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A COMPETITIVE MODEL OF WORKER REPLACEMENT AND WAGE RIGIDITY

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We adapt the models of Menzio and Moen (2010) and Snell and Thomas (2010) to consider a labor market in which firms can commit to wage contracts but cannot commit not to replace incumbent workers. Workers are risk averse, so that there exists an incentive for firms to smooth wages. Real wages respond in a highly nonlinear manner to shocks, exhibiting downward rigidity, and magnifying the response of unemployment to negative shocks. We also consider layoffs and show that for a range of shocks labor hoarding occurs while wages are cut. We argue these features are consistent with recent evidence. (*JEL* E32, J41)

I. INTRODUCTION

In this article, we develop a model in which wages of new hires are not determined independently of those of ongoing employees. The implication is that if there is a reason for ongoing wages to be rigid, this may be transmitted to the wages of new hires if the firm is hiring. Even if the firm is laying off workers, we shall see that its layoff policy also depends on this link.

The crucial assumption that we make is that firms cannot commit to employment levels. That is, as with "at-will" contracting, they cannot commit *not* to layoff a worker. They *can* commit, however, to wage contracts, current and future. If the wage for new hires is below that of incumbents, the firm will have an incentive to replace its incumbents if it can find suitable applicants. Anticipating this, workers will have a preference for a contract in which wages of future hires are never below their own wages, so that the firm will have no incentive to attempt to replace them. It will turn out that firms offer such contracts because the ex ante costs of hiring are lower by a sufficient amount to offset having to forgo the potential benefit of a lower wage for new hires in some future states. That is, it will be optimal to satisfy a "no replacement constraint" that requires that the wage for new hires is never below that of incumbents.

We adapt the models of Menzio and Moen (2010) and Snell and Thomas (2010), henceforth MM and ST, respectively. In MM overlapping generations of two-period lived firms interact with infinitely lived workers, while in ST both firms and workers are infinitely lived.[1] A related argument has been used in the insider-outsider literature; see Gottfries and Sjostrom (2000).[2] We simplify these models to a two-period version that is more tractable for our purposes, but the basic ideas are similar.

In adverse future states, because of the no replacement constraint, the firm will trade-off a desire to smooth the wages of workers in ongoing employment, with the benefits from

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1. MM's model concerns a frictional labor market; in ST, however, the context is of a perfectly competitive labor market, and we adapt the latter approach.

2. They allow for separation payments to workers (which we rule out) on the assumption that who instigates a separation is not contractible, but the fact of separation is. They show that under certain conditions optimal separation payments are zero, however, because positive payments would encourage workers to leave when this is undesirable for the firm.

ABBREVIATIONS

MM: Menzio and Moen (2010)
ST: Snell and Thomas (2010)

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cutting the wage for new entrants. Treated on their own merit, the latter would receive a lower wage, but this would take it below the optimal wage to be paid to incumbents. The upshot is that there is a degree of downward wage rigidity. The opposite is not true, however. In particularly good states, there is no problem in paying a higher wage to new entrants than to incumbents, so the rigidity only operates in a downward direction.[3]

Because the wage for new entrants is allocative, the downwardly rigid wage affects hiring, and increases the variability of both unemployment and vacancies in response to productivity shocks, a point made also by MM and ST.

Apart from providing an alternative, tractable, version of MM and ST, which captures a similar mechanism,[4] we believe that the model furnishes a number of new insights. One feature of our equilibrium which is a direct implication of the idea of equal treatment (i.e., when new hire and incumbent wages are tied together) but which we believe has not been formally analyzed before is that the downward pressure on wages exerted by negative shocks may diminish as shocks worsen. The idea is that once the no replacement constraint prevents wages from falling as far as they can, further negative shocks reduce the number of new hires, thus diminishing the benefits from wage cuts relative to the benefits of offering stable wages to incumbent workers.

A second implication arises from the fact we consider situations where firms are not hiring, which were not considered in MM and ST.[5] For a range of bad shocks, we find a zone of inactivity. At the point at which hiring falls to zero, wages of incumbents are perfectly smoothed (by the logic of the previous paragraph); this has the implication that the wage is *above* what incumbents might get if they are laid off. In slightly more adverse states, introducing layoffs would lead to a discrete drop in incumbents' utility should they be laid off, and from an ex ante perspective it will be preferable to cut the wages of incumbents. Thus wages fall but employment is unresponsive as shocks worsen over this range. We argue that recent experience in the United Kingdom is consistent with this.

Both of these results suggest a very nonlinear response of real wages to the state of the economy. When output is expanding wages are fully flexible upwards. In recessions real wages fall at first with no employment effects, but for more severe shocks wages are not only downward rigid but actually may rise relative to some less severe shocks. This leads to a potentially large employment response, larger than would occur in a first-best outcome (trivially so, as the latter involves full employment at a fixed labor supply). Beyond some point, however, wages will fall sharply again as firms seek to avoid layoffs.

When layoffs do occur they are involuntary (laid-off workers suffer a discrete drop in utility) and inefficient in that employment is below first-best levels. The fact that layoffs occur and are inefficient is perhaps surprising in view of the fact that information is symmetric and wages are contingent.[6] The assumption of at-will employment contracting—that the firm can decide on employment levels—is responsible for this outcome in two ways. It implies that the firm can only prevent layoffs by cutting the wage, as it cannot commit to avoid layoffs. This creates a trade-off between trying to smooth wages and trying to reduce layoffs, the outcome of which is a wage below the level in the previous period, but one which is insensitive to shocks in this range. Second, even over this range, the motive not to replace incumbents prevents firms from bringing in cheaper workers from outside, something that would be ex ante profitable if firms were able to commit not to replace incumbents.[7]

We argue that there is evidence that wages of new hires are not determined independently

3. We shall assume that it is costly or infeasible for workers who quit in the second period to find work; this implies that the firm does not have to respond to spot wages being above incumbent contract wages by raising the incumbent wage. ST allow for ex post worker mobility of the type assumed in (Beaudry and DiNardo 1991), which implies that wages rise to match outside options.

4. For example, MM are only able to numerically analyze a case where transitions between states have zero probability.

5. To solve their model, ST have to impose conditions that imply that hiring is always positive.

6. Thus the sort of mechanism underlying such layoffs is not the same as appealing to predetermined wages which do not respond to productivity or other shocks, as in Hall and Lazear (1984), for example.

7. Schmieder and von Wachter (2010) present the intriguing finding that workers who have received wage increases due to tight past labor market conditions, as in Beaudry and DiNardo (1991), face a higher risk of layoff. As they state, theirs is the first study " to provide direct evidence that downward wage rigidities—in the form of persistent effects of past favorable unemployment conditions on wages—lead to higher incidence of job displacement" (p. 3). This seems broadly consistent with the at-will contracting assumption that firms do not make employment commitments, but choose employment optimally given the wage, and suggests that downward rigidity is playing an important role in layoffs. It however leaves open the question of why firms are not able to cut the wages of such workers.

of those already employed in a firm. This point has been made for example by Bewley (1999), who has argued that internal equity considerations prevent new hires from being paid a wage below that paid to incumbents.[8] Gertler and Trigari (2009) also argue that wages of new hires do not appear to respond more to business cycle fluctuations than those of ongoing employees.[9] We return to this question at the end of Section II.

An outline of the article is as follows. We start by laying out the main assumptions of the model. We then consider the case where firms are hiring. We establish a degree of downward rigidity. Next, we consider the case where firms are neither hiring nor firing and show that firms cut wages to hold employment constant. We then consider an example and analyze layoff policy. Finally, Section III contains concluding comments.

