$$

$\tau=t+1,t+2,\ldots$, where x^τ is any history that starts with (x^t,x). By strict concavity of v this contract offers a higher discounted utility at $t+1$ than the initial contracts, costs the same at $t+1$, and must satisfy the no-replacement constraint after $x^{\tau'}$ since

$$\begin{aligned}E[\sum_{\tau=\tau'}^{T}(\beta_f(1-\delta))^{\tau-\tau'}w^c_\tau\mid x^{\tau'}]=\\ 0.5E[\sum_{\tau=\tau'}^{T}(\beta_f(1-\delta))^{\tau-\tau'}w_{\tau,\tau-t+i}\mid x^{\tau'}]+0.5E[\sum_{\tau=\tau'}^{T}(\beta_f(1-\delta))^{\tau-\tau'}w_{\tau,\tau-t}\mid x^{\tau'}]\\ \le E[\sum_{\tau=\tau'}^{T}(\beta_f(1-\delta))^{\tau-\tau'}w_{\tau,\tau-\tau'+1}\mid x^{\tau'}],\end{aligned}$$

using equation (41). As above, using the convex contract would allow the firm to cut costs, a contradiction.

B. The no-replacement is slack for this cohort at $t+1$. Then by the arguments in 1. and 2. above we have

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) = \beta_f v'\left(w_{t,i}(x^t)\right).$$

From (50) $\beta_w v'\left(w_{t+1,1}(x^t,x)\right) > \beta_w v'\left(w_{t+1,i+1}(x^t,x)\right)$, so we get

$$w_{t+1,1}(x^t,x) < w_{t+1,i+1}(x^t,x). \tag{51}$$

So the incumbent contract is cheaper, from $t+1$. To be cheaper, in view of (51), there must be a date $\tau \geq t+1$ and a continuation history $(\tilde{x}^\tau, x')$ with the property that the wage ranking reverses in the next period, i.e.,

$$w_{\tau,\tau-t}\left(\tilde{x}^\tau, x'\right) < w_{\tau,\tau-t+i}\left(\tilde{x}^\tau, x'\right) \tag{52}$$

and

$$w_{\tau+1,\tau-t+1}\left(\tilde{x}^\tau, x'\right) > w_{\tau+1,\tau-t+i+1}\left(\tilde{x}^\tau, x'\right), \tag{53}$$

and discounted costs for the incumbent contract are lower from $\tau+1$. However this implies that the constraint for the incumbent contract is slack at $\tau+1$, so from 2. above, (49) holds for the incumbent cohort (with $t=\tau$):

$$\beta_w v'\left(w_{\tau+1,\tau-t+i+1}\left(\tilde{x}^\tau, x'\right)\right) = \beta_f v'\left(w_{\tau,\tau-t+i}\left(\tilde{x}^\tau, x'\right)\right).$$

Then from (52) and (53),

$$\beta_w v'\left(w_{\tau+1,\tau-t+1}\left(\tilde{x}^\tau, x'\right)\right) < \beta_f v'\left(w_{\tau,\tau-t}\left(\tilde{x}^\tau, x'\right)\right),$$

which violates (47) and we have a contradiction. Thus only case A is possible, and $w_{t+1,i+1}(x^t,x) = w_{t+1,1}(x^t,x) < \tilde{w}_{t+1,i+1}\left(x^t\right)$.

4. Finally, suppose that

$$\beta_w v'\left(w_{t+1,1}(x^t,x)\right) \leq \beta_f v'\left(w_{t,i}(x^t)\right), \tag{54}$$

or equivalently, $\tilde{w}_{t+1,i+1} \leq w_{t+1,1}$. If

$$\beta_w v'\left(w_{t+1,i+1}(x^t,x)\right) > \beta_f v'\left(w_{t,i}(x^t)\right), \tag{55}$$

by part 2. the $t+1$ no replacement constraint is binding, and using (54),

$$w_{t+1,i+1}(x^t,x) < w_{t+1,1}(x^t,x). \tag{56}$$

We have two contracts costing the same; repeating the argument of part 3 case A, we can again show

$$w_{t+1,i+1}(x^t,x) = w_{t+1,1}(x^t,x),$$

which contradicts (56). Thus (55) cannot hold, and so, given (47), (49) holds.