II. THE MODEL

There are two periods $t = 1, 2$. We assume that each firm and worker lives for both periods with many firms and workers, with L the ratio of workers to firms. We identify each firm with an entrepreneur who owns it. In each period, a representative firm operates a decreasing returns technology producing a perishable good, with production function $f(N_t, x_t)$, where N_t is the current number of workers employed at the firm, $x_t \in X \subset \mathbb{R}_+$ is a productivity shock observable at the start of the period, and $\partial f/\partial N > 0$, $\partial^2 f/\partial N^2 < 0$ (hours per worker are not variable). Current profits are given by $f(N, x) - wN$, where w denotes the (real) wage paid in the current period (assuming a uniform wage, which need not hold in period 2). There is no disutility of work, and a worker who is unemployed in any period receives an income of b, so that they would prefer to work for any $w > b$. We assume that $\partial f/\partial N(L, x) > b$ for all $x \in X$ so that in a spot market the labor market would clear at full employment; also higher values of x correspond to better output shocks: $\partial f/\partial x(N, x) > 0$ and $\partial^2 f/\partial N \partial x > 0$. Each worker has a per-period utility of consumption function $v(c)$, $v' > 0$, and $v'' < 0$, where v' and v'' denote first and second derivatives, respectively. Workers cannot borrow or save, so consume all their current income; we assume for simplicity that there is no discounting of the future by workers. Entrepreneurs, however, are risk-neutral, but also have a zero discount rate. We assume that $x = x_1$ is fixed at $t = 1$, but at $t = 2$, x is a random variable, common across firms, with finite support. Henceforth x without a subscript will refer to the second period productivity shock.

A firm has a wage policy $(w_1, (w_2(x), w_h(x))_{x \in X})$ to which it commits: workers hired at $t = 1$ are offered a wage contract (w_1, w_2) and period 2 hires are offered w_h. A worker who accepts a contract at $t = 1$ suffers exogenous separation from the firm at the end of the first period, with probability δ. In the second period such unattached workers seek work. According to MM and ST, employment is assumed to be "at will," so in the second period (after observing x) the firm can dismiss a worker without compensation, and a worker can quit without penalty. We assume that such workers remain unemployed in their second period. Hence we rule out on-the-job search, which will imply that quitting is never optimal in period 2.

Thus workers face an exogenous separation risk, and also the possibility that the firm will dismiss them; in equilibrium we will find that the latter will only occur when the firm is laying off workers, but we shall see that the possibility that workers could be replaced by cheaper new hires will *constrain* the contract (as in MM and ST).

Simultaneously with committing to a wage policy at the start of period 1, firms plan how

8. According to Bewley (1999, 477): "Downward rigidity of the pay of new hires derives from that of existing employees, because all pay rates within a firm are tied together. Reduced hiring pay increases differentials between existing and new employees in each job, unless the pay of existing employees is cut as well." Bewley sees the maintenance of morale to be a major factor in avoiding pay differentials between new hires and incumbents. He does, however, find some evidence, in line with our hypothesis, that incumbent workers are concerned with replacement under two-tier wage systems (p. 146). A morale based theory of why new hires cannot be paid less than incumbents doing the same jobs, would have similar predictions to those we obtain here.

9. They estimate the cyclicality of hiring wages in the United States by using Survey of Income and Program Participation (SIPP) worker data collected at 4-month intervals during 1990–1996. They regress (log) real wage on the unemployment rate, the interaction of the unemployment rate with a new hire dummy variable, match dummy variables, and other controls including the current tenure of the match. The estimated coefficient of the interaction term is statistically insignificant, and they conclude that, "after we control for compositional effects, new hire wages appear no more cyclically sensitive than existing workers' wages" (pp. 73–4). Contrasting evidence is provided in Haefke, Sonntag, and vanRens (2013), who find the wage of those hired out of nonemployment to be quite sensitive to productivity changes. In our model, new hire wages *are* more variable than those of incumbents', but this is due to higher hiring wages in booms.

many *hires* n_t (layoffs if n_2 is negative, where $n_2 \geq -(1-\delta)n_1$) to make in period $t=1,2$; n_2 depends on x[10] (where there is no ambiguity we suppress x in what follows to simplify notation.). We assume that workers can observe n_1 when considering whether to accept a contract from the firm (this is only relevant for computing layoff probabilities). We denote the representative firm's period *t workforce* by N_t, so $N_1 \equiv n_1$ and $N_2 \equiv (1-\delta)n_1 + n_2$. We denote its *wage-employment policy* by $\sigma := (w_1, (w_2(x), w_h(x))_{x\in X}, n_1, (n_2(x))_{x\in X})$.

Let $Z_2(x)$ be the utility of a worker in period 2 looking for work in state x. The value to a worker at $t=1$ from being employed by a firm with policy σ then is

$$(1)\qquad V_1\left(\sigma; Z_2(\cdot)\right) := v\left(w_1\right) + E_x\left[\delta Z_2(x) + (1-\delta)\, v\left(w_2(x)\right)\right]$$

if the worker only faces a separation risk, where E_x denotes expectation with respect to the distribution of states x. However, if layoffs or replacement occur in some states, then in such states the final term inside the square brackets must be averaged with $(1-\delta)v(b)$ using the probability of this happening.

Markets are assumed to be competitive. The period 1 labor market clears but the period 2 market may not. We denote the equilibrium supply price of period 2 labor by w^s, that is any firm offering at least this amount can hire as much as it wants, otherwise it cannot hire at all. We look for a symmetric equilibrium and use a hat to denote equilibrium values of firms' choices. We define

$$(2)\qquad Z_2(x) := \left(\hat{\hat{n}}_2/(\delta L)\right) v\left(\hat{w}_h\right) + \left(1 - \hat{\hat{n}}_2/(\delta L)\right) v(b),$$

where to allow for layoffs we define $\hat{\hat{n}}_2 := \max\left\{\hat{n}_2, 0\right\}$, so $\hat{\hat{n}}_2/(\delta L)$ is the proportion of workers exogenously separated after period 1 who find work (recall that other separated workers remain unemployed). That is, if the labor market clears then $w^s = \hat{w}_h$ (i.e., the common hiring wage offered in a symmetric equilibrium) and $Z_2(x) = v\left(\hat{w}_h\right)$, while if it does not,[11] we assume that $w^s = b$ [12]; if $n_2 > 0$ then $Z_2(x)$ lies between $v\left(\hat{w}_h\right)$ and $v(b)$; if there are layoffs ($n_2 < 0$) then $Z_2(x) = v(b)$.

As mentioned above, at $t=2$ the firm can, by at will contracting, choose whether to replace its incumbents. If $w_h \geq w_2$ firms will have no incentive to do this, but for $w^s \leq w_h < w_2$, the firm will replace its incumbent workforce. If $w_h = w_2$ then the firm is indifferent, and we assume that replacement does not occur. In our framework, however, it is always weakly ex ante optimal to avoid replacement. In period 1 workers would anticipate replacement in period 2, and given they receive b in that case, they would weakly prefer to receive w_h as $w_h \geq w^s \geq b$. So the firm could reduce w_2 to w_h, which would not reduce the ex ante value of the contract $V_1(\sigma, Z_2)$ in Equation (1), and the firm would still be able to hire at w_h, so its profits cannot be reduced by this change. Given that wages are committed to ex ante, by setting $w_h \geq w_2$ in state x a firm effectively commits to *not replace* its incumbents, and we refer to this inequality as *the no replacement constraint.* We henceforth impose this constraint on the firm.[13,14]

If there are no layoffs in period 2 ($n_2(x) \geq 0$ for all x), a firm's profit is:

$$(3)\qquad \Pi(\sigma) = \left(f\left(n_1; x_1\right) - w_1 n_1\right) + E_x\left[\left(f\left((1-\delta)\,n_1 + n_2(x); x\right) - w_2(x)(1-\delta)\,n_1 - w_h(x)\,n_2(x)\right)\right].$$

Otherwise, in any state where layoffs occur, in the expression for second period profit wage costs are replaced by $w_2((1-\delta)n_1 + n_2)$. The firm solves

$$\max_{\sigma} \Pi(\sigma)$$

10. By the "at will contracting" assumption, the firm cannot commit to future employment levels, so we will require below that these must be ex post profit maximizing. We are ignoring the possibility of replacement in specifying these plans, as this will be shown to be suboptimal below.

11. Excess demand in the labor market cannot arise in this competitive model as a very small increase in w_h would allow an individual firm to hire as much as it wishes; such a deviation would always be profitable.

12. While this seems the natural assumption, results are not sensitive to this precise formulation.

13. Note that the firm can only be indifferent about satisfying the constraint if $w_h = b$, as otherwise period 1 hires are made strictly better off by the change, which would allow the firm to cut period 1 wages. We shall see that imposing the constraint leads to an optimal $w_h > b$, even when there is unemployment. In this case violating the constraint would strictly reduce profits.

14. Given that the no replacement constraint is imposed, although the firm cannot commit to period 2 employment, the utility of period 1 hires does not depend on planned employment provided there are no layoffs, that is, in this case there is no time consistency issue.

subject to

(4) $$V_1\left(\sigma, Z_2\right) \geq V_1\left(\widehat{\sigma}, Z_2\right),$$

(5) $$w_h(x) \geq w_2(x)$$

for all x, and, if $n_2(x) > 0$,

(6) $$w_h(x) \geq w^s(x),$$

while if $n_2(x) \leq 0$,

(7) $$n_2 \in \arg\max_{n_2} \left\{\left(f\left((1-\delta)n_1 + n_2\right) - w_2\left((1-\delta)n_1 + n_2\right)\right)\right\}.$$

Here, Equation (4) requires that the firm offers to period 1 hires at least what they can get elsewhere, Equation (5) is the no replacement constraint, Equation (6) requires that if the firm intends to hire in period 2, it must offer at least the supply price of labor, while Equation (7) requires that if there are layoffs that n_2 must be optimal given w_2 (ex post optimality is satisfied when $n_2 > 0$ and so does not need to be imposed as mentioned in Footnote 14). We define an equilibrium as follows:

DEFINITION 1 *A (symmetric) equilibrium consists of a wage-employment policy $\widehat{\sigma}$, which solves Problem A, and such that $n_1 = L$ and $n_2 \leq \delta L$, and where $Z_2(x)$ is given by Equation (2) for all x, and*

(8) $$w^s(x) = \begin{cases} \widehat{w}_h & \text{if} \quad n_2 = \delta L \\ b & \phantom{\text{if}} \quad n_2 < \delta L \end{cases}.$$

A. Positive Hiring

For the moment we shall ignore layoffs and consider a state x with $n_2 > 0$. This is appropriate provided that the period 2 shock x is not too bad and the rate of exogenous separation is sufficiently high. The explicit restrictions that imply $n_2 > 0$ will be established later.

Anticipating future results, aggregate period 2 labor demand N_2 will be $(f')^{-1}(w_h)$ (we suppress x) where $f' \equiv \partial f/\partial N(N, x)$, that is, will be on the standard labor demand curve, and we denote by $w^{spot}(x)$ the wage that would clear the market in state x, that is, $w^{\text{spot}}(x) := \partial f/\partial N(L; x) > b$.[15] Consequently either $w_h = w^{\text{spot}}$ and the labor market clears, in which case $w^s = w^{\text{spot}}$, or $w_h > w^{\text{spot}}$, the labor market fails to clear, in which case $w^s = b < w^{\text{spot}}$. We cannot have $w_h < w^{\text{spot}}$ as that would imply excess demand for labor, and as mentioned earlier, this cannot arise (see Footnote 11).

We proceed heuristically (see the Appendix for formal proofs).

In period 2 in any state x, given n_1 and w_1, following MM it can be shown that the firm must locally maximize ex post period 2 profits plus weighted incumbent utility. In particular, given it is optimal *not* to replace, and recall we are provisionally assuming that the shock is such that labor demand exceeds $(1-\delta)n_1$ (so that period 1 hires only face the exogenous separation risk), it must maximize

(9) $$\widehat{\Pi}(.) := f\left((1-\delta)n_1 + n_2; x\right) - w_2(1-\delta)n_1 - w_h n_2 + \left(1/v'\left(w_1\right)\right)n_1 \times \left((1-\delta)v\left(w_2\right) + \delta Z_2(x)\right)$$

with respect to n_2, w_h, w_2, $w_h \geq w_2$ subject to $w_h \geq w_h^s$. The last term in Equation (9) includes the continuation utility of an incumbent, taking into account the separation possibility, and multiplied by the number of period 1 hires. The intuition here is that any change which affects the utility of the firm's old workers can be offset by a change in the first period wage, leaving V_1 unchanged (and hence n_1). Multiplying the utility change through by the inverse of first period marginal utility then converts it (for a small change) to the first period wage saving per worker. If this was not satisfied then profits can be increased.

There are three cases to consider:

(Case A) The "no replacement constraint" $w_h \geq w_2$ is slack in state x. Given that $\partial\widehat{\Pi}/\partial w_h < 0$ and $w_h > w_2$, it would pay to cut w_h whenever $w_h > w^s$, and so $w_h = w^s$. Then differentiating Equation (9) with respect to w_2 yields the FOC:

(10) $$(1-\delta)n_1 = n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)v'\left(w_2\right)\right),$$

so that $w_1 = w_2$ $(< w_h)$. Intuitively the firm should stabilize the wages of the first period hires if there is no cost to doing this. In this case,

15. $\partial f/\partial n(L; x) > b$ by an earlier assumption. The intuition is clear: given the firm wants to hire and has committed to w_h, it will always hire up to the point where the marginal product of labor equals this wage. This is true whether the wage is against the no replacement constraint. As already remarked, in this case it does not matter that the firm cannot commit to period 2 employment as the utility of period 1 hires does not depend on n_2 (with layoffs the lack of commitment will affect the solution).

also differentiating Equation (9) with respect to n_2, we get

$$(11) \qquad f'\left(N_2\right) = w_h$$

where recall that $N_2 \equiv (1-\delta)n_1 + n_2$, and as $w_h = w^s$ we get that $w_h = w^{spot}$ (and there is full employment).[16]

Intuitively, labor demand in period 2 is sufficiently high that there is full employment and to hire firms must pay a wage above w_1. Then there is no reason not to offer incumbents a constant wage, $w_2 = w_1$, as this does not violate the no replacement constraint.

(Case B) Both $w_h \geq w_2$ and $w_h \geq w^s$ are binding at the optimum. So $w_h = w_2 = w^s$, and Equation (11) holds, and hence there is full employment ($w_h = w^s$ implies this). It must be that $w^s \leq w_1$ since otherwise from Equation (10) cutting w_2 would increase $\widehat{\Pi}$ without violating any constraints.

Intuitively this is an intermediate case, still with full employment, but now the spot wage is somewhat below w_1, and firms offer a new hire wage equal to the spot wage. To avoid violating the no replacement constraint incumbents are paid the same, that is, less than w_1, so wage smoothing is not perfect, but firms want to avail themselves of the opportunity to employ new hires at a wage below w_1.