(b) If $\beta_w v'(w_{t+1,1}) > \beta_f v'(w_{t,1})$, consider backloading the wages of cohort t (i.e., the cohort hired at t) and any other cohorts with $w_{t,i} = w_{t,1}$, with an increase in the wage at $t+1$ of Δ, offset by a decrease of η at t in the new state, so utility at t is unchanged, and also increase $w_{t+1,1}$ by Δ. Choose Δ sufficiently small that $w_{t,1} - \eta > w_{t,i}$ for all i such that $w_{t,i} < w_{t,1}$. Otherwise hold contracts and hiring constant. This change does not violate any constraints: at t and $t+1$ the cost of all the affected cohorts and new hires change by the same amount, while at t the cost of other cohorts remains less than that of new hires by part a) and $w_{t,1} - \eta > w_{t,i}$, and at $t+1$ the cost of new hires rises; so no replacement is satisfied. Moreover $V_{t,1}$ is unchanged so hiring is unaffected at t with an unchanged number of vacancies. By part (a), $w_{t+1,2} = w_{t+1,1}$, so $\beta_w v'(w_{t+1,2}) = \beta_w v'(w_{t+1,1}) > \beta_f v'(w_{t,1})$, and so following the logic of part 2. of (a) the firm's costs of employing cohort t and any other affected cohorts are reduced by the backloading. At $t+1$ however wage costs of new hires increase, and we need to show that the net hiring costs do not increase by more than the backloading savings. Suppose contrary to the proposition that $w_{t+1,1}$ lies on or below the commitment quasi-supply curve at n_{t+1}, that is, $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} \leq k$. The cost per new hire incurred at $t+1$ is $w_{t+1} + k/q$, which changes by $\eta\left(1 - k\tilde{q}'/q^2\right) \leq 0$ to a first-order approximation; the backloading produces a first-order reduction in costs (see part 2. of (a)) hence for Δ small enough, profits are increased, contradicting the assumed optimality of the original contract.

If $\beta_w v'(w_{t+1,1}) \leq \beta_f v'(w_{t,1})$, then by part (a) $w_{t+1,1} \geq w_{t+1,i}$ for all i. If $w_{t+1,1}$ lies below the commitment quasi-supply curve at n_{t+1}, that is, $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} < k$, then holding hiring constant at $t+1$, the cost per new hire incurred at $t+1$, $w_{t+1,1} + k/q$, is decreasing in $w_{t+1,1}$ (taking the first derivative). Raising $w_{t+1,1}$ cannot violate no replacement and would increase profits. If $w_{t+1,1}$ lies above the commitment quasi-supply curve at n_{t+1}, that is, $q^2\left(\tilde{q}'\left(w_{t+1,1}, x^{t+1}\right)\right)^{-1} > k$, and if $\beta_w v'(w_{t+1,1}) < \beta_f v'(w_{t,1})$, then cutting $w_{t+1,1}$ would increase profits (as $d(w_{t+1,1} + k/q)/dw_{t+1,1} > 0$) and since $w_{t+1,1} > w_{t+1,i}$ all i, no replacement is not violated for a sufficiently small cut; if $\beta_w v'(w_{t+1,1}) = \beta_f v'(w_{t,1})$ then there is a first-order reduction in hiring costs if $w_{t+1,1}$ is cut, and to avoid no replacement being violated at $t+1$, cut the wages at $t+1$ of all cohorts i with $w_{t,i} = w_{t,1}$ by the same amount, increasing wages at t to leave utilities unchanged; by initial optimality this frontloading will only have a second-order effect on costs so there is overall an increase in profits for a small enough change. Again this contradicts optimality of the original contract. ■

B Further Tables

Table A1: Number of Spells of Incumbent and Newly Hired Workers

Year	Newly Hired	Incumbents
1978	536,480	860,131
1979	580,482	1,070,423
1980	562,231	1,254,231
1981	472,966	1,423,195
1982	383,748	1,535,036
1983	384,038	1,607,852
1984	421,761	1,650,744
1985	433,296	1,703,623
1986	480,197	1,829,471
1987	467,208	1,925,379
1988	501,192	2,008,610
1989	580,223	2,080,315
1990	674,453	2,164,259
1991	651,557	2,284,766
1992	569,494	2,394,251
1993	482,607	2,431,712
1994	496,822	2,428,188
1995	516,571	2,416,687
1996	481,872	2,408,716
1997	481,019	2,405,614
1998	524,318	2,392,430
1999	580,765	2,385,722
2000	601,915	2,445,300
2001	558,655	2,454,149
2002	471,745	2,444,711
2003	424,415	2,505,278
2004	395,014	2,473,805
2005	391,361	2,443,718
2006	441,206	2,449,759
2007	487,477	2,465,401
2008	474,157	2,506,474
2009	400,230	2,502,328
2010	462,299	2,502,616
2011	444,522	2,409,295
2012	430,893	2,480,722
2013	418,203	2,519,325
2014	432,368	2,521,718
Total	18,097,160	79,785,954

Note: Newly hired workers identified using the first employment spell in a firm.