(Case C) If, on the other hand, $w_h \geq w_2$ is binding at the optimum but $w_h > w^s$, then setting $w_2 = w_h$ and differentiating Equation (9) with respect to w_h:

$$(12) \qquad n_1\left(1/v'\left(w_1\right)\right)\left((1-\delta)v'\left(w_h\right)\right) - N_2 = 0.$$

Differentiating Equation (9) with respect to n_2 we get Equation (11) again (consequently as noted earlier, period 2 employment is always on the labor demand function when $n_2 > 0$). Substitute for N_2 from Equation (11) in Equation (12), recalling that $N_2 = (1-\delta)n_1 + n_2$:

$$(13) \qquad 0 = \frac{n_1}{v'\left(w_1\right)}(1-\delta)v'\left(w_h\right) - \left(f'\right)^{-1}\left(w_h\right) =: g\left(w_h; n_1, w_1, x\right).$$

The first term in the definition of g captures the effect of raising period 2 incumbent wages by a small amount on reducing the period 1 wage bill; the second term is the aggregate period 2 labor demand, and so is the extra period 2 wage bill given that the new hire wage also rises by the same amount as the incumbent wage to avoid violating the no replacement constraint. If, say, $g > 0$, holding employment constant while increasing period 2 wages would lead to an overall lower wage bill, and vice versa if $g < 0$. Solving $g(w_h; n_1, w_1, x) = 0$ yields $w_h\left(= w_2\right) = w_h^d\left(n_1, w_1, x\right) := g^{-1}\left(0; n_1, w_1, x\right)$.

Intuitively, the spot wage is sufficiently low that firms do not want to cut wages that far. That is, because $w_h \geq w^s$ is not binding, wages can be cut, but there is a trade-off: cutting w_h obviously reduces the wages that must be paid to period 2 hires, but there is a cost imposed on period 1 hires who will also have their period 2 wages cut to avoid violating the no-replacement constraint, and thus must receive compensation for this variability in their wage profile. At w^d, these two effects just balance, and firms do not want to cut wages further, even though they could hire at a lower wage. This implies the wage exhibits downward rigidity (the value of w^s per se has no effect on it) and there is (involuntary) unemployment.

ASSUMPTION 1 *We assume henceforth* $\partial g(w^d; L, w_1, x)/\partial w_h < 0$.

This follows provided workers are sufficiently risk-averse and/or labor demand is sufficiently inelastic. It guarantees that w^d is unique.

We can summarize the Cases A, B, and C in a single proposition with the aid of a reference value for the new hire wage, $\underline{w}$, defined as follows. $\underline{w}$ solves Equation (13) but where the labor demand term $(f')^{-1}(w_h)$ is replaced by full-employment demand L:

$$(14) \qquad L/v'\left(w_1\right)(1-\delta)v'\left(\underline{w}\right) - L = 0,$$

that is

$$(1-\delta)v'\left(\underline{w}\right) = v'\left(w_1\right).$$

In the proposition, it is shown that Case C occurs when the spot wage is below $\underline{w}$, and Case B when the spot wage lies between $\underline{w}$ and w_1. It also strengthens the forgoing discussion (proof in the Appendix). Note that in equilibrium $n_1 = L$ so unemployment can only arise in period 2.

PROPOSITION 1 *Suppose, in an equilibrium, hiring is positive in state x at period 2. If in*

16. Recall $f^{'}(L) =: w^{\text{spot}}$, and $w^s > w^{\text{spot}}$ is impossible from Equation (8), while $w_h = w^s < w^{\text{spot}}$ implies, from Equation (11), $N_2 > L$, which is also impossible.

this state $w^{spot} \geq \widehat{w}_1$ *then* $\widehat{w}_h = w^{spot}$, $\widehat{w}_2 = \widehat{w}_1$ *and there is full employment,* $N_2 = L$ *(Case A). If* $\underline{w} \leq w^{spot} < \widehat{w}_1$ *then* $\widehat{w}_h = \widehat{w}_2 = w^{spot}$ *and* $N_2 = L$ *(Case B). If* $w^{spot} < \underline{w}$ *then* $\widehat{w}_1 > \widehat{w}_h = \widehat{w}_2 = w^d\left(L, \widehat{w}_1, x\right) > \underline{w}$ *and* $N_2 < L$ *(Case C). Hiring is positive in state x if and only if labor demand setting* w_h *equal to* $\widehat{w}_1$ *(i.e., not necessarily at the equilibrium level) exceeds* $(1-\delta)L$.

Thus, given $\widehat{w}_1$, in any state for which labor demand is high enough that the spot wage w^{spot} would exceed $\widehat{w}_1$, new hire wages will be at the spot wage and the incumbents' wages are perfectly smoothed: $\widehat{w}_2 = \widehat{w}_1$; there will be full employment.[17] This is how a policy would look even if the firm could commit to not replace incumbents and could therefore undercut their wages; in these states the replacement problem does not matter. For a somewhat lower spot wage, the new hire wage will equal the incumbent wage and be at the spot wage. The costs of reducing the incumbent's wage below $\widehat{w}_1$ is when w^s is close to $\widehat{w}_1$, whereas the cost of setting the new hire wage w_h above the supply price w^s is a first-order cost, so given the constraint $w_h \geq w_2$, $w_h = w_2 = w^{\text{spot}}$ is optimal.

However, for low enough labor demand, $w^{\text{spot}} < \underline{w}$; as explained above, the costs of reducing wage smoothing further would more than offset the gain from cheaper new hires, and w_h $(= w_2)$ is set above w^{spot} and there is unemployment. Moreover, the wage is *above* $\underline{w}$. The intuition is as follows. Start from that x for which $w^{\text{spot}} = \underline{w}$, so there is (just) full employment. At a lower value of x, as hiring is lower than would be the case at full employment, the trade-off between lowering the wage to benefit from cheaper new hires and the desire to stabilize wages, moves in favor of the latter, since firms make fewer hires and so benefit less from a lower wage.

This intuition suggests that in region C the wage *might increase as the productivity shock worsens,* as again at lower period 2 employment levels the gain from a lower wage is diminished given that a smaller number of new hires is being made. This is confirmed in the next proposition (proof in the Appendix):

PROPOSITION 2 *For any two states at date 2, x, x′ with* $x < x'$, *and such that for both states hiring is positive and* $w^{spot} < \underline{w}$, *(i.e., we are in Case C), then* $w^d\left(L, \widehat{w}_1, x\right) > w^d\left(L, \widehat{w}_1, x'\right)$.

A final general result when there is positive hiring at date 2 in all states is that $\widehat{w}_1$ is no lower than the marginal product of labor in period 1.

PROPOSITION 3 *Suppose in an equilibrium, hiring is positive in each state x at period 2. Then* $\partial f\left(n_1, x_1\right)/\partial N \leq \widehat{w}_1$. *This is strict whenever* $\widehat{w}_h > \widehat{w}_2$ *in at least one state at date 2.*

The intuition is that in bad states of the world the no replacement constraint binds and so $w_h = w_2$; there is no benefit or cost to carrying an extra worker into period 2 as one new hire less or more can be made at the same wage. In good states when $w_h > w_2$, however, there is a benefit in having an extra incumbent worker given that they will receive a lower wage than new hires. It follows that the value to hiring an extra worker in period 1 may be greater than their period 1 product, and the wage may not be allocative.