Table A2: GDP, CPI, Population, and Unemployment Rate

Year	Nominal GDP (in Mill. Euros)	CPI	Population (in 1,000)	Unemployment rate (in %)
1978	678,940	47.6	61,322	4.3
1979	737,370	49.5	61,439	3.8
1980	788,520	52.2	61,658	3.8
1981	825,790	55.5	61,713	5.5
1982	860,210	58.4	61,546	7.5
1983	898,270	60.3	61,307	9.1
1984	942,000	61.8	61,049	9.1
1985	984,410	63.0	61,020	9.3
1986	1,037,130	63.0	61,140	9
1987	1,065,130	63.1	61,238	8.9
1988	1,123,290	63.9	61,715	8.7
1989	1,200,660	65.7	62,679	7.9
1990	1,306,680	67.5	63,726	7.2
1991	1,415,800	70.2	64,485	6.2
1992	1,485,759	73.8	65,289	6.4
1993	1,503,858	77.1	65,740	8.0
1994	1,556,575	79.1	66,007	9.0
1995	1,606,164	80.5	66,342	9.1
1996	1,625,847	81.6	66,583	9.9
1997	1,664,512	83.2	66,688	10.8
1998	1,711,722	84.0	66,747	10.3
1999	1,751,665	84.5	66,946	9.6
2000	1,799,706	85.7	67,140	8.4
2001	1,856,557	87.4	65,323	8.0
2002	1,879,896	88.6	65,527	8.5
2003	1,888,205	89.6	65,619	9.3
2004	1,933,051	91.0	65,680	9.4
2005	1,960,396	92.5	65,698	11
2006	2,038,803	93.9	65,667	10.2
2007	2,142,032	96.1	65,664	8.3
2008	2,180,829	98.6	65,541	7.2
2009	2,088,073	98.9	65,422	7.8
2010	2,191,138	100.0	65,426	7.4
2011	2,298,449	102.1	64,429	6.7
2012	2,345,295	104.1	64,619	6.6
2013	2,401,853	105.7	64,848	6.7
2014	2,483,514	106.7	65,223	6.7

Note: Identified downswing years are indicated in bold year numbers. Real GDP per capita calculated using nominal GDP, CPI, and population. Sources for the nominal GDP for West Germany: German Federal Statistical Office & the Federal Statistical Offices of the Federal States. Source German CPI: Federal Reserve Bank of St. Louis (FRED Economic Data). Source West German Population: German Federal Statistical Office. Source West German unemployment rate (in % of total civilian workforce): Sachverständigenrat.

Table A3: Classification of Economic Activities, Edition 2008 (WZ 2008)

Industry	WZ 2008	
	Section	Description
1	A	AGRICULTURE, FORESTRY AND FISHING
2	B	MINING AND QUARRYING
-	C	MANUFACTURING
3	C 10-12	Manufacture of: food products / beverages / tobacco products
4	C13-15	Manufacture of: textiles / wearing apparel / leather and related products
5	C16+31	Manufacture of: wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials / furniture
6	C17-18	Manufacture of paper and paper products / Printing and reproduction of recorded media
7	C19-25	Manufacture of: coke and refined petroleum products / chemicals and chemical products / basic pharmaceutical products and pharmaceutical preparations / rubber and plastic products / other non-metallic mineral products / basic metals / fabricated metal products, except machinery and equipment
8	C26-28	Manufacture of: computer, electronic and optical products / electrical equipment / machinery and equipment n.e.c.
9	C29-30	Manufacture of: motor vehicles, trailers and semi-trailers / other transport equipment
10	C32	Other manufacturing
11	C33	Repair and installation of machinery and equipment
12	D	ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY
13	E	WATER SUPPLY; SEWERAGE, WASTE MANAGEMENT AND REMEDIATION ACTIVITIES
14	F	CONSTRUCTION
15	G	WHOLESALE AND RETAIL TRADE; REPAIR OF MOTOR VEHICLES AND MOTORCYCLES
16	H	TRANSPORTATION AND STORAGE
17	I	ACCOMMODATION AND FOOD SERVICE ACTIVITIES
18	J	INFORMATION AND COMMUNICATION
19	K	FINANCIAL AND INSURANCE ACTIVITIES
20	L	REAL ESTATE ACTIVITIES
21	M	PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACTIVITIES
22	N	ADMINISTRATIVE AND SUPPORT SERVICE ACTIVITIES
23	O	PUBLIC ADMINISTRATION AND DEFENCE; COMPULSORY SOCIAL SECURITY
24	P	EDUCATION
25	Q	HUMAN HEALTH AND SOCIAL WORK ACTIVITIES
26	R	ARTS, ENTERTAINMENT AND RECREATION
27	S	OTHER SERVICE ACTIVITIES
28	T	ACTIVITIES OF HOUSEHOLDS AS EMPLOYERS; UNDIFFERENTIATED GOODS- AND SERVICES-PRODUCING ACTIVITIES OF HOUSEHOLDS FOR OWN USE
29	U	ACTIVITIES OF EXTRATERRITORIAL ORGANIZATIONS AND BODIES

Note: n.e.c. = not elsewhere classified. Source: Destatis.