EXAMPLE 1 *Assume each firm has technology given by, at time* $t = 1, 2$:

$$f\left(N_t, x_t\right) = x_t \log\left(N_t\right), \tag{15}$$

where $x_1 > 0, x_2 = x > 0$, and recall x_1 is fixed. Assume also that workers have per-period utility functions of the constant relative risk aversion family with coefficient $\gamma > 1$, described by $v(c) = c^{1-\gamma}/(1-\gamma)$, with $v'(c) = c^{-\gamma}$. In this case then $w^{\text{spot}}(x) = x/L$.

In Case C, we have $w^{\text{spot}} < \underline{w}$, $w_h = w_2 = w^d$ and $N_2 < L$. We can write Equation (13) for $n_1 = L$ as

$$L(1-\delta)\, w_h^{-\gamma}/\widehat{w}_1^{-\gamma} - x/w_h = 0,$$

and solving for w_h we get

$$w_h = w^d = \left((1-\delta)\, L\widehat{w}_1^{\gamma}/x\right)^{1/\gamma - 1}.$$

Also

$$\underline{w} = \widehat{w}_1\,(1-\delta)^{1/\gamma}.$$

Moreover, Assumption 1 is satisfied, given $\gamma > 1$.[18]

17. If such a state was anticipated with a sufficiently high probability, then it might appear that a worker who quit at the same time as the exogenous separations occur could be better off as they would receive a higher period 2 wage. While we have ruled this out by assumption, it could be justified by supposing there is a high enough mobility cost incurred by the worker. Going to the other extreme and allowing full ex post mobility of labor, as in Snell and Thomas (2010), would eliminate region A as it would impose an extra constraint, $w_h \leq w_2$ (so that $w_h = w_2$ always). We conjecture however that the other qualitative results would still hold.

18. After manipulation we get $\partial g\left(w_h = w^d; n_1, w_1, x\right)/\partial w_h = -(\gamma-1)\,x\left(\frac{L(1-\delta)w_1^{\gamma}}{x}\right)^{-\frac{2}{\gamma-1}} < 0.$

B. No Hiring

For a bad enough shock in period 2, the firm may prefer not to hire. This occurs, from Proposition 1, if labor demand at $w_h = \widehat{w}_1$ is no greater than $(1-\delta)L$.[19] Suppose that $x = x^*$ is such that we are just on the limit of the positive hiring regime, that is, when labor demand at $w_h = \widehat{w}_1$ equals $(1-\delta)L$.

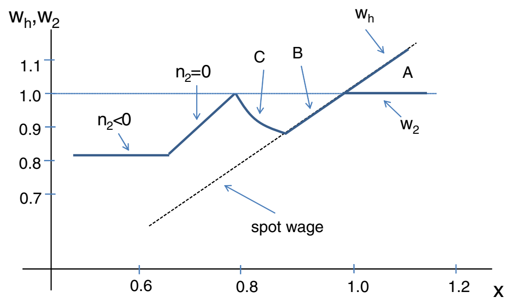
PROPOSITION 4 *There exists an interval of values for x,* $[\underline{x}, x^*]$ *with* $\underline{x} < x^*$*, such that* $\widehat{n}_2(x) = 0$ *for any* $x \in [\underline{x}, x^*]$.

At x^*, we have $w^d = \widehat{w}_1$. Then for $x \in [\underline{x}, x^*]$ there is neither hiring nor firing, so that as x falls, the wage falls from $\widehat{w}_1$ to maintain labor demand at $(1-\delta)L$.[20] Over this range we observe "labor hoarding" in the sense that worse states do not lead to layoffs (although employment remains below the first-best level). The intuition is as follows. At x^*, $\widehat{w}_2 = \widehat{w}_1$ and $\widehat{n}_2 = 0$. For x slightly below x^*, if the firm maintained $w_2 = \widehat{w}_1$ then there would be layoffs, $n_2 < 0$. However, the firm would suffer only a second-order loss were it to increase employment (reduce layoffs from this point); however, incumbents would get a first-order benefit as their layoff probability falls (their utility falls when laid off as $w_2 > b$). The firm cannot commit to do this, but it can reduce the wage slightly from $\widehat{w}_1$ to achieve the same employment outcome; this also only has a second-order cost for wage smoothing. Thus the firm can strictly increase profits by holding $n_2 = 0$ in this way. Only when productivity falls sufficiently below x^*, and w_2 is much lower than $\widehat{w}_1$ so the wage smoothing costs of further wage cuts become too large, will the firm start laying off its workforce.

When layoffs occur, the same logic implies that $\widehat{w}_2 < \widehat{w}_1$: if to the contrary $\widehat{w}_2 = \widehat{w}_1$ then a small cut in the wage would increase profits by diminishing the layoff rate.

There is some evidence that, consistent with this range, during the great recession in the United Kingdom substantial wage cuts went alongside the unusually small reductions in employment given the path of output. Crawford, Jin, and Simpson (2013) argue that the evidence is consistent with labor hoarding, particularly by smaller and medium-sized firms.[21]

FIGURE 1
Wages and Productivity ($\gamma = 2$, $\delta = 0.2$)



C. Numerical Example

To illustrate the solution, in the example suppose that $L = 1$, $\widehat{w}_1 = 1$, the coefficient of relative risk aversion $\gamma = 2$, $\delta = 0.2$, $b = 0.6$. The value for δ is plausible on an annual basis, given that it effectively excludes quits due to on-the-job search and layoffs. The value for b gives a replacement ratio of 60% relative to period 1 wages. The solution for period 2 wages as a function of possible values for x is illustrated in Figure 1.

Here the spot wage equals x. Starting from the top, when $x > 1$ we are in Case A, and the spot wage exceeds $\widehat{w}_1$ so $\widehat{w}_h = x > \widehat{w}_2 = 1$. In Case B, the spot wage below $\widehat{w}_1$ reduces both $\widehat{w}_h$ and $\widehat{w}_2$ as the no replacement constraint bites. For values of x below $\underline{w} = 0.894$, optimal wages are above the spot wage and there is involuntary unemployment. The gap between the wage and the (dotted) spot wage line determines the amount of unemployment, given that the spot wage line shows the level of wages needed for full employment. As explained earlier, wages rise as x falls in Case C. Thus unemployment rises as x falls in this region. For still lower values of x, we have a range of (employment) inactivity, where wages fall sharply enough to just maintain employment

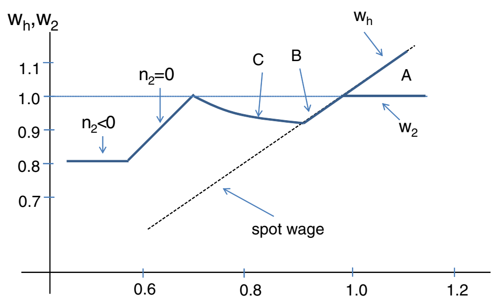
19. In the example, this is when $x/\widehat{w}_1 \leq (1-\delta)L$.

20. Given we are analyzing a finite set of states, this proposition might be empty if $X \cap [\underline{x}, x^*]$ is itself empty. In a continuous state version of the model, however, the same result would hold and sufficiently adverse shocks would imply that this region occurs with positive probability. Moreover, $[\underline{x}, x^*]$ depends only on $\widehat{w}_1$, not on X, and provided there are uniform bounds on X, we can show that for sufficiently dense X a similar conclusion applies in the finite state case.

21. Of course in our model employment is also constant at full employment. However, in a model with an upward sloping labor supply curve this would not be the case, while the "labor hoarding" region we have identified would still imply employment that is unresponsive to a range of productivity shocks. The point is that in the latter case, we are " off the labor supply curve."

FIGURE 2

Wages and productivity ($\gamma = 4$, $\delta = 0.3$)



of all incumbents; equivalently unemployment is constant. Finally layoffs occur, and we find that wages are constant as x falls further so in this range negative shocks feed directly in to increases in layoffs and further increases in unemployment.[22]

We stress three points: first, the period 2 wage is allocative in all regions, and whenever it lies above the spot wage line (the 45 degree line), unemployment exists and is involuntary, and employment is inefficiently low; employment is excessively sensitive to productivity shocks in Case C and when $n_2 < 0$; this is trivial here as employment is constant at L in the first best allocation. Second, and related, is that not only is there inefficiently low hiring over some range where hiring is positive (i.e., in Case C), but also employment is inefficiently low over the whole range where $n_2 = 0$ and where layoffs occur (recall that provided $x > b = 0.6$, the first-best allocation would have full employment). Third, even when $n_2 \leq 0$, the no-replacement constraint matters because it prevents firms from bringing new hires in at a lower wage.[23]

In Figure 2, risk aversion is higher ($\gamma = 4$) and the rate of separation is somewhat higher, at 30% ($\delta = 0.3$). While γ is high for measured risk aversion, estimated elasticities of intertemporal consumption (the inverse of γ) are frequently very low (e.g., Hall 1988). In this case, wages fall less far before Case C occurs; intuitively the higher risk aversion means that the incentive to stabilize wages overrides the benefit of cheaper new hire wages when the spot wage is closer to w_1.

There is little empirical work that breaks down the response of real wages to different parts of the cycle. One recent exception is Snell and Stüber (2013), who analyze wages from the IAB Employee History File (BEH)—a comprehensive Social Security based administrative dataset for Germany. Controlling for match quality (see for example Hagedorn and Manovskii 2013) at a detailed level (worker-firm-occupation) they extract year effects from a panel regression of real wages on match dummies and other controls for the years 1977–2008. They find a strong asymmetry in the response of these real wage year effects to the aggregate unemployment rate; when unemployment is below its long run mean the response is around -0.25 and insignificant while when unemployment is above its long run mean the response is strongly significant at -1.84. Strong upward movements of wages in upswings with rigidity in downswings is reasonably consistent with the predictions of our model. In a further analysis, they find the incremental cyclical response for new hires and for low tenure workers, respectively, are both insignificant—in fact these incremental year effects are virtually white noise. This second finding lends some support to our theoretical prediction that new hire wages do not undercut those of incumbents in downswings.[24]

III. CONCLUDING COMMENTS

It has been argued that the assumption that firms cannot commit *not to replace* has major implications for the nature of employment contracts. Not only does this lead to firms choosing not to pay new hires less than ongoing employees, but it also implies that real wages have a substantial degree of downward rigidity. This implies excessive sensitivity of employment to shocks. Nevertheless the relationship is highly nonlinear. At the point where new hiring falls to zero, and for a range of worse shocks, it is optimal to offer incumbents (i.e., those who were not exogenously separated from the previous period) job

22. Solving Equation (A10) in the Appendix for the functional forms of Example 1, we get $b^{\gamma}\left((\gamma-1)\,w_1^{-\gamma}-\gamma w_2^{-\gamma}\right)+b/w_2=0$, so the solution for w_2 is independent of x.

23. In fact, in the absence of at-will contracting, that is, if it could commit to employment levels, the firm would have no layoffs, given the first best involves full-employment. It would pay its incumbents the same wage as in period 1, *and* it would hire in new workers at a potentially lower wage.

24. This second empirical feature is inconsistent with our result that $w_h > w_2$ in Case A. However, as discussed in Footnote 3, modifying the model to allow for incumbents to costlessly move to other firms in period 2 would imply $w_h = w_2$ for high shocks, but otherwise we conjecture would leave the main results unchanged.

security and cut wages to preserve employment. We term this labor hoarding. For shocks such that layoffs occur, however, wages remain rigid, and again employment is excessively sensitive to shocks.

In future work, we aim to test the more nuanced relationship between shocks and wages/employment that the model predicts. Further theoretical extensions that are desirable include considering asymmetric shocks, frictional labor markets, and a multi-period model with the same characteristics.

APPENDIX

PROOF OF PROPOSITION 1

We derive necessary conditions for the firm by considering the following Lagrangian for Problem A, assuming for ease of presentation an interior solution with $n_2(x) > 0$, all x.[25]

$$\mathcal{L}\left(w_1, n_1, \left(w_h(x), w_2(x), n_2(x), \lambda, \mu_x, \gamma_x\right)_{x \in X}\right)$$
$$= \left(f\left(n_1; x_1\right) - w_1 n_1\right) + E_x\left[f\left((1-\delta) n_1 + n_2(x)\right)\right.$$
$$\left. - w_2(x)(1-\delta) n_1 - w_h(x) n_2(x)\right] +$$

$$\lambda\left[v\left(w_1\right) + E\left[\delta Z_2(x) + (1-\delta) v\left(w_2(x)\right)\right] - V_1\left(\widehat{\sigma}, Z_2\right)\right]$$
$$+ E_x \mu_x\left[w_h(x) - w_2(x)\right] + E_x \gamma_x\left[w_h(x) - w^s(x)\right],$$

where $\lambda, \mu_x, \gamma_x \geq 0$. This leads to the FOCs:

$$\text{(A1)} \qquad w_1 : -n_1 + \lambda v'\left(w_1\right) = 0,$$

$$\text{(A2)} \qquad n_1 : f'\left(n_1; x_1\right) - w_1 + E\left[(1-\delta) f'\left(N_2\right) - w_2(1-\delta)\right] = 0,$$

$$\text{(A3)} \qquad w_h : -n_2 + \mu_x + \gamma_x = 0,$$

$$\text{(A4)} \qquad w_2 : -(1-\delta) n_1 + \lambda(1-\delta) v'\left(w_2(x)\right) - \mu_x = 0,$$

and

$$\text{(A5)} \qquad n_2 : f'\left(N_2\right) - w_h = 0.$$

Clearly $\lambda > 0$ (a).[26] Suppose that $\mu_x = 0$ and $\gamma_x > 0$. From Equations (A1) and (A4),

$$\text{(A6)} \qquad v'\left(w_1\right) = v'\left(w_2(x)\right) - \mu_x / \lambda(1-\delta),$$

and consequently

$$v'\left(w_1\right) = v'\left(w_2(x)\right),$$

so that $w_h(x) = w^s(x) \geq w_2(x) = w_1$.

(b) Suppose that $\mu_x > 0$ and $\gamma_x > 0$. From Equation (A6), $w_1 > w_2(x) = w_h(x) = w^s(x)$.

(c) Suppose that $\mu_x > 0$ and $\gamma_x = 0$. Then, using Equations (A1), (A3), (A5), and $w_h = w_2$ in Equation (A4):

$$-(1-\delta) n_1 + (1-\delta) n_1 v'\left(w_h(x)\right) / v'\left(w_1\right)$$
$$-\left(\left(f'\right)^{-1}\left(w_h\right) - (1-\delta) n_1\right) = 0,$$

which yields Equation (13) in the text. And from Equation (A6), $w_1 > w_2(x) = w_h(x) \geq w^s(x)$.

Next, consider equilibrium. From Equation (A5), period 2 total labor demand $N_2(x)$ is on the usual labor demand (marginal productivity) curve, so that if (I) $\widehat{w}_h(x) > w^{\text{spot}}(x)$, $N_2(x) < L$ and hence $w^s(x) = b < w^{\text{spot}}(x) < \widehat{w}_h(x)$ (the first inequality by assumption), while if (II) $\widehat{w}_h(x) = w^{\text{spot}}(x)$, $N_2(x) = L$ and hence $w^s(x) = \widehat{w}_h(x) = w^{\text{spot}}(x)$. $\widehat{w}_h(x) < w^{\text{spot}}(x)$ cannot arise in equilibrium, as discussed in the text. Note then that $\widehat{w}_h(x) = w^s(x)$ implies Case (II) while $\widehat{w}_h(x) > w^s(x)$ implies Case (I).

Consider now the three possibilities addressed in the proposition. First suppose that in equilibrium, in state x, $w^{\text{spot}}(x) \geq \widehat{w}_1$. Case (b) implies $\widehat{w}_1 > \widehat{w}_2(x) = \widehat{w}_h(x) = w^s(x)$, which from the above paragraph implies (II), so $\widehat{w}_h(x) = w^{\text{spot}}(x)$, but then we get $\widehat{w}_h(x) = w^{\text{spot}}(x) \geq \widehat{w}_1 > \widehat{w}_h(x)$, a contradiction. Likewise, Case (c) implies either $\widehat{w}_1 > \widehat{w}_2(x) = \widehat{w}_h(x) = w^s(x)$, so the same argument implies a contradiction, or $\widehat{w}_1 > \widehat{w}_2(x) = \widehat{w}_h(x) > w^s(x)$, so Case (I), and $w^s(x) = b < w^{\text{spot}}(x) < \widehat{w}_h(x)$, but then $\widehat{w}_1 > \widehat{w}_h(x) > w^{\text{spot}}(x)$, a contradiction. We conclude that only Case (a) is compatible.

Next suppose that $w^{\text{spot}}(x) < \widehat{w}_1$. This is incompatible with Case (a) as the latter implies that $\widehat{w}_h(x) = w^s(x) \geq \widehat{w}_2(x) = \widehat{w}_1$, and so Case (II), $w^s(x) = \widehat{w}_h(x) = w^{\text{spot}}(x)$; another contradiction.

If $\underline{w} < w^{\text{spot}}(x) < \widehat{w}_1$, then at $w_h = \underline{w}$, as, by $\underline{w} < w^{\text{spot}}(x)$, $\left(f'\right)^{-1}\left(\underline{w}\right) > L$, we have $g\left(\underline{w}; L, \widehat{w}_1, x\right) < 0$ in view of Equations (13) and (14). By Assumption 1, $w_h^d\left(L, \widehat{w}_1, x\right) < \underline{w}$, so if we are in Case (c), then we would have $\widehat{w}_h(x) = w_h^d\left(L, \widehat{w}_1, x\right) < w^{\text{spot}}(x)$, which is impossible; so we are in Case (b).

Finally, if $w^{\text{spot}}(x) \leq \underline{w}$: then at $w_h = \underline{w}$, as, by $w^{\text{spot}}(x) \leq \underline{w}$, $\left(f'\right)^{-1}\left(\underline{w}\right) \leq L$ and comparing Equations (13) and (14), we have $g\left(\underline{w}; L, \widehat{w}_1, x\right) \geq 0$. Suppose we are in Case (b), which recall implies Case (II), and so $\widehat{w}_h(x) = w^{\text{spot}}(x) \leq \underline{w}$. By Assumption 1, $g\left(\widehat{w}_h(x); L, \widehat{w}_1, x\right) \geq 0$ (as otherwise there would exist a solution to Equation (13) with $\partial g(w^d; n_1, w_1, x)/\partial w_h > 0$), that is

$$\text{(A7)} \quad (1-\delta) n_1 v'\left(\widehat{w}_h(x)\right) / v'\left(\widehat{w}_1\right) - \left(f'\right)^{-1}\left(\widehat{w}_h(x)\right) \geq 0.$$

Using Equations (A1), (A3), (A5), and $\widehat{w}_h = \widehat{w}_2$ in Equation (A4),

$$(1-\delta) n_1 v'\left(\widehat{w}_h(x)\right) / v'\left(\widehat{w}_1\right) - \left(f'\right)^{-1}\left(\widehat{w}_h(x)\right) + \gamma_x = 0,$$

which contradicts Equation (A7) as $\gamma_x > 0$ in Case (b). So we are in Case (c) and $\widehat{w}_h = \widehat{w}_2 = w_h^d\left(L, \widehat{w}_1, x\right)$. If $w^{\text{spot}}(x) = \underline{w}$, then $\widehat{w}_h = \underline{w}$ as $\left(f'\right)^{-1}\left(\underline{w}\right) = L$, so in view of Equations (13) and (14) we have $g\left(\underline{w}; L, \widehat{w}_1, x\right) = 0$. Thus $N_2 = L$. If $w^{spot}(x) < \underline{w}$, then at $w_h = \underline{w}$, we have $g\left(\underline{w}; L, \widehat{w}_1, x\right) > 0$,

25. If $n_2 \leq 0$ in states other than x, the Lagrangian is modified as in the proof of Proposition 4, and the argument below is unaffected since the FOCs arising in other states at $t = 2$ are not used.

26. Cases (a)–(c) correspond to (A)–(C) in the text, except for the knife-edge case where $w^{\text{spot}} = w$.

using the argument above Equation (A7). By Assumption 1 and continuity of g, a solution to $g\left(w_h; L, \widehat{w}_1, x\right) = 0$ must then have $w_h = w^d > \underline{w}$. $w^d > \underline{w}$ implies $\left(f'\right)^{-1}\left(\widehat{w}^d\right) < L$ so $N_2 < L$.

To prove the final claim of the proposition, consider the case $w^{\text{spot}}(x) < \underline{w}$ (other cases are straightforward). Note that $\left(f'\right)^{-1}\left(\widehat{w}_1\right) > (1-\delta)L$ implies that at $w_h = \widehat{w}_1$, $g\left(\widehat{w}_1; L, \widehat{w}_1, x\right) = (1-\delta)L - \left(f'\right)^{-1}\left(\widehat{w}_1\right) < 0$. Given that $g\left(\underline{w}; L, \widehat{w}_1, x\right) > 0$, then by continuity a solution for $g\left(w_h; L, \widehat{w}_1, x\right) = 0$ exists between $\underline{w}$ and $\widehat{w}_1$, and so at w^d, total labor demand $(f')^{-1}(w^d) > (1-\delta)L$ and hiring is positive. Conversely, suppose that $\left(f'\right)^{-1}\left(\widehat{w}_1\right) \leq (1-\delta)L$. Then $g\left(\widehat{w}_1; L, \widehat{w}_1, x\right) = (1-\delta)L - \left(f'\right)^{-1}\left(\widehat{w}_1\right) \geq \dot{0}$. Given Assumption 1, no solution exists between $\underline{w}$ and $\widehat{w}_1$, contradicting the solution for the case $w^{spot}(x) < \underline{w}$. ■

PROOF OF PROPOSITION 2

Totally differentiating Equation (13) with n_1 set equal to L and noting that w_1 will be constant, yields

$$dw^d/dx = \left(\partial^2 f/\partial N \partial x\right) / \left(1 - (1-\delta)\left(v'\left(\widehat{w}_1\right)\right)^{-1}\right.$$
$$\left. v''\left(w^d\right)\partial^2 f/\partial N^2\right).$$

By Assumption 1, $\partial g(w^d; n_1, w_1, x)/\partial w_h = (1-\delta)(v'(w_1))^{-1} v''(w^d) - (\partial^2 f/\partial N^2)^{-1} < 0$. Given that $\partial^2 f/\partial N \partial x > 0$ and $\partial^2 f/\partial N^2 < 0$, it follows straightforwardly that $dw^d/dx < 0$, which establishes the claim of the proposition. ■

PROOF OF PROPOSITION 3

We see that $E[(1-\delta)f'(N_2) - w_2(1-\delta)] \geq 0$ given that $f'(N_2) = w_h$ from Equation (A5) and $w_h \geq w_2$ from the no replacement constraint. Thus from Equation (A2) $\partial f(n_1, x_1)/\partial N \leq w_1$. This is strict whenever $w_h > w_2$ in at least one state. ■

PROOF OF PROPOSITION 4

For states with $n_2 \leq 0$ the relevant terms inside the expectation operator in the Lagrangian in the proof of Proposition 1 are replaced by

$$\left(f\left(\left(f'\right)^{-1}\left(w_2\right)\right) - w_2\left(\left(f'\right)^{-1}\left(w_2\right)\right)\right)$$
$$+ \lambda\left(\delta Z_2 + \left(1 - \delta - \left(f'\right)^{-1}\left(w_2\right)/n_1\right)v(b)\right.$$
$$\left. + \left(\left(f'\right)^{-1}\left(w_2\right)/n_1\right)v\left(w_2\right)\right)$$
$$+ \phi_x\left((1-\delta)n_1 - \left(f'\right)^{-1}\left(w_2\right)\right),$$

and $\mu_x = \gamma_x = 0$. Note that w_h no longer appears, we have added a constraint that $n_2 \leq 0$ with multiplier $\phi_x \geq 0$, and we have substituted $N_2 = (f')^{-1}(w_2)$ from Equation (7) (i.e., to reflect the at-will employment assumption implies that the firm cannot commit to n_2 but must choose optimally given w_2). In the expression for expected worker utility, $(1-\delta-(f')^{-1}(w_2)/n_1)$ is the probability of layoff while $(f')^{-1}(w_2)/n_1$ is the probability of being retained. We get the first-order conditions

$$\text{(A8)} \quad -N_2 + \lambda\left(N_2/n_1\right)v'\left(w_2\right)$$
$$+ \left(f'\left(N_2\right) - w_2 + \lambda\left(-n_1 v(b) + n_1 v\left(w_2\right)\right) - \phi_x\right)$$
$$\left(d\left(f'\right)^{-1}\left(w_2\right)/dw_2\right) = 0.$$

We are in equilibrium, so $n_1 = L$. From Equation (A1) $\lambda = L/v'(w_1)$, and noting that $f'(N_2) - w_2 = 0$, Equation (A8) can be rewritten as:

$$\text{(A9)} \quad \left(f'\right)^{-1}\left(w_2\right)\left(\frac{v'\left(w_2\right)}{v'\left(w_1\right)} - 1\right)$$
$$+ \left(\left(v'\left(w_1\right)\right)^{-1}\left(-v(b) + v\left(w_2\right)\right) - \phi_x\right)$$
$$\times\left(d\left(f'\right)^{-1}\left(w_2\right)/dw_2\right) = 0.$$

For Equation (A9) to have a solution with $\phi_x = 0$, we must have

$$\text{(A10)} \quad \left(f'\right)^{-1}\left(w_2\right)\left(v'\left(w_2\right) - v'\left(w_1\right)\right)$$
$$= -\left(v\left(w_2\right) - v(b)\right)\left(d\left(f'\right)^{-1}\left(w_2\right)/dw_2\right).$$

At x^*, there are neither layoffs nor new hires: $n_2 = 0$, that is, $N_2 = \delta L$, and $w_2 = w_1$. Consider $x < x^*$. By the last claim in Proposition 1, $n_2 \leq 0$. It cannot be that $w_2 \geq w_1$, as this implies that $(v'(w_2)/v'(w_1) - 1) \leq 0$ and since $d(f')^{-1}(w_2)/dw_2 < 0$ we have from Equation (A9),

$$\left(v'\left(w_1\right)\right)^{-1}\left(-v(b) + v\left(w_2\right)\right) - \phi_x \leq 0,$$

and given that $v(b) < v\left(\widehat{w}_1\right) \leq v\left(w_2\right)$, $\phi_x > 0$ (so $N_2 = \delta L$), but $N_2 = (f')^{-1}(w_2) < \delta L$ by $\partial^2 f/\partial N \partial x > 0$, a contradiction.

Next, consider a sequence such that $x^r \to x^*$ from below; if there are solutions to the first-order conditions with $\phi_x = 0$ at each x^r then there exist solutions $w_2^r \leq w_1$ to Equation (A10) for each x^r satisfying

$$\text{(A11)} \quad \left(f'\right)^{-1}\left(w_2\right) \leq (1-\delta)L;$$

in this case $w_2^r \to w_1$ (otherwise—along a subsequence—$\lim w_2^r < w_1$ and so $\lim\left(f'\right)^{-1}\left(w_2^r\right) > (1-\delta)L$, and thus Equation (A11) is eventually violated along the sequence). By continuity of all the functions in Equation (A10) in x and w_2, the L.H.S. converges to zero, the R.H.S. to a positive number, so Equation (A10) is eventually violated along the sequence, a contradiction. Hence no such sequence exists, and so there exists an interval of values for x, $\left(\underline{x}, x^*\right)$ say, such that for $x \in \left(\underline{x}, x^*\right)$, $\phi_x > 0$, and the claim of the proposition follows (for a closed interval, as $\phi_x > 0$ at x^*). ■

REFERENCES

Beaudry, P., and J. DiNardo. "The Effect of Implicit Contracts on the Movement of Wages over the Business-cycle—Evidence from Micro Data." *Journal of Political Economy*, 99, 1991, 665–88.

Bewley, T. F. *Why Wages Don't Fall During a Recession*. Cambridge, MA: Harvard University Press, 1999.

Crawford, C., W. M. Jin, and H. Simpson. "Productivity, Investment and Profits During the Great Recession: Evidence from UK Firms and Workers." *Fiscal Studies*, 34(2), 2013, 153–77.

Gertler, M., and A. Trigari. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Journal of Political Economy*, 117(1), 2009, 38–86.

Gottfries, N., and T. Sjostrom. "Insider Bargaining Power, Starting Wages, and Involuntary Unemployment." *Scandinavian Journal of Economics*, 102, 2000, 669–88.

Haefke, C., M. Sonntag, and T. vanRens. "Wage Rigidity and Job Creation." *Journal of Monetary Economics*, 60(8), 2013, 887–99.

Hagedorn, M., and I. Manovskii. "Job Selection and Wages Over the Business Cycle." *American Economic Review*, 103(2), 2013, 771–803.

Hall, R. E. "Intertemporal Substitution in Consumption." *Journal of Political Economy*, 96(2), 1988, 339–57.

Hall, R. E., and E. Lazear. "The Excess Sensitivity of Layoffs and Quits to Demand." *Journal of Labor Economics*, 2(2), 1984, 233–57.

Menzio, G., and E. R. Moen. "Worker Replacement." *Journal of Monetary Economics*, 57(6), 2010, 623–36.

Schmieder, J. F., and T. vonWachter. "Does Wage Persistence Matter for Employment Fluctuations? Evidence from Displaced Workers." *American Economic Journal: Applied Economics*, 2(3), 2010, 1–21.

Snell, A., and H. Stüber. "Downward Real Wage Rigidity and Equal Treatment Wage Contracts: Evidence from a Large German Panel Dataset." Nuremberg, Germany: mimeo, Institute for Employment Research (IAB), 2013.

Snell, A., and J. P. Thomas. "Labor Contracts, Equal Treatment and Wage-Unemployment Dynamics." *American Economic Journal: Macroeconomics*, 2(3), 2010, 98–127.